Non-Linearities in the Relationship between House Prices and Interest Rates: Implications for Monetary Policy

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Abstract
Understanding the impact of changes in interest rates on house prices is important for managing house price bubbles and ensuring housing affordability. This paper investigates the effect of interest rates on regional house price to income measures based on a non-linear smooth transition VAR model of inter-regional house price dynamics. To minimize the impact of housing mix changes on estimated effects, we apply the model to an Australian dataset of regional hedonic house price indices that account for both changes in housing mix and quality over time. The empirical analysis provides evidence that house price to income ratios depend non-linearly on interest rates, and moreover that there is an interest rate ‘transition point’ below which a house price bubble is probable. We investigate the implications for monetary policy of stable and unstable house price regimes and propose a housing lending rate lower bound that achieves long-run house price stability in the presence of regime uncertainty. To check the generality of the result, we also apply the model to aggregate Australian and US data.

**JEL classification:** C30, E43, E52, G21, R10, R31

**Keywords:** House prices, interest rates, monetary policy, nonlinear VAR, housing affordability, bubbles
1 Introduction

The link between monetary conditions, mortgage borrowing, and house price appreciation has attracted extensive attention in recent years (Taylor 2007; Bernanke 2010; Leamer, 2015). This is largely unsurprising given widespread talk of pre- and post-2007/08 housing booms and busts, and the current status quo of record low interest rates. There is considerable contention, however, regarding the significance of the relationship between house prices and interest rates. Numerous researchers find that the impact of monetary policy shocks on house prices is relatively small (Del Negro and Otrok, 2007; Goodhart and Hofmann, 2008; Glaeser, Gottlieb and Gyourko, 2010). Conversely, others contend that there is a clear relationship between house price booms and busts, and interest rates (Taylor, 2007 and 2009; Jordà, Schularick and Taylor, 2015). The latter authors, in particular, argue that loose monetary conditions are causal for house price booms.

Although reconciling the competing findings regarding the importance of interest rates for house prices is not a straightforward exercise, we identify three possible reasons for the different findings that we attempt to address in this paper. The first pertains to the potential loss of information associated with the use of an aggregate house price variable. There is little doubt that regional house price differences exist, raising the risk that aggregating these differences can mitigate the relationship between interest rates and house prices. If housing investors consider regional differences in their investment decision then interest rate conditions may affect certain regions more than others. These impacts may offset each other when restricting examination to a single national housing market thereby yielding incorrect inferences about the impact of interest rates on house prices. Labour market mobility is also likely to be relevant, with regional differences in labour demand potentially resulting in regional house price dynamics that partially offset each other in the presence of interstate migration. More broadly, Beraja et al. (2015) highlight the importance of regional heterogeneity in collateral values in determining

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1 This typically involves the incorporation of a national house price indicator into a structural VAR in order to identify the impact of monetary shocks on the housing market.
the impact of monetary policy, finding that the impact of monetary policy declines when collateral values become depressed. Given these factors, there appears to be a non-negligible risk that discarding regional information may limit the model’s capacity to identify the relationship between interest rates and house prices.

Second, the assumption of a linear relationship between interest rates and house prices may be an over-simplification. Himmelberg, Mayer and Sinai (2005), for example, suggest that house prices are theoretically more sensitive to fundamentals when rates are already low. Kuttner (2012) also describes a theoretical ‘over-reaction’ point where house prices tend to over-react to interest rates below this point, arguing that such a point is necessary in order to validate the hypothesis that expansionary monetary policy leads to housing market bubbles. In the absence of a model allowing for non-linearities between house prices and interest rates, the generality of findings of a weak relationship between house prices and interest rates is in doubt.

Third, the data used to measure house prices may not be optimal for investigating the dynamics of house prices or their relationship to interest rates. In almost all cases, house price indices are constructed by reference to factors that do not consider the quality change in housing that is observed over time. Changes in factors such as housing structure and land size are typically not accounted for in deriving house price changes, thereby producing units of comparison that can be substantially different over time. In so far as quality adjustments take place during periods where the cost of borrowing is low and housing investment is high (for example, the sub-division of land into smaller higher density dwellings), the failure to adjust prices for changes in these factors may obfuscate the capacity to accurately measure the impact of interest rates on housing prices.

This paper attempts to overcome these weaknesses by adopting a framework which allows for potential non-linearities in the relationship between the house price to income

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2Median or mix-adjusted measures are often used. Whilst the latter account for changes in compositional mix they do not account for changes in quality. Some articles use repeat-sales measures to account for quality adjustments, although these measures are inefficient (omitting sales data for non-repeat sales), in addition to requiring the strong assumption that older houses are not renovated or updated between selling periods.
ratio and interest rates, and which also allows for regional spillovers. The adoption of the house price to income ratio as the dependent variable follows Jordà, Schularick and Taylor (2015) and allows us to consider the impact of interest rates on relative house prices.

The framework is a non-linear smooth transition vector-autoregression model of inter-regional house price dynamics. Non-linearities in the relationship between "affordability" ratios and interest rates are captured using a logistic transition function with endogenously estimated smoothness and threshold parameters. These parameters determine whether there is an interest rate ‘over-reaction’ point where the response of relative house prices to interest rates exhibits a significant change, and whether the point is ‘hard’ or ‘soft’ (in other words, whether the transition from one set of dynamics to another upon breaching the over-reaction point is abrupt or smooth).³

The model is estimated using Australian regional house price to income ratios that are constructed using hedonic indices of house prices that account for inter-temporal changes in both compositional structure and housing quality. The choice of case study is partly pragmatic (regional hedonic indices are available for each of the five major states) and partly because the Australian economy and housing markets exhibit clear regional differences. To examine the impact of aggregation (and the potential generality of the findings), we also confirm that the results are not contingent on the adopted form of disaggregation, by estimating the model using aggregate data.⁴

Our results show that regional spillovers change substantially around the threshold, reflecting a transition from stable to unstable housing conditions. The threshold is shown to be a ‘soft’ threshold such that the transition is not abrupt, but is instead a function of both the extent to which rates are below the threshold and the period of time that rates stay below the threshold. It is also shown that the unstable dynamics accommodate house price boom and bust conditions that justify interpretation of the

³The framework thus provides a basis for testing the predictions stemming from theoretical models that establish a link between asset price bubbles and fundamental drivers (such as interest rates), by essentially identifying the interest rate point (if it exists) at which bubble formation can occur (Froot and Obstfeld, 1991).

⁴To support the generality of the methodology, the model is also estimated on US data.
threshold as an over-reaction point. The results are particularly important for monetary policy setting since easing beyond a certain point, and for a long enough period, creates real risks of housing market instability. We establish long-run lending rate conditions for ensuring house price stability in the presence of stochastic stable and unstable housing regimes.

The paper is structured as follows. Section 2 presents the econometric model used to examine non-linearities in house prices that depend on interest rates. Section 3 contains the empirical analysis. The first sub-section describes the data used in the paper, and the process for creating the regional house price measures. The remaining sub-sections present the results, examine their robustness and consider the implications of estimating the model using aggregate housing data. Monetary policy ramifications are considered in Section 4, with particular focus on the conduct of policy in the face of regime uncertainty. Section 5 concludes.

2 An econometric model of the relationship between house prices and interest rates

To examine the relationship between house prices and interest rates we estimate a smooth transition VAR (Granger and Terasvirta, 1993) that allows for asymmetries in house price dynamics that are induced by interest rate movements. The formal specification is:

\[ h_t = G(r_t; \gamma, c) \times [\Pi_1 (L) h_t + X_t \beta_1] + (1 - G(r_t; \gamma, c)) \times [\Pi_2 (L) h_t + X_t \beta_2] + e_t \]  

(1)

\[ G(r_t; \gamma, c) = 1 - (1 + \exp \{-\gamma (r_t - c)\})^{-1}, \quad \gamma > 0, c \geq 0 \]  

(2)

\[ e_t \sim MVN(0, \Sigma) \]  

(3)

where \( h_t = [h_{1t}, ..., h_{nt}]' \) is a vector of house price to income ratios for each of the \( n \) regions, \( X_t \) is a matrix of exogenous demographic and macroeconomic regressors and \( r_t \)
is the lending rate for housing loans which is common to all markets. The dynamics in the model are governed by the $\Pi_1$ and $\Pi_2$ matrices (which determine the inter-temporal spillovers between each of the markets), and the $\beta_1$ and $\beta_2$ vectors (which specify each market’s relationship with demographic and macroeconomic conditions in $X_t$).

The framework assumes that there are two house price regimes. The weight associated with the alternatives $(\Pi_1, \beta_1)$ and $(\Pi_2, \beta_2)$ is determined by $G(r_t; \gamma, c)$ which is a logistic transition function of interest rates $r_t$. Our specification of $G(r_t; \gamma, c)$ captures the effects of interest rates on housing dynamics by reference to the threshold $c$ and the scale (or smoothness) parameter $\gamma$. Rather than calibrate the threshold and scale parameters, the parameters are treated as estimable parameters. Consequently, we allow the data to determine the interest rate point (if it exists) at which there is a change in house price dynamics.

The benefit of the adoption of a ‘smooth transition’ probability function $G$ is that we avoid imposing the binary situation where any value of $r_t$ less than or equal to $c$ forces the regime dynamics $(\Pi_1, \beta_1)$ to prevail with probability one, and any value of $r_t$ exceeding $c$ implies that the probability associated with $(\Pi_2, \beta_2)$ is unity. This restrictive scenario is, nevertheless, accommodated if $\gamma$ is estimated to be a large number. Instead, as $r_t$ falls below $c$, the probability function $G$ rises above 0.5 (such that the first regime is more probable than the second regime), with $\gamma$ determining the ‘speed’ at which $G$ approaches unity.

Thus, the transition function $G(r_t; \gamma, c)$ allows for non-linearities in the relationship between nominal lending rates and housing prices (Brunnermeier and Julliard, 2008).\(^5\) This is because the effect of inter-temporal dynamics, and other exogenous (demographic and economic) variables, on house prices depends indirectly on interest rates through the potentially time-varying $G(r_t; \gamma, c)$.

We obtain maximum likelihood estimates of $\theta = \{\gamma, c, \Pi_1(L), \Pi_2(L), \beta_1, \beta_2, \Sigma\}$ by \(^5\)Muellbauer and Murphy (1997) also find that the real interest rate has no power in explaining the variation in real residential house prices.
solving
\[ \max_\theta \log L = -\frac{TN}{2} \ln 2\pi - \frac{T}{2} \ln |\Sigma| - \frac{1}{2} \sum_{t=p+1}^T \hat{\epsilon}_t' \Sigma^{-1} \hat{\epsilon}_t \] (4)

where \( p \) is the number of lags \( L \) in the model’s VAR component (see, further, Terasvirta, 1994).

Given the large number of parameters in the model, it is important to obtain sensible starting estimates. We do this by first solving the following non-linear least squares problem for \( \gamma \) and \( c \)
\[ \min_{\gamma,c} \sum_{i=1}^N \sum_{t=p+1}^T (h_{it} - z_{it}' \psi)^2 \] (5)

where \( z_{it} = [Gh_{t-1}, ..., Gh_{t-p}, Gx_{t}, (1 - G) h_{t-1}, ..., (1 - G) h_{t-p}, (1 - G) x_{t}]' \) and \( \psi = [\Pi_{i1}, ..., \Pi_{1p}, \beta_1, \Pi_{21}, ..., \Pi_{2p}, \beta_2]' \). Conditional on \( \gamma \) and \( c \), it is straightforward to obtain \( \psi \) as the solution to an OLS problem by the appropriate vectorization of \( h_{it} \) and \( z_{it} \). It can be shown that (5) provides a consistent, but not efficient, estimate of \( \theta \) (Potscher and Prucha, 1997; Leybourne et al., 1998). To obtain an efficient estimate of \( \theta \), we use the \( \gamma, c \) and \( \psi \) estimates from (5) as starting values for the maximum likelihood problem. Furthermore, we concentrate \( \Sigma \) out of the likelihood function by replacing it with the consistent estimator \((T - p)^{-1} \sum_{t=p+1}^T \hat{\epsilon}_t \hat{\epsilon}_t'\).

3 Empirical Analysis

3.1 Data

The house price measure \( h_{it} \) is constructed on a quarterly basis for Australia’s five major capital cities (\( i = 1 \) to \( n, n = 5 \), corresponding to Sydney, Melbourne, Brisbane, Adelaide and Perth respectively) over the period December 1995 to June 2015. This produces \( T = 79 \) observations for each of the five capital cities. The measure is created as the ratio of dwelling prices to household disposable income
\[ h_{it} = \frac{p_{it}}{y_{it}} \]
where $p_{it}$ is the dwelling price and $y_{it}$ is household disposable income.

To obtain $p_{it}$ we adjust median dwelling prices as at 31 July 2015 based on the changes in the CoreLogic RP Data Daily Home Value Index. This hedonic index accounts for time-varying changes in the attributes of properties thereby better reflecting changes in the value of dwellings. In the absence of such an adjustment, a shift in housing structure (for example, smaller houses) will obfuscate the true value of dwellings over time.\(^6\)

The denominator $y_{it}$ is based on annual household disposable income per capita for each capital city. The per capita value is transformed to a household level value using the average number of household members for each city and converted to a quarterly basis by allocating the annual change in household disposable income by reference to the proportion of annual total wage compensation attributed to the relevant quarter.\(^7\)

The latter variable is chosen on the basis that wages have historically constituted about 80 per cent of total income for (non-retired) households across the five capital cities that we consider.\(^8\) Given the smoothness of total wage compensation, this procedure produces a quarterly allocation of the annual change in household disposable income that is mostly equal for each quarter in the relevant fiscal year. We also constructed $y_{it}$ by assuming that household disposable income for the relevant fiscal year grows at a constant growth rate in each quarter with little change to $h_{it}$ or the results.

The indicators are presented in Figure (1). With the exception of Sydney, the indicators exhibit relatively normal growth until 2000 when affordability levels decline substantially across all capital cities. A general correction is observed in 2003 for all areas except Perth which appears to be related to a surge in prices associated with the commodities ‘boom’. During this period, Perth overtakes Sydney as the least affordable

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\(^7\)Average household members for each capital city are obtained from the Australian Bureau of Statistics’ (ABS) Surveys of Income and Housing over the period 1994-95 to 2011-12. The average household size across the five capital cities exhibits a small decline over time, but has remained fairly flat since 2002-03 at about 2.5 persons per household.

\(^8\)For example, according to the 2011-12 ABS Survey of Income and Housing, wages and salaries constituted 78 per cent of total income (for non-retired households) for Sydney, 75 per cent for Adelaide and 79 per cent for Melbourne, Brisbane and Perth.
of the major capital cities in Australia. In the last ten years, affordability levels appear to be related to business cycle conditions, particularly for Melbourne and Sydney. The latter two cities have exhibited a significant decline in affordability levels in the last few years, in contrast to the remaining cities.

![Figure 1: House price to income levels $h_{it}$ for Australia’s five major capital cities over the period December 1995 to June 2015.](image)

The lending rate $r_t$ for housing loans is chosen as an alternative to the formal cash rate since it accounts for periods when banks increase or decrease their lending rates irrespective of any changes to the cash rate.\(^9\) In particular, we find that although lending rates typically follow the path of the official cash rate (which targets the Consumer Price Index), there are periods when bank lending rates do not co-move with changes in the cash rate. This occurs particularly during the periods that are of greatest interest (being crises periods). The use of the cash rate to proxy for lending rates may therefore be inaccurate.

We also adopt the nominal lending rate in order to account for possible money illusion effects. This point is important, given the specification of a threshold. Kuttner (2012) argues that, notwithstanding any relationship between interest rates and house prices, it does not follow that low interest rates cause bubbles (see, also, Glaeser et al.,\(^9\)

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\(^9\)We use the quarterly standard variable bank lending rate for housing loans computed by the Reserve Bank of Australia.
In particular, he argues that the latter conclusion requires showing that house prices tend to over-react to rate reductions. In this respect, a possible motivation for a house price over-reaction to low interest rates is that the impact of money illusion on housing affordability is non-linear and greatest when rates are below the nominal threshold $c$. This explanation is supported by Brunnermeier and Julliard (2008) who find money illusion effects in the US, UK and Australian house price to rent ratios, and provide evidence that the ratios are affected by nominal rather than real interest rates.

The exogenous regressors $X_t = [x_{1t}, \ldots, x_{nt}]'$ are specified using

$$x_{it} = [1, g_{it}, u_{it}, s_{it}]', \quad i = 1, \ldots, 5 \quad (6)$$

where $g_{it}$ is the log of the quarterly growth rate of the resident population in the relevant state, and $u_{it}$ represents the change in the quarterly state-specific unemployment rate (see, further, Poterba (1991) regarding the importance of population factors).\(^{10}\) We augment the demographic and macroeconomic indicators with household sentiment data $s_{it}$ regarding whether it is a ‘good’ time to buy a dwelling in the relevant state.\(^{11}\) This data is intended to provide information about economic expectations and user costs as they pertain to the housing market that is not captured by historical population or employment conditions. As a whole, the exogenous regressors aim to parsimoniously reflect the impact of demographic and economic conditions and expectations on house prices. We also consider the impact of additional regressors on the results, and this is discussed further in Section 3.4.

\(^{10}\)This data is obtained from the Australian Bureau of Statistics. The quarterly unemployment rate is obtained as the average of the monthly seasonally adjusted unemployment rates for the relevant quarter.

\(^{11}\)The data is from the ‘Time to buy a Dwelling’ index associated with the Melbourne Institute - Westpac Consumer Sentiment Survey and is converted to a quarterly basis by averaging over the relevant months. The index is constructed for each state and is based on household responses regarding whether it is a good or bad time to buy a dwelling.
3.2 Testing for linearity and lag length

Prior to model estimation, we test for the linearity of the specification (1) using the method proposed in Terasvirta and Yang (2014). The test of linearity is essentially a test of the equivalence of \((\Pi_1, \beta_1)\) and \((\Pi_2, \beta_2)\), but can alternatively be considered a test of \(\gamma = 0\). The test rejects the null hypothesis of linearity at the 0.01 level rendering it clear that asymmetries are present in the behaviour of the housing price to income ratio at different interest rates. The strength of this rejection indicates that the inter-temporal dynamics of the housing measures, and/or their relationship with demographic and economic conditions, changes markedly as interest rates move above or below some unknown threshold.

To estimate the model, we adopt a lag length of \(p = 2\) with standard diagnostic tests indicating that little autocorrelation is observed in the residuals after accounting for the first two lags. The choice of \(p = 2\) is also consistent with the preferred lag length indicated by the Akaike Information Criterion (AIC).

We also test for the presence of any further non-linearity associated with interest rates (for example, there being more than one transition function \(G\)) by applying the Lagrange Multiplier test for additional additive non-linearity proposed in Terasvirta and Yang (2014). The test depends on both the functional form and parameterisation of the transition function and the associated test statistic provides no evidence of any additional additive non-linearity. Our choice of a single logistic transition function \(G(r_t; \gamma, c)\) with unrestricted scale and threshold parameters therefore appears to be consistent with the non-linearities in house prices that are associated with interest rates.

\begin{itemize}
  \item The Lagrange multiplier test statistic for the null hypothesis of \((\Pi_1, \beta_1) = (\Pi_2, \beta_2)\) is 112.17 (\(p<0.01\)).
  \item Given the large number of parameters, we restrict the VAR matrices for lags greater than unity to be diagonal for both testing and estimation purposes. The Ljung-Box Q statistic fails to reject the null hypothesis of no serial dependence in the residuals. Furthermore, Engle’s ARCH test does not indicate any significant conditional heteroscedasticity in the squared residuals.
  \item The test statistic is 82.11 (\(p=0.1167\)).
\end{itemize}
3.3 Housing asymmetries and interest rates

The model proposes understanding the dynamics of the relationship between house prices and income by reference to exogenous economic and demographic variables (as measured by $\beta_1, \beta_2$) and regional spillovers (as reflected in the autoregressive matrices $\Pi_1, \Pi_2$), with the observed outcome determined by the time-varying weight $G$ (which is a function of the lending rate). We discuss each of these in turn.

The impact of economic and demographic conditions

The $\beta_1$ and $\beta_2$ coefficients provide information about the degree of asymmetry in the relationship between the house price measures and demographic or economic conditions as interest rates move across the threshold $c$. The estimated parameters are

$$x'_{it}\hat{\beta}_1 = -1.0159 - 0.1458g_{it} - 0.0571u_{it} - 0.1449s_{it},$$

$$x'_{it}\hat{\beta}_2 = 0.1093 + 0.0780g_{it} + 0.0307u_{it} - 0.0598s_{it},$$

where the values in parentheses are associated test statistics and, as noted above, $g_{it}, u_{it}$ and $s_{it}$ are respectively: log quarterly growth in estimated resident population for the relevant state, change in the quarterly unemployment rate and quarterly percentage change in dwelling purchase sentiment.

In the second regime, which dominates when $G(r_t; \gamma, c)$ is close to zero and interest rates exceed the threshold $c$, the relationship between house prices and the exogenous regressors is statistically significant, with coefficients that are plausible and have the desired sign. Greater population growth leads to greater housing demand, resulting in a statistically significant increase in the house price to income ratio. Similarly, rising unemployment dampens household income and also produces a statistically significant increase in the house price measure.\footnote{It is also noted that a likelihood ratio test of a common intercept cannot be rejected at the 0.05 level such that we present the model with a common intercept rather than an individual intercept for each capital city.}
In the first regime, however, a statistically significant relationship with the drivers is no longer discernible. In this regime, the nature of the relationship between the house price to income ratio and the economic or demographic variables is difficult to rationalise. At the least, it is clear that the linearity of the relationship between the house price to income ratio and the exogenous regressors breaks down.

**Spillover Effects**

The autoregressive dynamics $\Pi_1, \Pi_2$ represent the intertemporal relationships and spillovers between the housing measures across the five regions. It is evident that, just as the impact of economic and demographic conditions varies, the spillover dynamics change noticeably as $G$ approaches unity. The parameter estimates are shown in Table 1.

<table>
<thead>
<tr>
<th>$\Pi_{1,1}$</th>
<th>$\Pi_{1,2}$</th>
<th>$\Pi_{1,3}$</th>
<th>$\Pi_{1,4}$</th>
<th>$\Pi_{1,5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9496</td>
<td>0.0385</td>
<td>0.1063</td>
<td>0.0125</td>
<td>-0.0351</td>
</tr>
<tr>
<td>0.0271</td>
<td>0.9866</td>
<td>0.0598</td>
<td>0.0432</td>
<td>-0.0997</td>
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<tr>
<td>0.1431</td>
<td>0.0417</td>
<td>0.8290</td>
<td>0.0182</td>
<td>-0.0811</td>
</tr>
<tr>
<td>-0.1747</td>
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<td>0.3401</td>
</tr>
<tr>
<td>0.0160</td>
<td>0.0452</td>
<td>0.0629</td>
<td>0.0399</td>
<td>0.8578</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\Pi_{2,1}$</th>
<th>$\Pi_{2,2}$</th>
<th>$\Pi_{2,3}$</th>
<th>$\Pi_{2,4}$</th>
<th>$\Pi_{2,5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6543</td>
<td>0.2464</td>
<td>0.3294</td>
<td>0.2983</td>
<td>0.3961</td>
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<tr>
<td>-0.3023</td>
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<td>0.0977</td>
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<tr>
<td>0.2974</td>
<td>0.2281</td>
<td>-0.2228</td>
<td>-0.0026</td>
<td>1.3559</td>
</tr>
</tbody>
</table>

House price dynamics appear to be stable when $\Pi_2$ prevails, with the roots of $\det (I_n - \Pi_2(1)z - \Pi_2(2)z^2) = 0$ all lying outside of the complex unit circle. The dynamics change significantly when $\Pi_1$ prevails with the VAR system becoming unstable;
the roots of \( \det(I_n - \Pi_1(1)z - \Pi_1(2)z^2) = 0 \) now do not all lie outside the unit circle and are, in fact, explosive (i.e. some of the roots are less than unity).

The resulting dynamics suggest that, in ‘normal’ conditions, the ratio of house prices to household disposable income is stationary with a predictable long-run mean. Below a given interest rate, however, the relationship between house prices and household income breaks down. At this point, the cost of borrowing is sufficiently low that household income levels are no longer an appropriate anchor for the dynamics of house prices. Technically, at below the threshold \( c \), it is also not possible to identify a long-run mean in the ratio of house prices to household income nor is it possible to identify the inter-temporal impact of a housing shock over a long time horizon.

Given that the roots of \( \det(I_n - \Pi_1(1)z - \Pi_1(2)z^2) = 0 \) are explosive, a large \( G \) results in dynamics that encapsulate both house price bubbles and boom and bust cycles in house prices. In this respect, the relationship between the roots of an autoregressive process and the presence of bubbles is detailed in Evans (1991) and Phillips, Wu and Yu (2011), and is consistent with the treatment of \( c \) as a house price over-reaction point.

Transition between regimes

Figure (2) shows the estimated transition function \( G(r_t; \hat{\gamma}, \hat{c}) \) which includes four spikes that are associated with the Asian Financial Crisis in the late 1990s, the 2001 US recession, the Global Financial Crisis (GFC) and the ‘zero-lower-bound’ period resulting from the cumulative impact of the GFC and the European debt crisis. The latter three periods produce a greater than 50 per cent probability associated with the unstable (lower interest rate) regime.

The transition function’s scale (or smoothness) parameter \( \gamma = 1.922 \ (p = 0.039) \) indicates that transitions between the two states are fairly smooth (in contrast, large values of \( \gamma \) imply abrupt shifts between the regimes). The smoothness in the transition function suggests that there is no ‘hard’ threshold below and above which the unstable or stable regimes (as represented by the \((\Pi_1,\beta_1)\) and \((\Pi_2,\beta_2)\) parameters respectively) unambiguously prevail. Instead there is a ‘soft’ threshold with house price dynamics
approaching the two extremes \((\Pi_1, \beta_1)\) and \((\Pi_2, \beta_2)\) as interest rates move away from
the threshold.

![Figure 2: Plot of transition function \( G(r_t; \gamma, c) \) representing the probability associated with the unstable house price regime. The four major episodes are: (1) declining rates following the Asian Financial Crisis; (2) the 2001 US recession; (3) the Global Financial Crisis; and (4) the ‘zero lower bound’ period. The dashed line represents the end of the estimation sample period.](image)

To place the estimated value of \(\gamma\) in perspective, we note that a range of papers calibrate \(\gamma\) to 1.5 when using moving averages of output growth as the input variable into the transition function (in order to capture expansionary and contractionary periods) (Auerbach and Gorodnichenko, 2012; Berger and Vavra, 2014).

The results indicate that house prices exhibit an asymmetric relationship with interest rates. As the transition function approaches unity, the system moves away from stable house prices towards an unstable system. However, the presence of a ‘soft’ threshold also negates the hypothesis of an immediate or sudden price response to interest rates. Instead, both \(\gamma\) and the spillover dynamics \(\Pi_3\) imply that any house price instability due to interest rates is the result of interest rates set at below the threshold \(c\) for a sufficiently long period of time.

Figure (3) shows the threshold, which is significantly different to zero at \(c = 6.1256\) \((p < 0.01)\), in the context of the lending rate movements observed over the period of interest. According to the empirical distribution of interest rates over the relevant period, interest rates are only below \(c\) approximately 11.25 per cent of the time. It therefore appears sensible to interpret \(c\) as the threshold below which ‘normal’ or typical
housing spillovers and relationships with demographic and economic variables no longer hold.

*The threshold as an over-reaction point*

This paper shows that the house price over-reaction to interest rates described by Kuttner (2012) can take place, but that the over-reaction appears to be conditional on both the level of interest rates and the period of time that interest rates remain at (or below) that level (given that \(\gamma\) provides little evidence of an immediate or sudden onset of instability). We have shown empirically that, when lending rates are below the threshold \(c\), there can be a transition from the stable dynamics implied by \(\Pi_2\) to the unstable dynamics \(\Pi_1\). It is also shown that the relationship between relative house prices, population growth and economic conditions becomes unclear when lending rates fall below a certain point.

Himmelberg, Mayer and Sinai (2005) also discuss non-linearities in the discounting of rents that can lead to over-reactions to changes in interest rates. In particular, the authors provide theoretical evidence that the sensitivity of house prices to interest rates changes when rates are low. The authors also suggest that the sensitivity of house prices to fundamentals increases with low interest rates. We fail to observe this, however, with the demographic and economic drivers becoming insignificant when rates fall sufficiently low. The inability to find statistically significant fundamental drivers when the first
regime prevails is, however, in general agreement with Shiller (2007; 2008) who finds that boom-bust cycles have frequently had little to do with economic fundamentals, instead being driven by investor sentiment.

In general terms, whether the over-reaction point is considered from the perspective of the explosiveness of a process (Phillips, Wu and Yu, 2011), or the absence of a relationship with fundamental drivers (Garber, 2004), both conditions are satisfied when rates fall below the threshold. This suggests that, at least for the data considered, the two conditions are not mutually exclusive.

In 2015, the Australian lending rate was below the threshold (see Figure (2)), implying a probability in favour of the over-reaction regime of around 75 per cent. In such circumstances, if the aim of regulators is to prevent a housing bubble, monetary and macroprudential policy should aim at tightening bank lending rates. Before we discuss the monetary policy implications of the analysis, however, we present various checks for the robustness of the model.

3.4 Robustness

Supply conditions, interest rates and stockmarket returns as regressors

We first test the sensitivity of the parameter estimates by considering whether the addition of housing supply information markedly impacts on the results. Glaeser, Gyourko and Saiz (2008) provide evidence that the elasticity of housing supply impacts on the frequency and magnitude of housing bubbles which suggests that supply conditions are important in both normal or low interest rate conditions, and that rates should have little impact on prices in elastically supplied markets.

In particular, we construct a supply side variable using residential dwelling commencement data for each state. Using national data on average dwelling construction periods, we construct the residential supply variable as a weighted average of residential dwelling commencements lagged two and three quarters. The weights account for the typical time required to transition from the dwelling commencement stage to dwelling
completion and therefore broadly capture actual new supply at a given time period.\textsuperscript{16} We use the annual change in the resulting weighted average thereby capturing the additional new supply relative to the same period last year. Note also that we have considered different weighted averages and alternative lag structures but we find that the resulting variables exhibit a smaller level of statistical significance than the preferred supply variable.

Our findings do not change significantly following the inclusion of this variable. Although the coefficient is statistically significant \((p < 0.05)\) in the ‘normal’ regime (regime 2), it ceases to be significant in the lower interest rate regime (although it continues to have the ‘correct’ negative sign). The remaining parameters exhibit little change. In particular, the smoothness and threshold parameters in the transition function increase slightly \((\gamma \text{ rises from 1.92 to 2.32 whilst } c \text{ goes from 6.13 to 6.34})\) but the transition function is highly similar. Likewise, the inclusion of the supply variable has little impact on the parameters in \(\Pi_1, \Pi_2\) and on the explosive nature of the parameters in the first regime.

We also consider the impact of linearly incorporated lagged lending rates into the regression equation, such that lending rates enter Eq. (1) both linearly and non-linearly. We find that, although the coefficients are significantly negative (with a percentage point increase in lending rates having a statistically significant but relatively small marginal impact on \(h_t\) that averages around -0.03 across the regions), the transition function parameters do not change substantially.\textsuperscript{17} This result is consistent with previous research that highlights the relatively small linear impact of interest rates (nominal or real) on house prices (Muellbauer and Murphy, 1997; Brunnermeir and Julliard, 2007). In particular, \(c\) continues to remain at approximately 6.13 while \(\gamma\) falls from 1.92 to 1.65. Furthermore, the VAR parameters remain unstable in the first regime, with the regres-

\textsuperscript{16}ABS data on residential dwelling construction periods indicates that the majority of residential dwellings are built within two and three quarters (with the average being closer to two). Construction periods are similar across the major states, although Queensland times are somewhat smaller. In any case, adjusting the weights for Queensland produces little change in the results.

\textsuperscript{17}The transition function remains similar if the lagged cash rate is included as a regressor instead of the lending rate. However, although the coefficients on the cash rate continue to be negative they are no longer statistically significant.
sors continuing to exhibit a significant shift when moving from the ‘normal’ interest rate regime to the first regime.

Based on the results that we observe regarding the linear relationship between interest rates and house prices, it is relatively unsurprising that a range of papers that assume a linear relationship have found the impact of interest rates on $h_{it}$ to be statistically significant but small. The results in this paper indicate that the non-linear impact of interest rates is of key importance in determining the overall impact of interest rates on house prices.

Finally, we consider the addition of stockmarket returns into the vector of regressors $x_{it}$ but find no statistically significant relationship between house prices and stockmarket performance in either regime.

*Uniqueness of the threshold*

To ensure robustness of the transition function parameters, we consider the extent to which the threshold and the smoothness parameters are uniquely identified. Auerbach and Gorodnichenko (2012) point out that the advantage of using a smooth-transition VAR is that the regime dependent parameter estimates use all the data. Consequently, the problem of having relatively few observations above or below the threshold is accounted for. However, due to the non-linearities in the model it is also possible there may be a number of local optima that the optimisation function may converge to (see, also, Terasvirta and Yang, 2013).

To examine this possibility, we adopt the following approach to identify the basins of attraction for the $\gamma, c$ parameters in the transition function. We randomly draw starting points for $\gamma^* \in (0, 20), c^* \in (0, 11)$. Conditional on the draws, we obtain least squares estimates of the model’s remaining parameters by solving (5) subject to $\gamma = \gamma^*, c = c^*$. The latter parameters coupled with the draws $\gamma^*, c^*$ are used as starting values for the maximum likelihood problem (4). The procedure is repeated 500 times to ascertain the points of convergence. The results, presented in Fig. (4), indicate that for nearly every starting point $\gamma^*, c^*$ (represented by the small solid dots in Fig. (4)) the model converges
to the result $\gamma = 1.922, c = 6.1256$ (represented by the star in Fig. (4), which is also the point at which the likelihood function is maximised) indicating that the smoothness and threshold parameters in the transition function are well identified.\textsuperscript{18}

*Transformations*

To examine whether the results are robust to alternative transformations, we re-estimate the model by replacing $h_t$ with the high-frequency component stemming from the Hodrick-Prescott (HP) filter and by using the residuals from a second-order detrending of $h_t$.\textsuperscript{19} Both transformations attempt to remove underlying trends in the dataset. We find that the transformations have little impact on the transition function or the stability of the VAR component in the two regimes. The transition function’s parameters are $\gamma = 5.05, c = 6.27$ for the HP-filtered dataset and $\gamma = 3.67, c = 6.28$ for the

\textsuperscript{18}In a small number of cases, the parameters converge to a flat portion of the likelihood function characterised by a threshold $c$ that is implausibly high or low (eg. a threshold that is lower than the smallest lending rate in the dataset).

\textsuperscript{19}The Hodrick-Prescott filter was applied with a smoothing parameter of 1600 which is typical for a quarterly dataset.
second-order detrended dataset. Given that the smoothness parameter $\gamma \in [0, \infty)$, there is compelling evidence the threshold is a ‘soft’ threshold. It is also fairly clear that $c$ is not particularly sensitive to reasonable transformations of $h_t$. The interpretation of $c$ as a point of instability is also maintained, with the transformations not producing a change in the instability/stability dichotomy observed for $\Pi_1$ and $\Pi_2$ respectively. In all cases, when interest rates fall substantially below $c$ the price dynamics $\Pi_1$ are unstable and contain explosive roots.

*Impulse Responses*

The model estimates indicate that the observed outcome for the house price to income ratio is a weighted average of two possible regimes - one stable and one unstable. If housing is fully weighted to regime $\Pi_2$, impulses will be stable; conversely, if housing is fully weighted to regime $\Pi_1$, impulses will be unstable. In reality, impulses will be a weighted average of the two.

For completeness, and to illustrate the importance of regional considerations, Figure (5) shows each capital city’s generalised impulse response to a housing shock (defined as a shock producing a standard deviation increase in the house price to income ratio) when dynamics are determined by $\Pi_2$ for two shocks - one emanating from Sydney and the other from Perth.

As shown, a shock stemming from Sydney leads to a rise in house price to income levels for all capital cities (except Adelaide which becomes more affordable) that typically subsists for about three years. Shocks in Melbourne and Brisbane produce a similar response.

For the case of a shock emanating from Perth (viz. a commodity price shock), the system is also stable, but the dynamics are substantially different. In this case, the shock to Perth causes an immediate drop in affordability for Perth but a substantially smaller initial impact for the remaining capital cities. There also appears to be some

\[20\] The results are similar if the supply regressor is included. All results are available on request from the corresponding author.

\[21\] The instability stems largely from Sydney and Brisbane, which continue to be explosive even after transformation.
initial counter-cyclicality with Perth and Adelaide house prices increasing as Melbourne, Sydney and Brisbane prices decline.

Figure 5: Generalised impulse responses to housing shocks (figure (a) pertains to a 1 standard deviation shock in the Sydney house price to income ratio; figure (b) is for a 1 standard deviation shock in the Perth house price to income ratio).

**Testing the model using aggregate data**

To determine the impact of aggregation on the findings, we re-estimate the model using aggregate Australian data. We also consider the generality of the methodology by estimating the model on US house price data and find statistically significant evidence of an interest rate threshold in both US and Australian aggregate house prices (refer to the Appendix for further information).

The estimated threshold parameter for the aggregate Australian dataset is 7.03 per cent which is considerably higher than the 6.13 per cent estimated using the regional dataset. The aggregate model’s persistence parameters are 1.03 and 0.75 for the first
and second regimes respectively indicating that the stability dichotomy continues to be applicable. In this respect, the Terasvirta and Yang (2014) test of the null hypothesis of linearity is rejected at the 0.05 level indicating that a threshold is also evident in the aggregate dataset (the null hypothesis is rejected at the 0.01 level for the regional dataset).

The threshold for the aggregate dataset, however, corresponds to the 45th percentile of the empirical distribution of Australian mortgage lending rates and does not appear to be meaningfully associated with periods of sharp changes in affordability. In particular, the associated transition probabilities do not appear to accurately correspond with the periods of sharp capital appreciation observed in Figure (1). In contrast, the transition probabilities estimated using the regional dataset correspond closely with the periods where affordability levels declined sharply. The results suggest that information regarding regional spillovers is critical in determining the threshold at which stability breaks down, with the adoption of aggregate data clearly over-estimating the probability of being in the unstable regime.

4 Monetary policy and the threshold

Several papers have investigated the relationship between monetary policy and house prices in an attempt to deduce the impact of monetary policy shocks on house prices, and to determine the extent to which monetary policy levels can be used to diffuse house price instability (Del Negro and Otrak 2007; Goodhart and Hofmann, 2008; Jarocinski and Smets 2008; Allen and Rogoff 2011; Glaeser, Gottlieb and Gyourko; 2010; Williams 2011; Kuttner 2012; Adam and Woodford 2013). Relying predominantly on structural VARs, the papers suggest that expansionary monetary policy leads to a small but statistically significant increase in house prices for both the U.S. and a significant number of other industrialized countries.\(^\text{22}\)

Our empirical analysis regarding the presence of non-linearities in the relationship

\(^{22}\text{Several papers indicate that a 25 basis point decline in the policy rate leads to a house price increase anywhere between 0 and 1 per cent.}\)
between house prices and interest rates suggests three further considerations that are embodied in the time-varying weight function \( G(r_t; \gamma, c) \). Namely, it suggests that the response of house prices to changes in interest rates depends on: first, the level of the interest rate \( r_t \); second, whether the rate is above or below the critical threshold \( c \); and third, on the parameter \( \gamma \) which determines the speed at which house prices destabilise.

For our case study, when the outcome for \( G \) is large, \( \Pi_1 \) becomes dominant and house prices exhibit explosive behaviour. Conversely, when \( G \) is small \( \Pi_2 \) becomes dominant and the trajectory of house prices is stable. A bubble becomes a certainty only when \( G(r_t; \gamma, c) \to 1 \). Thus, to the extent that changes in the official cash rate cause changes in lending rates, it is important for central bankers to monitor the pass-through of policy changes to mortgage rates, to ascertain whether the resultant lending rates are above or below the critical threshold \( c \) and to assess the probability of the unstable regime.

*The impact of regime uncertainty*

Consider now a policy maker who influences the housing lending rate with the objective of maintaining house price and financial stability (see, further, Notarpietro and Siviero, 2015). The focus of the policy maker is to observe interest rates and to assess whether they are above or below the economy dependent threshold \( c \) (which is 6.1256 for Australia, but will differ for other economies). The policy maker will also observe \( h_t \), but will not be able to determine which regime - stable or unstable housing dynamics - will prevail because the smoothness parameter \( \gamma \) is relatively small and the observed outcome for house prices is a weighted average of the two regimes.\(^{23}\) Over time, the policy maker may be able to make inferences regarding \( \Pi_1 \) and \( \Pi_2 \) by observing the path of \( h_t \) but it would be desirable to take action before the housing market tips into the unstable regime. The importance of early action is highlighted by the finding that the spillovers implied by \( \Pi_1 \) render the house price response to a standard monetary policy shock unclear.

The aim then is to adopt a lending rate target in order to ensure that changes in the dynamics of house price to income ratios will always be stabilising. To identify the

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\(^{23}\)In contrast, the policy maker is able to precisely determine the relevant regime if \( \gamma \to \infty \).
target, we construct the following matrix of autoregressive dynamics

\[ \Pi (L) = G(r; \gamma, c) \Pi_1 (L) + (1 - G(r; \gamma, c)) \Pi_2 (L) \]

and solve for the lowest lending rate \( r = r^* \) for which the roots of

\[ \det (I_n - \Pi_1 z - \Pi_2 z^2) = 0 \]

are all outside the complex unit circle.

This lending rate \( r^* \) is the critical rate at which long-run house price to income levels are always stable in the presence of uncertainty regarding the prevailing regime. We find that \( r^* = 7.28 \) such that an equilibrium lending rate at or above this point is a necessary condition for stable long-run housing affordability dynamics. At \( r^* \), we also have \( G(r^*; \gamma, c) = 0.112 \) implying that, in the long run, the probability in favour of \( \Pi_1 \) should not exceed 11.2 per cent in order to prevent house price instability. Given that \( c = 6.13 \), it follows that the cost of regime uncertainty is non-negligible, resulting in an estimated 115 bps risk premium to the threshold \( c \) in order to produce stable house price to income conditions in the long-run. We note that as \( \gamma \) increases we have \( r^* \rightarrow c \) such that the cost of regime uncertainty approaches zero.

**Impulse Responses**

To illustrate the implementation of the strategy, consider our case study where lending rates are at the value observed at the end of the estimation period (\( r = 5.95 \)). Since \( r \) is below the threshold \( c = 6.13 \), the central bank knows that there is a non-zero probability of an incipient bubble and hence policy should aim to raise housing lending rates. To ensure that housing affordability dynamics are stable in the long-run we assume that the policy function for lending rates follows an error correction form

\[ \Delta r_t = \phi (r_{t-1} - r^*) + \varepsilon_t \]

where \( \varepsilon_t \sim N(0, \sigma^2) \) and \( \phi < 0 \).
Equation (7) is largely agnostic about the short run dynamics underpinning housing lending rates but ensures that the speed at which $r_t$ converges to the target $r^*$ is consistent with the historical stickiness observed in lending rates. $^24$ Given the target, the model’s VAR dynamics are such that $G(r; \gamma, c)$ converges to $G(r^*; \gamma, c)$ so that affordability dynamics are stable in the long run for every region. $^25$ Prior to convergence, however, the VAR dynamics can be temporarily unstable in which case the resulting impulse responses are a function of both stable and unstable dynamics in the short or medium term, although eventually becoming stable in accordance with the lending rate target $r^*$.

The impulse responses to a housing shock are obtained by projecting $r_t$ forward from its last actual value in the dataset. Accordingly, the unstable regime is initially more probable than the stable regime, with $\Pi(L)$ also being unstable. The resulting impulse responses in Figure (6a) are therefore indicative of a system that is initially persistently unstable and only becomes stable in the presence of the imposed lending rate target $r^*$. The lack of initial stability is, however, difficult to discern from the impulse responses alone and this highlights the policy maker’s problem in evaluating the house price response to low lending rates prior to the onset of instability. $^26$

We also derive impulse responses by assuming that interest rates are at the level $r^*$ at the time of the housing shock. In the absence of any interest rate shocks, lending rates therefore remain at $r^*$. $^27$ It is clear from Figure (6b) that the magnitude and persistence of the impulse responses declines substantially when the shock occurs during a period where lending rates are greater than or equal to $r^*$. Furthermore, as the gap between the initial lending rate and $r^*$ becomes larger, both the magnitude and persistence of

$^24$ We obtain the least squares estimate $\phi = -0.09$.

$^25$ We assume that $X_t$ is weakly stationary and does not prohibit the stability of the smooth transition VAR.

$^26$ Note that these impulse responses take into account the presence of two regimes, unlike Figure (5) which assumes that the economy is exclusively in the stable regime 2. A comparison of the two responses shows that the presence of regime uncertainty yields impulse responses that can subsist for a substantially longer period of time, particularly when lending rates are initially well below their long-run target. In the case where lending rates are initially at 5.95 per cent, the impact of a shock subsists for approximately twice the duration that is observed when there is no regime uncertainty.

$^27$ The results are equivalent to assuming an interest rate adjustment parameter $\phi = -1$ whereby interest rates adjust immediately to the stable value $r^*$.
Figure 6: Generalised impulse responses to an housing shock when the long term lending rate is set in order to achieve stable long-run affordability dynamics in an environment with regime uncertainty (impulse responses are for a 1 std deviation shock to the Sydney house price to income ratio). Figure (a) is contingent on a shock occurring at the actual lending rate observed at the end of the estimation sample (5.95 per cent), which converges to the long-run rate (7.28 per cent) at a speed consistent with historical stickiness. Figure (b) is for a shock when lending rates are already at the long-run stable rate at the time of the shock.

the impulse responses continue to decline with Figure (5) representing the limiting case in which the impulse responses under regime uncertainty correspond to those where the stable regime prevails with probability 1.

5 Conclusion

This paper addresses the issue of non-linearities in the relationship between house prices and interest rates. Our empirical analysis indicates that structural dynamics are non-linear and, moreover, that regional differences (and spillovers) matter. Using regional house price information, we find that the autoregressive dynamics for house prices and their sensitivity to fundamental information depend significantly on interest rate levels
and are regime-dependent.

In particular, we estimate an interest rate threshold that represents a tipping point for changes in house price dynamics. When interest rates fall below this level, we find that house prices become unstable and no longer exhibit ‘normal’ levels of sensitivity to fundamental drivers. Instead, fundamental drivers cease to be statistically significant when rates cross below the threshold. This failure to reflect fundamentals has substantial welfare ramifications, including misdirecting the decision of households regarding where to live and whether or not to rent or purchase (Glaeser, Gyourko and Saiz, 2008).

The threshold is shown to act as an over-reaction point in the vein of Kuttner (2012), with house prices being open to boom and bust type behaviour when the threshold is breached. We find, however, that the onset of house price instability does not occur abruptly, but depends on the extent to which rates fall below the threshold and the period of time that rates remain below this point. In the absence of other controls, the threshold effectively constitutes a housing lending rate lower bound.

The results have substantial implications for monetary policy setting and macro-prudential policies regarding the regulation of housing lending rates, particularly given recent evidence that house price booms are predictive of future financial crises (Jordà, Schularick and Taylor, 2015). The empirical analysis suggests that, when interest rates are below the over-reaction point, there is a risk that the stimulus stemming from expansionary monetary policy will be offset by house price instability.

The difficulty for policy makers is that there is uncertainty regarding whether rates are sufficiently low such as to lead to house price instability. We suggest a simple calculation for the lower bound of the lending rate in the presence of regime uncertainty. In other words, policies that maintain the housing lending rate at or above this critical level ensure that housing dynamics remain stable and render the likelihood of a bubble negligible.
References


Appendix: Comparing Australian and US house price dynamics

The empirical analysis of regional Australian house prices produces a key result regarding the existence of a threshold that characterises stable and unstable price dynamics that depend on mortgage rates. To assess the robustness of the result, we also apply the model to Australian and US aggregate house prices. The US house price to income ratio is obtained as the quarterly ratio of median house prices to median household income over the period December 1983 to December 2014. As with the Australian analysis, the US estimation also proceeds with the inclusion of US unemployment, population and consumer sentiment data.28

To examine the relationship between (US and Australian) lending rates and house price dynamics we first consider model estimation for a fixed threshold (with the remaining parameters unrestricted) and observe changes in the persistence of the aggregate house price measures (calculated as the sum of the first and second order autoregressive parameters) above and below the fixed lending rate thresholds.

Table A1 shows two common properties in the Australian and US aggregate measures that are observed as the lending rate threshold declines. The first is that the persistence of house prices in the first regime (being the low interest rate regime) tend to increase as the threshold declines. Second, the gap between the persistence levels in the two regimes tends to increase with smaller thresholds. In both cases, as the threshold declines the house price measure becomes explosive in the first regime, but is stationary in the second regime. Interestingly, the explosiveness of the US housing measure in the low interest rate regime exceeds that of Australia. These results suggest that the stability implications of the low interest rate regime are not Australian specific and are likely to be applicable to the US housing market as well.

\(^{28}\)Median household income and house prices are obtained from the FRED database. Annual household income is linearly interpolated to quarterly values. Lending rates are based on the Freddie Mac Primary Mortgage Market Survey. Population and sentiment data are obtained from the National Monthly Population Estimates and the Michigan Consumer Sentiment Survey, with the civilian unemployment rate obtained from the Bureau of Labour Statistics.
Table A1: Persistence in aggregate Australian and US house price to income ratios for a given lending rate threshold

<table>
<thead>
<tr>
<th>Lending rate threshold c</th>
<th>Australia</th>
<th>US</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Regime 1</td>
<td>Regime 2</td>
</tr>
<tr>
<td>7.00</td>
<td>1.006</td>
<td>1.016</td>
</tr>
<tr>
<td>6.75</td>
<td>1.020</td>
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<tr>
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<td>1.009</td>
</tr>
<tr>
<td>5.00</td>
<td></td>
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<tr>
<td>3.00</td>
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</tr>
</tbody>
</table>

Note: Regime 1 is more probable when lending rates are below the threshold c. The inability to observe a second regime for Australia at thresholds at or below 5.75 per cent is due to the thresholds being below the lower bound of the Australian mortgage rate variable used in the dataset. Regimes cannot be reliably estimated for thresholds above 7 per cent for Australia and above 5.5 per cent the US.

Estimation of the model on the aggregate US dataset yields a maximum likelihood estimate of the threshold parameter of 3.98 per cent. As is the case for the Australian dataset, the autoregressive parameters exceed unity when lending rates are below the threshold. In contrast, the parameters are below unity when lending rates are above the threshold. The null hypothesis of linearity (based on the test proposed in Terasvirta and Yang (2014)) is also rejected for the US dataset at the 0.10 level. Similar results are obtained if the data are first detrended prior to estimation. In both the Australian and

29The sum of the autoregressive parameters is 1.25 when lending rates are below the threshold and 0.54 when rates are above the threshold.
US cases, the thresholds obtained using aggregate data are below the median lending rate but, given the results for the Australian regional model, it is difficult to infer the reliability of the US threshold, with the true level of the mortgage rate threshold likely to be substantially smaller if regional spillovers are accounted for. In any case, the results provide evidence for both the Australian and US aggregate datasets of an over-reaction point at which the dynamics of house prices exhibit a significant change.