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Abstract

Motivated by the increasing use of forward guidance, we consider DSGE models in which the central bank holds the policy rate fixed for an extended period of time. Private agents' beliefs about how long the fixed-rate regime will last influences current output and inflation. We estimate the structural parameters for US data and infer the expected duration of the zero lower bound regime. Our results suggest that the average expected duration is around 3 quarters and has varied significantly since the onset of the zero lower bound regime, with changes that can be related to the Federal Reserve's forward guidance.

JEL classification: E52, E58

Keywords: Zero lower bound, forward guidance

1 Introduction

To combat the recent financial crisis and the resulting economic downturn, the Federal Reserve and many other central banks in advanced economies pushed their policy interest rates close to the zero lower bound and turned, among other policies, to forward guidance. Forward guidance refers to announcements about the future path of the policy rate. This communications policy has received increasing attention in the press and the academic literature. In particular, while some central banks have previously given guidance about the direction or timing of future policy rates, these recent announcements have been interpreted as an explicit attempt to influence expectations so as to increase the current degree of monetary policy accommodation.[1]

There is a good argument in theory why forward guidance can alleviate the contractionary impact of the zero lower bound. In forward-looking models the current stance of monetary policy depends on the expected path of the nominal interest rate, and therefore forward guidance can, in principle, stimulate aggregate demand to the extent it lowers private agents' forecasts of future nominal interest rates. So, a credible commitment to maintain interest rates at zero for longer than would have otherwise been implied by the zero bound itself represents an additional channel of monetary stimulus. Eggertsson and Woodford [2003], Jung et al. [2005] and more recently Werning [2012] all make this point: monetary policy can stimulate the economy by creating the right kind of expectations about the way the policy rate will be used once the constraint ceases to bind.[2]

Since December 2008, the Federal Reserve has made use of forward guidance. As is evident from FOMC statements, its forward guidance evolved over time and it is likely that the public's interpretation has changed as well. From early 2009 to mid 2011 the statements were somewhat vague, as is the case for example in the December 2009 statement which reads:

> "*The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions, [...], are likely to warrant exceptionally low levels of the federal funds rate for an extended period.*"

Then, from August 2011 to October 2012 the statements gave more precision about the "extended period" and the language changed, as the October 2012 statement shows:

> "*In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015.*"

But then, starting in December of 2012, the FOMC statements provided clearer state-contingent conditions linking the path of interest rates to the state of the economy, as is the case of the June 2013 statement which reads:

[1] See Woodford [2012].

[2] Krugman [1998] was the first to recast the liquidity trap as an expectations-driven phenomenon.

"[...] the Committee currently anticipates that this exceptionally low range for the federal funds rate will be appropriate at least as long as the unemployment rate remains above 6-1/2 percent, [...]".

The existing empirical literature on forward guidance is sparse. Swanson and Williams [2012], for example, use high-frequency data to study the effects of the zero lower bound on interest rates of longer maturities and find that market participants often expected the zero bound to constrain policy for only a few quarters. Bauer and Rudebusch [2013] use a shadow rate affine dynamic term structure model that accounts for the zero lower bound to infer expected future policy and estimate the future lift-off date. They find that the expected duration of the zero interest rate policy was quite short prior to mid 2011, when it noticeably increased. Campbell et al. [2012] study the response of asset prices and private macroeconomic forecasts to FOMC forward guidance before and after the crisis and conclude that the zero lower bound has not prevented the Federal Reserve from communicating future policy intentions.

Our approach here is different. We build on Kulish and Pagan [2012] and construct the likelihood function for the case in which monetary policy switches at the zero lower bound from following a standard Taylor-type rule to forward guidance. We use the model of Ireland [2004] as a benchmark because it is the closest estimated specification to the simple type of New Keynesian model used in the theoretical literature on forward guidance. For robustness and to show that our approach is feasible with larger models, such as those typically used at policy institutions, we also estimate the model of Smets and Wouters [2007]. This model, which has been widely used, has additional frictions and is estimated with a larger set of observable variables.

Using Bayesian methods we estimate, for the period 1983Q1-2013Q4, both the structural parameters and the expected duration of the zero interest rate policy in each quarter since the beginning of 2009. To the best of our knowledge, this paper is the first to provide the tools to estimate with full-information methods a DSGE model with a change in the monetary regime at the zero lower bound. The estimation challenge that the zero lower bound raises has not been addressed in the literature before and is a central contribution of our paper.

In measuring the impact of communications on expectations of future interest rates and the economy, it is useful to distinguish between two kinds of forward guidance: a *conditional forward guidance* in which the central bank publicly states its forecasts and anticipated policy actions based on its own objectives and an *unconditional forward guidance* in which the central bank publicly commits to a particular course of action.[3] These two kinds of forward guidance should have quite different effects. To illustrate, take the case of the Bank of England which on 7 August 2013 stated that it would probably not raise its policy rate until unemployment fell to 7 percent, subject to a number of

[3]Campbell et al. [2012] use the terms Delphic and Odyssean forward guidance to draw a somewhat similar distinction. Optimal monetary policy under commitment with a zero lower bound on interest rates is not the same as unconditional forward guidance. Although both involve a pre-commitment, optimal policy with commitment is history-dependent; as the commitment to prolonging zero interest rates depends on past conditions it is not equivalent to an unexpected unconditional promise of the central bank. Our terminology is useful to distinguish whether or not future policy actions are contingent on the state of the economy.

other conditions too.[4] According to the financial press, the very conditionality of this forward guidance diluted the effectiveness of the pre-commitment to keep interest rates low for longer.[5]

In our analysis, we let the data speak through the lens of a model in which forward guidance is unconditional. During the zero lower bound regime, agents base current expectations on the assumption that the central bank will unconditionally keep to its zero interest rate policy for a certain number of quarters in the future, after which it will revert back to its temporarily abandoned Taylor rule. We achieve identification because variation in the expected duration gives rise to distinct dynamics of the observable variables and also because the sub-sample prior to the zero lower bound helps us identify competing sources of exogenous variation. It is well known that in models with rational expectations, unconditional forward guidance is powerful in the sense that it can generate very large responses of aggregate variables, a phenomenon del Negro et al. [2012] call the 'forward guidance puzzle'.[6] For the purposes of econometric identification, however, this sensitivity of aggregate variables to forward guidance turns out to be quite useful in pinning down the expected durations in estimation. But one should bear in mind that an absence of 'unreasonably large' fluctuations in the data does not necessarily imply short expected durations. It is possible that the powerful effects of forward guidance have served to offset some of the impact of the very large shocks that led to the Great Recession in the United States.

We find that including the zero lower bound regime in our sample and estimating the expected durations of the zero lower bound produces structural parameter estimates that are in line with those found in the literature on the pre-crisis data only, such as those found by Ireland [2004] and by Smets and Wouters [2007]. We find the mean expected duration for the zero lower bound regime to be between 3 and 4 quarters with statistically significant changes in the expected duration over the zero lower bound regime.

For the zero lower bound regime, we compute shadow policy rates – i.e., the policy rate that would have prevailed in the absence of the zero constraint. Shadow rates allow us to assess whether increases in the expected duration are likely to reflect forward guidance about keeping rates fixed at zero for longer than would have otherwise been implied by the zero bound itself or simply reflect negative shocks making the constraint bind for longer. In the benchmark case, we find positive shadow rates after the last quarter of 2009, which suggests that forward guidance has been at work in the United States. The fact that the policy rate remained at zero when the shadow rate was positive is in line with the optimal monetary policy prescription of keeping the interest rate at zero once the constraint ceases to bind. As additional evidence that forward guidance has been effective, we find that the expected duration increases exactly in the quarter when the

[4] See News Release - Bank of England for more details.

[5] See "When will interest rates rise?", September 2013, FT.com. A possible argument, one imagines, goes like this: if the public were to expect low interest rates for long, then inflation would rise, but as the commitment to low interest rates is conditional on the path of inflation and the commitment is void if inflation were to breach the inflation target, then announcements to keep low interest rates for long lack credibility.

[6] Carlstrom et al. [2012] argue that an announcement to keep the interest rate at zero for more than 8 quarters delivers 'unreasonably large' responses in the Smets and Wouters model.

Federal Reserve made significant changes to its communications strategy, the quarter when it introduced calendar-based guidance.

We carry out additional exercises to assess our benchmark results. First, we consider an informative prior over sequences of expected durations. This prior is designed to capture that one might anticipate, absent further announcements, that the expected duration of the regime should gradually decrease over time. Second, as forward guidance is in part intended to stimulate the economy by lowering long-term interest rates, we consider using yield curve data in estimation.[7] Finally, we also estimate the expected durations with Smets and Wouters [2007] model. We find that although the informative prior shrinks the expected duration, we can detect an increase in the quarter when calendar based guidance is introduced. Adding the yield curve up to a maturity of 8 quarters does not materially change our inferences. Using the Smets-Wouters model the average expected duration is estimated to be moderately longer. The shadow rate obtained is persistently negative, and therefore in this instance changes in the expected duration may better thought of as reflecting revisions about how long the zero lower bound is expected to bind rather than being the result of forward guidance.

The rest of the paper is structured as follows. Section 2 presents the benchmark model, then in Section 3 we discuss how we solve for equilibria under forward guidance. Section 4 discusses the estimation methodology. Section 5 contains the main results, and section 6 presents the sensitivity analysis.

2 The Model

In our empirical analysis, we focus on a small New-Keynesian model, akin to that used in the theoretical literature on forward guidance, namely a modified version of Ireland [2004]. We allow lagged inflation to partly determine the cost of price adjustment. We remove the output gap from the Taylor rule so that the Federal Funds rate responds only to observable variables. Such specification of policy, we think, is more realistic because in practice there is significant uncertainty around the definition and the measurement of potential output, and in any case, the response to the output gap estimated by Ireland [2004] is small. Despite these modifications, the details of the model are well-established,

[7] However, how longer term rates respond to forward guidance is an empirical issue. As Werning [2012] highlights, the response of the yield curve to forward guidance is not straightforward because a commitment to zero interest rates may lower medium maturity rates but it may not necessarily translate into lower long term interest rates.

so we just list the linearised equations for sake of brevity:

$$\hat{x}_t = -(\hat{r}_t - \mathbb{E}_t\hat{\pi}_{t+1}) + \mathbb{E}_t\hat{x}_{t+1} + (1-\omega)(1-\rho_a)\hat{a}_t \tag{1}$$

$$\hat{\pi}_t = \frac{1}{1+\beta\alpha}(\alpha\hat{\pi}_{t-1} + \psi\hat{x}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} - \hat{e}_t) \tag{2}$$

$$\hat{r}_t = \rho_r\hat{r}_{t-1} + \rho_\pi\hat{\pi}_t + \rho_g\hat{g}_t + \varepsilon_{r,t} \tag{3}$$

$$\hat{x}_t = \hat{y}_t - \omega\hat{a}_t \tag{4}$$

$$\hat{g}_t = \hat{y}_t - \hat{y}_{t-1} + \varepsilon_{z,t} \tag{5}$$

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon_{a,t} \tag{6}$$

$$\hat{e}_t = \rho_e\hat{e}_{t-1} + \varepsilon_{e,t} \tag{7}$$

All variables are in percentage deviations from their steady state values; with this in mind, $\hat{x}_t$ is the output gap, the deviation of output from a socially efficient level of output, $\hat{\pi}_t$ is inflation, $\hat{r}_t$ is the one-period nominal interest rate, $\hat{g}_t$ is the growth rate of output, $\hat{y}_t$ is the stochastically detrended level of output (labour-augmenting technology follows a unit root with drift and innovation $\varepsilon_{z,t}$). The autocorrelated process, $\hat{a}_t$ and $\hat{e}_t$, are demand and cost push shocks with persistence ρ_a and ρ_e and innovations $\varepsilon_{a,t}$ and $\varepsilon_{e,t}$ respectively. Finally, β is the discount factor, ψ the slope of the Phillips curve, α measures the degree of indexation and ω governs the Frisch elasticity of labour supply.

The measurement equations include:

$$\pi_t = \pi + \hat{\pi}_t \tag{8}$$

$$g_t = g + \hat{g}_t \tag{9}$$

$$r_t = r + \hat{r}_t \tag{10}$$

where π is the steady state rate of inflation and g the steady state growth rate of real output per person. Below, we estimate π and g and then set β so that the sample mean of the nominal interest rate satisfies $r = \pi g/\beta$.

3 Solution with Forward Guidance

To solve for equilibria under forward guidance we use a special case of the solution developed by Kulish and Pagan [2012] for forward-looking models in the presence of possibly anticipated structural change. That solution has more general application than the context we are considering here, so we provide a simplified discussion to highlight some important features.

To discuss the solution, we introduce notation. Below we take a sample of data of size T to estimate the model. For presenting the solution under forward guidance, however, it is useful to take the start of the zero lower bound regime to be at $t = 1$. In Figure 1, the zero lower bound starts in period 1 and lasts for d periods and conventional policy is assumed to resume out of sample.

In the form of Binder and Pesaran [1995], the system of linearised equations can be written as

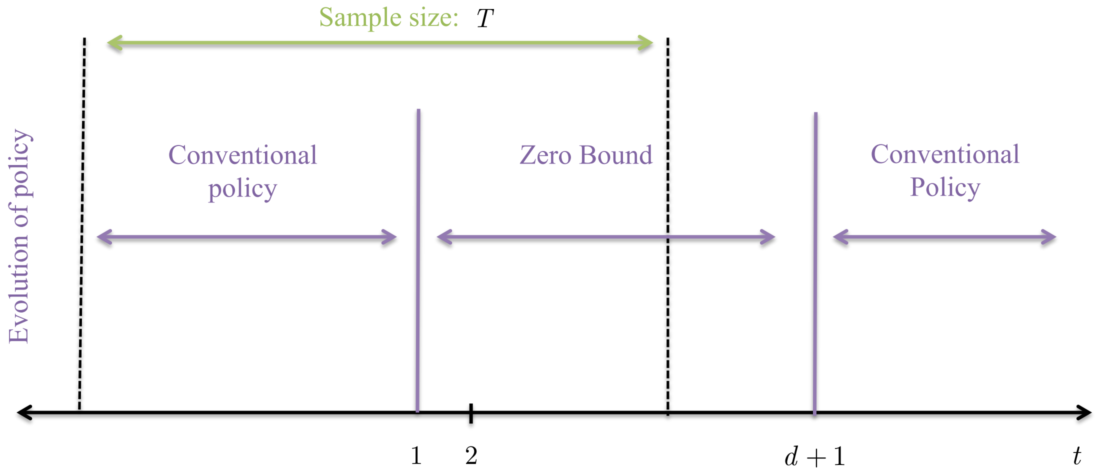
$$y_t = Ay_{t-1} + B\mathbb{E}_t y_{t+1} + D\varepsilon_t \tag{11}$$

where y_t is an $n \times 1$ vector of state and jump variables and with no loss of generality ε_t is a $l \times 1$ vector of white noise shocks; the matrices A, B and D are of conformable dimensions.

Prior to the zero lower bound, the economy follows equation (11) and the standard rational expectations solution applies. If the solution exists and is unique, then y_t follows the VAR process

$$y_t = Qy_{t-1} + G\varepsilon_t \tag{12}$$

Figure 1: Timing of Events



Assume now that at $t = 1$ the implementation of equation (3) would have implied $\hat{r}_t < -r$. As this is not possible, monetary policy sets its policy rate to its lower bound, $\hat{r}_t = -r$, and communicates its intentions to revert back to conventional policy at a later time, $t = d^e + 1$.[8] This later date may or may not coincide with the realised duration of the zero lower bound regime. If agents, in fact, expect conventional policy to resume at that time, then the expected duration of the zero lower bound regime in period $t = 1$ is given by d^e. During the zero lower bound regime, the structural equations are given by

$$y_t = \bar{A}y_{t-1} + \bar{B}\mathbb{E}_t y_{t+1} + \bar{D}\varepsilon_t \tag{13}$$

and monetary policy now follows $\hat{r}_t = -r$. A monetary policy rule that fixes the nominal interest rate would give rise to indeterminacy if agents indeed expected this rule to be implemented indefinitely. Alternatively, if monetary policy is expected to adopt a rule consistent with a unique equilibrium in the future, then, as shown in Cagliarini and Kulish

[8] In our empirical application, we assume that this conventional policy which is reverted back to includes an inflation target of 2 per cent per annum.

[2013], a rule like $\hat{r}_t = -r$ can be temporarily consistent with a unique equilibrium as well. Suppose then that monetary policy will indeed revert back to conventional policy by $t = d^e + 1$, so we assume $d^e = d$. For periods $t = 1, 2, ..., d$ the solution for y_t becomes a time-varying coefficient VAR process

$$y_t = Q_t y_{t-1} + G_t \varepsilon_t. \tag{14}$$

which implies that

$$\mathbb{E}_t y_{t+1} = Q_{t+1} y_t \tag{15}$$

Using equations (15) and (13), it is possible to establish via undetermined coefficients that

$$(I - \bar{B}Q_{t+1})^{-1}\bar{A} = Q_t \tag{16}$$
$$(I - \bar{B}Q_{t+1})^{-1}\bar{D} = G_t \tag{17}$$

Starting from the solution to the final structure, $Q_{d+1} = Q$, equation (16) determines via backward recursion the sequence $\{Q_t\}_{t=1}^{d}$. With the sequence $\{Q_t\}_{t=1}^{d}$ in hand, equation (17) yields the sequence $\{G_t\}_{t=1}^{d}$.

The sequence of time-varying reduced form matrices, $\{Q_t\}_{t=1}^{d}$, provide the solution for the case in which the expected duration of the zero lower bound follows $d, d-1, d-2, ..., 1$. In the sequence $\{Q_1, Q_2, ..., Q_d\}$, the matrix associated with an expected duration of d quarters is Q_1, the matrix associated with an expected duration of $d-1$ quarters is Q_2 and so on. This sequence, however, can be thought of in two ways: as one announcement made in $t = 1$ and carried out as announced or as a sequence of announcements with expected durations $d, d-1, d-2, ..., 1$. This implies, for instance, that if in every period monetary policy were to announce, or alternatively, if agents were to expect zero interest rates to last for d^e periods then the resulting sequence of reduced form matrices would simply be $\{Q_1, Q_1, ..., Q_1\}$. The point is that a sequence of expected durations maps uniquely into a sequence of reduced-form matrices.

4 Estimation

We use Bayesian methods, as is common in the estimated DSGE model literature.[9] Our case, however, is non standard in a few ways. First, forward guidance implies a form of regime change as we have described above. Second, it is necessary to adjust the Kalman filter to handle missing observations. Because the federal funds rate has no variance at the zero lower bound, it must be removed as an observable to prevent the variance-covariance matrix of the one-step ahead predictions of the observable variables from becoming singular.[10] Third, we jointly estimate two sets of distinct parameters: the

[9]See An and Schorfheide [2007]).

[10]See Appendix A for a description of how this is implemented. One could alternatively allow for measurement error in the observation equation of the federal funds rate. Although there is little variation of the federal funds rate throughout the zero lower bound regime, we do not consider this variation to be a form of measurement error.

structural parameters of the model, θ, that have continuous support and the sequence of expected durations, $\{d_t^e\}$, that have discrete support. In other words, the sequence of expected durations can take on only integer values and have to be treated differently. For notational convenience we will denote a sequence of expected durations hereafter simply by $\mathbf{d}$.

Next, we describe how we construct the joint posterior density of θ and $\mathbf{d}$:

$$p(\theta, \mathbf{d}|Z) \propto \mathcal{L}(Z|\theta, \mathbf{d})p(\theta, \mathbf{d}), \tag{18}$$

where $Z \equiv \{z_t\}_{t=1}^T$ is the data and z_t is a $n_z \times 1$ vector of observable variables. The likelihood is given by $\mathcal{L}(Z|\theta, \mathbf{d})$, the priors for the structural parameters and the sequence of expected durations are assumed to be independent, so that $p(\theta, \mathbf{d}) = p(\theta)p(\mathbf{d})$. Our baseline results are based on a flat prior for $\mathbf{d}$ such that $p(\mathbf{d}) \propto 1$, which is proper given its discrete support.

4.1 The Likelihood with Forward Guidance

The sample runs from 1983q1 to 2013q4 and has two distinct sub-samples, one before and one after the zero lower bound. Before the zero lower bound, from 1983q1 to 2008q4, we postulate a constant regime so that the reduced-form solution that governs the system for those periods is $y_t = Qy_{t-1} + G\varepsilon_t$. Once the zero lower bound regime is in place, for 2009q1 to 2013q4, the reduced-form solution follows Equation (14), that is $y_t = Q_t y_{t-1} + G_t\varepsilon_t$, where the sequence of reduced-form matrices is determined solely by the expected duration that prevails at each quarter.

Before the zero lower bound, the model variables, y_t, are related to the observable variables, z_t, via the measurement equation

$$z_t = Hy_t + v_t \tag{19}$$

For the zero lower bound regime, we define a new vector of observables, $\bar{z}_t \equiv Wz_t$, where W is an $(n_z - 1) \times n_z$ matrix that selects a subset of the observable variables in z_t. In our case, $n_z = 3$ and we drop the federal funds rate which if ordered first in z_t implies $W = [\mathbf{0}_{2\times1}, I_{2\times2}]$. Defining $\bar{H} \equiv WH$ and $\bar{v}_t \equiv Wv_t$, the model variables relate to the subset of the observables during the zero lower bound regime by

$$\bar{z}_t = \bar{H}y_t + \bar{v}_t \tag{20}$$

Equations (12) and (14) from the previous section summarize the evolution of the state and Equations (19) and (20) the evolution of the measurement equations. Together they form a state space model to which the Kalman filter can be applied to construct the likelihood, $\mathcal{L}(Z|\theta, \mathbf{d})$, as described in Appendix A.

4.2 The Prior

As mentioned above, the joint prior for the parameters is split into two independent priors, one for the structural parameters, $p(\theta)$, and one for the sequence of expected durations. We discuss each in turn.

4.2.1 Priors on the structural parameters

The joint prior for the structural parameters is factorized into independent priors for each structural parameter. The prior mean for each parameter is chosen with reference to the maximum likelihood estimates of Ireland [2004] or a training sample analysis in the case of the standard deviation of the structural shocks.[11] The priors, together with the posterior estimates, are given in Table 2.

As is well-known, the parameter that governs the slope of the Phillips curve, ψ, in New Keynesian models is not well identified. We follow Ireland [2004] and calibrate it to 0.1.[12] We also calibrate ω, which governs the Frisch-elasticity of labour supply. As the output gap does not enter the policy rule in our case and is also not included as an observable variable in estimation, it can be seen from Equation (1) that the way ω enters the system is by scaling the variance of the demand shock, $\hat{a}_t$. It is therefore not separately identified from the variance of the demand shock, σ_a. We set ω to 0.1. The priors for the monetary policy rule parameters, ρ_r, ρ_π and ρ_g, are relatively standard and place considerable weight on interest rate smoothing. At the prior mean, policy responds in the long-run more aggressively to inflation than to output growth. As the Federal Reserve has recently made its inflation target explicit, we assume that the out-of-sample monetary policy rule to which the Federal Reserve reverts back to has an inflation target of 2 per cent per annum.

We consider a relatively loose prior for α, the parameter that governs the degree to which the Phillips curve is backward looking, allowing for the possibility of a sizable backward-looking component. Similarly, the priors on the autoregressive parameters for the shocks, ρ_a and ρ_e, have a mean of around 0.7, but place considerable weight on all values greater than 0.25. Plots of the priors of all of the structural parameters, together with their posteriors, can be found in the next section.

4.2.2 Prior on the Sequence of Expected Durations

We set an upper bound of d^* on the maximum possible expected duration. If the zero lower bound regime lasts for L quarters, there are $(d^*)^L$ admissible sequences of expected durations. With an uninformative prior, every admissible sequence, $\{d_1, d_2, ..., d_L\}$, has equal probability given by $1/(d^*)^L$.

Because the expected durations are treated as parameters for estimation, there is nothing in the model linking their values from one period to the next. Following a one time credible announcement about a fixed policy regime and in the absence of future announcements, however, the expected duration of the regime should decrease by one every period as implied by our solution. Additionally, given that the Federal Reserve FOMC statements generally change little from month to month one might anticipate the expected durations to change reasonably smoothly over time. Therefore we explore the sensitivity of our baseline results to allowing for an informative prior on expected

[11] The training sample analysis is to estimate the standard deviations of the shocks using maximum likelihood over 1959:Q3 to 1982:Q4, with the other structural parameters set at the mean of their priors.

[12] The slope of the Phillips curve depends on the size of the cost of price adjustment. This value, as Ireland [2004] explains, would correspond to the case with Calvo-pricing where prices are reset on average every 3.74 quarters - or just a little more frequently than once per year.

durations that implies a tendency for the expected duration to decrease by one every period all else equal and to change only smoothly over time.

Letting d_1 be the expected duration in the first quarter of the zero lower bound regime, d_2 be the expected duration in the second quarter, and so on up until d_L, which is the expected duration in the last quarter of the zero lower bound, the informative prior is specified based on a conditional factorization as follows:

$$p(\mathbf{d}) = p(\{d_1, d_2, ..., d_L\}) = p(d_1)\, p(d_2|d_1)\, p(d_3|d_2) \ldots p(d_L|d_{L-1}),$$

so that the prior at a particular quarter during the zero lower bound regime is conditional on the expected duration held in the previous quarter. In the first period of the zero lower bound regime, we place a uniform prior over all possible expected durations. That is, for all admissible values of d_1, $p(d_1) = 1/d^*$. For any subsequent quarter k, however, we set the mode of the conditional prior, $p(d_k|d_{k-1})$, to $d_{k-1} - 1$ with probability given by the parameter *peak*. The remaining probability $(1 - peak)$ is distributed to admissible durations that are no more than the parameter *width* away from the mode, with weights inversely proportional to the distance from the mode. Assuming that all of the durations within the *width* distance from the mode are admissible, the conditional prior will look like a peaked pyramid with the relative height of the mode depending on *peak*. However, if any of the durations within the *width* distance from the mode are inadmissible, the probability that would have been assigned to the inadmissible durations is reallocated proportionately to the tails of the distribution.[13] The probability assigned to the mode, if admissible, is always *peak*. In such a case, the conditional prior will look like a truncated pyramid.

4.3 The Posterior Sampler

To simulate from the joint posterior of the structural parameters and the expected durations of the zero lower bound, $p(\theta, \mathbf{d}|Z)$, we use the Metropolis-Hastings algorithm. As we have two distinct sets of parameters we consider a slight modification to the standard setup for estimating DSGE models. We separate the parameters into two natural blocks: the expected durations of the zero lower bound policy and the structural parameters. To be clear, though, our sampler delivers draws from the joint posterior of both sets of parameters.

The first block of the sampler is for the expected duration of the zero lower bound, $\mathbf{d}$. It is possible to update the entire sequence of expected durations at each iteration of the sampler. However, preliminary estimation with simulated data suggested that more accurate results were obtained when only a subset of the parameters were possibly updated at each iteration. The approach we use is to randomize both the *number* of expected durations in $\mathbf{d}$ to be updated, and *which* particular expected durations in $\mathbf{d}$ to update. Our approach is motivated by the randomized blocking scheme developed for DSGE models in Chib and Ramamurthy [2010]. For this block, we use a uniform proposal density.

[13]The duration assigned to be the mode would be zero when $d_{k-1} = 1$, but since zero is inadmissible we reallocate the *peak* probability proportionately to the tails of the distribution.

To be specific, the algorithm for drawing from the expected durations block is given as follows: Initial values of the expected durations, $\mathbf{d}_0$, and the structural parameters, θ_0, are set. Then, for the j^{th} iteration, we proceed as follows:

1. randomly sample the *number* of quarters to update in the proposal from a discrete uniform distribution $[1, d^*]$

2. randomly sample without replacement *which* quarters to update in the proposal from a discrete uniform distribution $[1, L]$

3. randomly sample the corresponding elements of the proposed sequence of durations, $\mathbf{d}_j^{'}$, from a discrete uniform distribution $[1, d^*]$ and set the remaining elements to their values in $\mathbf{d}_{j-1}$

4. calculate the acceptance ratio $\alpha_j^{\mathbf{d}} \equiv \frac{p(\theta_{j-1}, \mathbf{d}_j^{'}|Z)}{p(\theta_{j-1}, \mathbf{d}_{j-1}|Z)}$

5. accept the proposal with probability $\min\{\alpha_j^{\mathbf{d}}, 1\}$, setting $\mathbf{d}_j = \mathbf{d}_j^{'}$, or $\mathbf{d}_{j-1}$ otherwise.

The second block of the sampler is for the n_s structural parameters.[14] It follows a similar strategy to the expected-durations-block described above - we randomize over the number and which parameters to possibly update at each iteration. One difference, however, is that the proposal density is a multivariate Student's $t-$ distribution.[15] Once again, for the j^{th} iteration we proceed as follows:

1. randomly sample the *number* of parameters to update from a discrete uniform distribution $[1, n_s]$

2. randomly sample without replacement *which* parameters to update from a discrete uniform distribution $[1, n_s]$

3. construct the proposed $\theta_j^{'}$ by drawing the parameters to update from a multivariate Student's $t-$ distribution with location set at the corresponding elements of θ_{j-1}, scale matrix based on the corresponding elements of the negative inverse Hessian at the posterior mode multiplied by a tuning parameter $\kappa = 0.2$, and degree of freedom parameter $\nu = 12$.

4. calculate the acceptance ratio $\alpha_j^{\theta} \equiv \frac{p(\theta_j^{'}, \mathbf{d}_j|Z)}{p(\theta_{j-1}, \mathbf{d}_j|Z)}$ or set $\alpha_j^{\theta} = 0$ if the proposed $\theta_j^{'}$ includes inadmissible values (e.g. a proposed negative value for the standard deviation of a shock) preventing calculation of $p(\theta_j^{'}, \mathbf{d}_j|Z)$

5. accept the proposal with probability $\min\{\alpha_j^{\theta}, 1\}$, setting $\theta_j = \theta_j^{'}$, or θ_{j-1} otherwise.

[14] In our benchmark application $n_s = 12$.

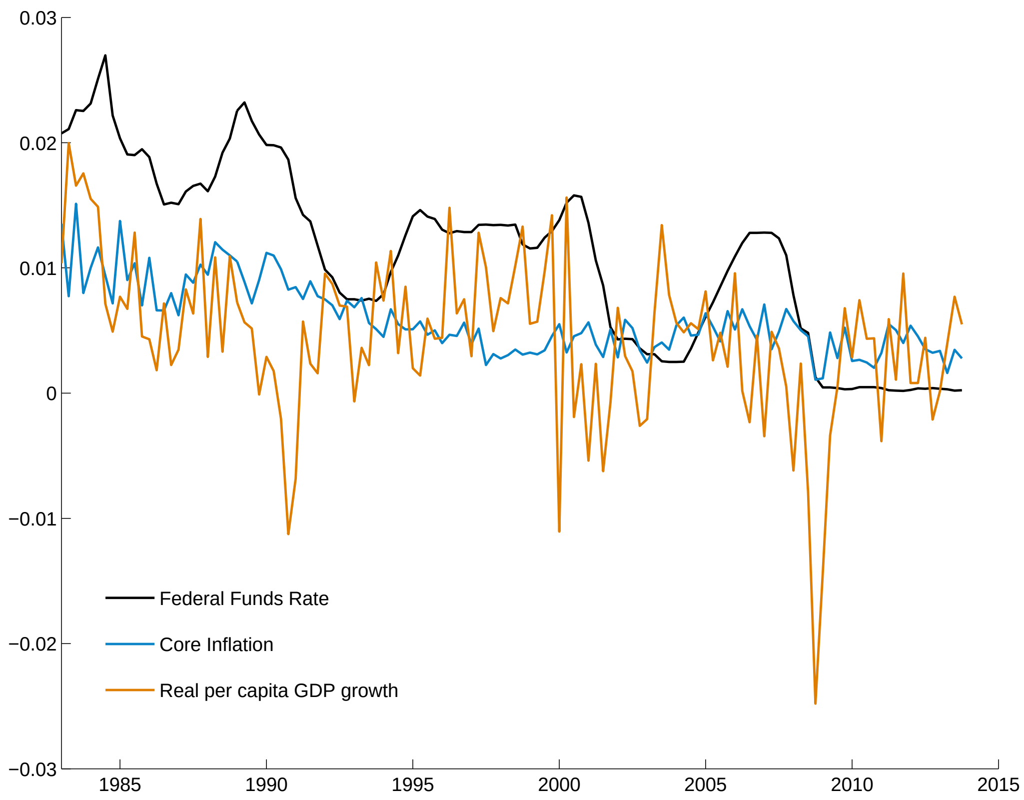
[15] For computational efficiency, the hessian of the proposal density is computed at the mode of the structural parameters rather than at each iteration as in Chib and Ramamurthy [2010].

We use this multi-block algorithm to construct a chain of 575,000 draws from the joint posterior, $p(\theta, \mathbf{d}|Z)$, throwing out the first 25 per cent as burn in (approximately 140,000 draws). The chain exhibits some persistence, although this in part reflects that blocking of the parameters, and simple trace plots suggest that the estimates of the structural parameters mix well.

4.4 The Data

We use three observable variables in estimation: the Effective Federal Funds rate r_t^{obs}, Real GDP per capita growth, Δy_t^{obs} and Core personal consumption expenditure chain-type price index (excluding food and energy), π_t^{obs}. These are denoted collectively as the vector $z_t \equiv (r_t^{obs}, \pi_t^{obs}, \Delta y_t^{obs})'$.[16] The sample is the period after the Volcker disinlflation, 1983q1:2013q4, during which the objectives of US monetary policy are likely to be relatively stable. The data are shown in Figure 2.

Figure 2: Observable variables



[16]The data were obtained from the FRED database of the St. Louis Federal Reserve. The mnemonics are: FEDFUNDS, JCXFE, GDPC1 and CNP16OV.

5 Results

There are two sets of parameter to estimate, the expected durations and the structural parameters. We discuss each in turn.

We focus on the results with an uninformative prior on the expected durations and relegate results based on an informative prior to the sensitivity analysis section.

5.1 Expected Durations

Table 1 shows summary statistics of the posterior distribution of the expected durations.

The mean estimate of the expected duration that the zero interest rate policy will continue is between 3 and 4 quarters throughout most of the zero lower bound regime. Prior to the June quarter of 2011, these results are in line with the Blue Chip survey of professional forecasters reported by Swanson and Williams [2012] that shows that "the expected number of quarters until the first federal funds rate increase above 25 basis points" fluctuates between 3 and 4 quarters.[17] Bauer and Rudebusch [2013], in the context of an affine term structure model, estimate the lift-off date from the zero lower bound policy. And their estimates are also consistent with our findings, as their median expected duration fluctuates around 3 to 4 quarters prior to mid 2010.

In mid 2011 the FOMC shifted to calendar-based forward guidance. Our estimates suggest that at this time the expected duration that the Federal Funds rate will be held at zero increased markedly. An increase in the expected duration at this time also is evident in the results of Bauer and Rudebusch [2013], where it is even more marked, and in the survey data reported by Swanson and Williams [2012]. The increase in the expected duration is evident in the marginal distribution of the expected duration, which shifts considerably in the June quarter of 2011 (Figure 3) when the FOMC introduced calendar based guidance. In the June quarter of 2011 the mode of the marginal distribution jumps to 3 quarters and remains elevated for the next five quarters. The mode of the expected duration falls in the last quarter of 2012 when the FOMC linked its guidance to the path of unemployment.

Table 1 may give the impression that the sequence of expected durations given by the mode of the marginal distribution of each quarter is very likely. But, as we show next, this particular sequence is very unlikely. The sequence of modes in Table 1 imply an average expected duration of 1.6 quarters. Figure 4, however, shows the posterior density of the average expected duration over the 20 quarters of the zero lower bound regime. The probability of a sequence of beliefs with an average expected duration of 1.6 quarters is negligible. Figure 4 also highlights that the standard deviation of each sequence $\mathbf{d}_i$ is typically considerably above zero - the mode of that distribution is around 2. Together, these figures imply that a sequence where the expected duration fluctuates only between 1 and 2 quarters is highly improbable. In fact, we estimate a posterior probability of over 50 per cent that the expected duration exceeded 7 quarters at least once during the zero lower bound regime and a posterior probability that it exceeded 8 quarters of over 25 per cent. However, the posterior probability that the expected duration ever exceeded

[17] See Figure 5 in Swanson and Williams [2012].

10 quarters is quite low, at around 5 per cent.

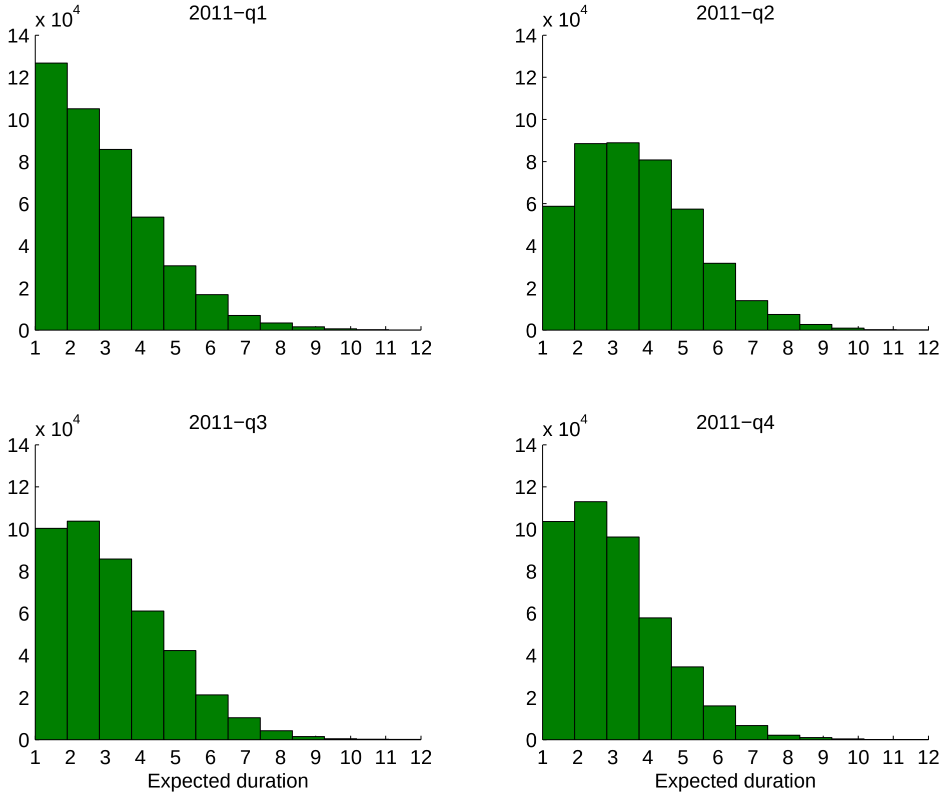
One possible interpretation of the fluctuations in the expected durations is that they reflect changes to expectations that the zero lower bound constraint will bind in the future. Put differently, the expected duration could increase, not because of forward guidance, but because of negative shocks that make the zero lower bound constraint more binding. This would be the case when the estimated Taylor rule, which captures Federal Reserve's past behavior, implies more negative interest rates. Alternatively, if past behavior suggests the Federal Reserve would have increased the policy rate, an increase in the expected duration can be thought to reflect forward guidance.

Table 1: Estimated Expected Duration of the Zero Lower Bound

	Expected Duration (qtrs)		
Quarter	Mode	Mean	Std. Dev.
2009Q1	1	2.37	1.55
2009Q2	3	3.81	2.17
2009Q3	1	2.92	1.86
2009Q4	3	3.84	2.04
2010Q1	1	2.70	1.70
2010Q2	1	2.81	1.69
2010Q3	1	2.64	1.62
2010Q4	1	2.58	1.55
2011Q1	1	2.69	1.67
2011Q2	3	3.47	1.80
2011Q3	2	2.96	1.73
2011Q4	2	2.77	1.57
2012Q1	2	3.03	1.67
2012Q2	2	2.79	1.66
2012Q3	2	2.73	1.56
2012Q4	1	2.74	1.68
2013Q1	1	2.95	1.83
2013Q2	1	2.53	1.61
2013Q3	2	3.07	1.79
2013Q4	1	2.71	1.79

To disentangle these two possible interpretations of changes in expected durations we construct a shadow federal funds rate. We do this by adding a second policy rate to the model, governed by a Taylor rule which is identical to Equation 3 without the monetary policy shock, but which is not subject to the zero lower bound constraint and has no feedback into the rest of the model. The resulting shadow rate is shown in Figure 5. There is a distribution of shadow rates reflecting uncertainty stemming from both the shocks and the estimated structural parameters of the model.

Figure 3: Histogram of Expected Durations: 2011q1-2011-q4



Only for a relatively short period – namely 2009 – was there any sizable probability of a negative shadow rate. A negative shadow rate suggests that, faced with the estimated shocks, the Federal Reserve usually would have conducted more expansionary policy according to its historical behavior. It is also evident from Table 1 and Figure 5 that increases in the expected duration tend to coincide with increases, not decreases, in the shadow rate, most notably in the mid-2011 episode. Our interpretation is that these are circumstances in which historical behavior would suggest that the Federal Reserve would be tightening monetary policy, but forward guidance has instead successfully extended expectations that low interest rates will be maintained. During this period the Federal Reserve obviously not only used forward guidance, but engaged in quantitative easing, considerably increasing its balance sheet. In this model there is no direct expansionary impact of such policy; however, quantitative easing can be thought of as a means of forward guidance, along the lines suggested by Bernanke and Reinhart [2004], as it demonstrates a commitment to holding rates low for an extended period. In the last quarter of 2009 the FOMC indicated that most special liquidity facilities would expire on February 1, 2010, but also noted that it was prepared to modify these plans if necessary to support financial stability and economic growth.[18]

[18]Prior to the last quarter of 2009, FOMC statements were vague in this respect stating that "*The*

Figure 4: Posterior Distributions

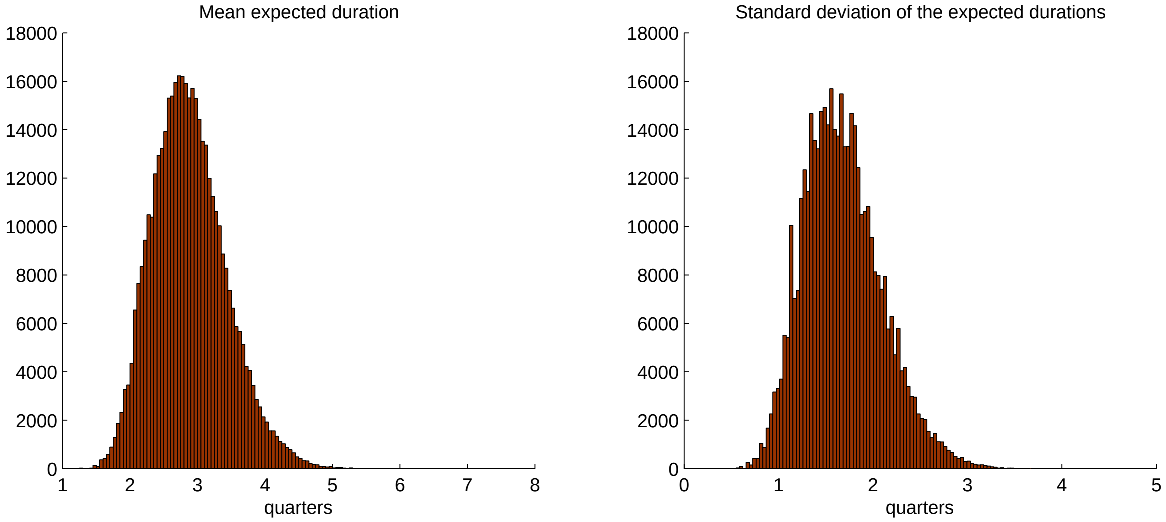
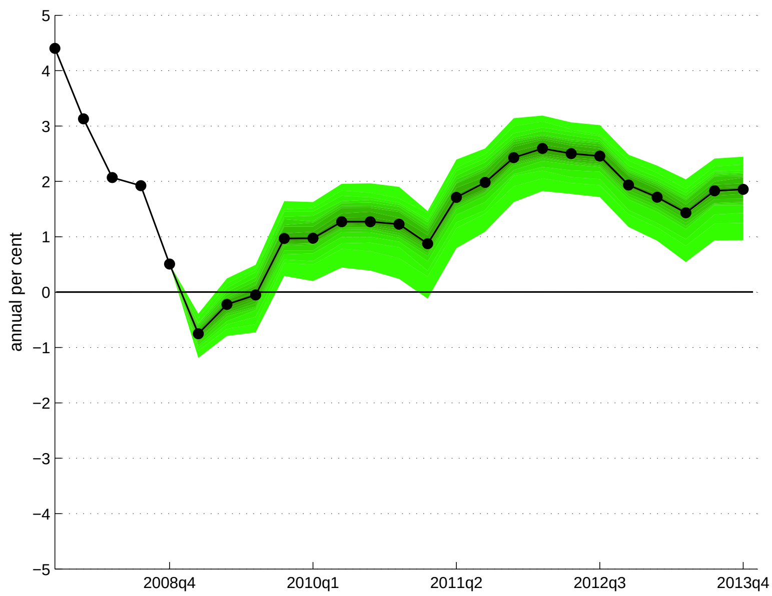


Figure 5: Shadow Federal Funds Rate



Federal Reserve is monitoring the size and composition of its balance sheet and will make adjustments to its credit and liquidity programs as warranted."

In summary, our results suggest that the expected duration for which the federal funds rate will be maintained at zero typically is around 3 quarters in the zero lower bound regime – the average estimated duration in most quarters is close to 3 quarters, although there is considerable posterior mass on longer durations. The shift to calendar-based forward guidance is estimated to have increased the expected duration of the low interest rate policy. Our results suggest that the Federal Reserve's forward guidance is best characterized as a commitment to follow more expansionary policy than what would be historically normal rather than as a simple acknowledgment of the zero lower bound constraint on interest rates holding in the future.

5.2 Structural Parameters

As mentioned above, we estimate the structural parameters jointly with the sequence of expected durations for the zero lower bound. Relative to previous studies, we also extend the sample to include observations from the zero lower regime. Because of these two reasons, the estimates of the structural parameters may be different from those previously found. Table 2 and Figures 6 show the posterior estimates of the structural parameters, which are broadly in line with previous findings.

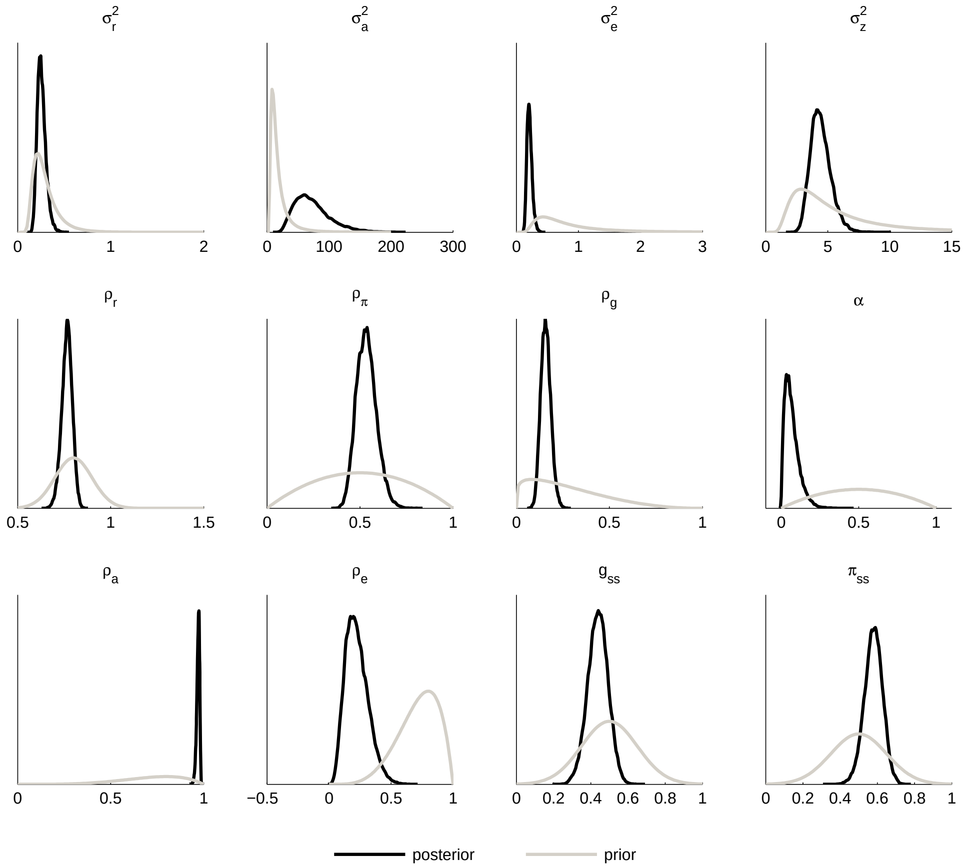
Table 2: Structural Parameters Posterior Estimates†

Parameter	Posterior Mode	Posterior Mean	90 percent Credibility Interval		Prior Distribution	Prior Mean
Variances of the innovations to the shocks						
σ_r^2	0.25	0.26	0.19	0.34	Inv. Gamma (6,1.5)	0.30
σ_a^2	61.42	73.26	35.47	129.20	Inv. Gamma (2.5,30)	20.00
σ_e^2	0.20	0.21	0.15	0.29	Inv. Gamma (2.5,1.5)	1.00
σ_z^2	4.11	4.43	3.24	5.89	Inv. Gamma (2.5,10)	6.67
Structural Parameters						
ρ_r	0.76	0.77	0.72	0.81	Normal (0.8,0.1)	0.80
ρ_π	0.54	0.54	0.45	0.63	Beta (2,2)	0.50
ρ_g	0.16	0.16	0.12	0.21	Beta (1.2,3.2)	0.27
α	0.03	0.07	0.01	0.16	Beta (2,2)	0.50
ρ_a	0.97	0.97	0.96	0.98	Beta (5,2)	0.71
ρ_e	0.17	0.22	0.09	0.39	Beta (5,2)	0.71
g	0.44	0.44	0.35	0.53	Normal (0.5,0.15)	0.50
π	0.59	0.58	0.50	0.66	Normal (0.5,0.15)	0.50

† Moments multiplied by 10^{-5}.

Considering first monetary policy, the mean of the posterior distributions of the Taylor rule parameters are broadly similar to the prior means, although the rule responds slightly more aggressively to inflation (and less to output growth). The mean posterior estimate of the standard deviation of the monetary policy shock is approximately 50 basis points.

Figure 6: Structural Parameter: Prior and Posterior Densities



The posterior of α suggests that price-setting is predominantly forward looking, consistent with Ireland [2004] and Galí and Gertler [1999]. ω, which governs the Frisch elasticity of labour supply, is not identified separately from the variance of the demand shock, and consequently it is calibrated to 0.1, similar to the estimates obtained by Ireland [2004].

The preference shock is found to be highly persistent, which was also found by Ireland [2004]. The standard deviation of the innovation to the preference shock is large and imprecisely estimated. The cost-push shock, alternatively, is found to be considerably less persistent – the mean of its posterior is less than 0.5. Our sample, however, starts after the Volcker disinflation and excludes the 1970's where cost-push shocks are likely to have been at work.

6 Sensitivity Analysis

6.1 An Informative Prior on the Expected Duration

One of the advantages of the methods developed in this paper is that they allow us to directly estimate, and therefore place priors on, the sequences of expected durations of the zero lower bound policy. In the results presented above an uninformative prior was used. We now instead use the prior described in Section 4 with *peak* set at 0.4 and *width* at 5. This prior is motivated by the idea that the expected duration should decline each period, all else equal, and that large changes in the expected durations from quarter to quarter are less likely than small changes. It is apparent from the results in Table 3 that the informative prior tends to reduce the estimates of the expected duration of the zero lower bound policy. This is not surprising, given the estimates from the baseline model and that the prior effectively penalizes large changes in the expected duration. The introduction of the calendar-based forward guidance still results in an increase in the mean estimate of the expected duration, but the increase is more muted.

6.2 With Yield Curve Data

Our second sensitivity analysis uses yield curve information to inform our estimates of the expected durations. We add yields at different maturities as observable variables to the benchmark model. To do this we extend the set of first order conditions to allow for an explicit consideration of long-term nominal interest rates. In the model the expectations hypothesis holds and implies:

$$\hat{r}_{j,t} = \frac{1}{j}\mathbb{E}_t\left(\hat{r}_t + \hat{r}_{t+1} + \cdots + \hat{r}_{t+j-1}\right) \quad \text{for} \quad j = 2, 3, ..., m \tag{21}$$

There is no feedback from these longer yields to the rest of the model.

Drawing on Graeve et al. [2009], we relate the model-implied yields, $\hat{r}_{j,t}$, to observed ones, $r_{j,t}$, as follows:

$$r_{j,t} = \hat{r}_{j,t} + r + c_j + \eta_t + \varepsilon_{j,t} \quad \text{for} \quad j = 2, 3, ..., m \tag{22}$$

$$\eta_t = \rho_\eta \eta_{t-1} + \varepsilon_{\eta,t} \tag{23}$$

where r is the steady state of the one-period nominal interest rate and $r + c_j$ is the steady state of the observed yield with j periods to maturity.[19] $\varepsilon_{j,t}$ is an idiosyncratic *i.i.d.* shock to the observed yield of j periods to maturity while the persistence 'term premium' shock, η_t, is common to all observed maturities.[20]

[19] We add to the observed variables the six-month, one-year and two-year Treasury constant maturity rates ($r_t^{2\ obs}$, $r_t^{4\ obs}$ and $r_t^{8\ obs}$). These data were also obtained from the FRED database of the St. Louis Federal Reserve, and their mnemonics are: DGS2, DGS1 and DGS6MO.

[20] Graeve et al. [2009] do not allow for persistence in the measurement equation as we do with η_t but allow for correlation between the idiosyncratic measurement errors. That is, they treat term premia as a constant and measurement error, with the measurement error uncorrelated across time, but correlated across maturities.

Table 3: Sensitivity Analysis

	Expected Duration (qtrs)								
	Baseline Model			Informative Prior			Using Yield Curve Data		
Quarter	Mode	Mean	Std. Dev.	Mode	Mean	Std. Dev.	Mode	Mean	Std. Dev.
2009Q1	1	2.37	1.55	1	1.93	1.15	1	3.25	2.24
2009Q2	3	3.81	2.17	1	2.17	1.19	2	4.06	2.63
2009Q3	1	2.92	1.86	1	1.84	1.05	1	3.28	2.34
2009Q4	3	3.84	2.04	2	2.20	1.09	2	4.34	2.49
2010Q1	1	2.70	1.69	1	1.73	0.92	1	3.23	2.22
2010Q2	1	2.81	1.68	1	1.75	0.94	1	3.43	2.32
2010Q3	1	2.64	1.62	1	1.73	0.89	1	3.85	2.44
2010Q4	1	2.58	1.55	1	1.68	0.83	1	3.81	2.43
2011Q1	1	2.69	1.67	1	1.64	0.86	1	3.59	2.52
2011Q2	3	3.47	1.80	1	1.97	1.01	2	4.53	2.63
2011Q3	2	2.96	1.73	1	1.72	0.90	2	4.32	2.66
2011Q4	2	2.77	1.57	1	1.73	0.89	3	4.37	2.54
2012Q1	2	3.02	1.67	1	1.80	0.88	2	4.77	2.69
2012Q2	2	2.79	1.66	1	1.70	0.87	1	4.18	2.67
2012Q3	2	2.73	1.56	1	1.73	0.87	1	4.09	2.66
2012Q4	1	2.75	1.68	1	1.66	0.86	1	3.81	2.66
2013Q1	1	2.95	1.82	1	1.73	0.90	1	3.75	2.71
2013Q2	1	2.53	1.61	1	1.65	0.85	1	3.59	2.57
2013Q3	2	3.07	1.79	1	1.88	0.98	1	4.07	2.76
2013Q4	1	2.71	1.79	1	1.78	1.00	1	3.84	2.69

The equations that relate model yields to observed ones contain additional parameters. We place loose priors on the constants, c_j, with higher means at longer maturities. For the persistent component of the term premia, ρ_η we use the same prior as for the structural shocks. A loose prior is placed on the standard deviations of the innovations.

The results presented in Table 3 suggest that if yield curve data are used in estimation, the mean estimates of the expected duration increases. The variability of the estimates also increases slightly. Nevertheless, the estimates are qualitatively similar to before, increasing in mid 2011.

The persistent component of the term premia displays considerable autocorrelation, with the mean of the posterior of ρ_s approximately 0.8 (see Appendix B). More surprising is that the mean estimate of the average term premia for the six-month bond is negative, with the posterior for its constant term, c_2, having considerable mass at negative values. Alternatively, the mean of the posterior estimates for the longer bonds are positive and increase with maturity, as expected, although they are estimated less precisely.

The posterior for the structural parameters is not independent from that of the expected duration of the zero lower bound policy, and consequently it changes either when an informative prior is used or the yield curve data are included from estimation, although the changes are not large (see Appendix B).

Overall, it appears that the main result above, namely that the estimated expected duration that the zero lower bound will prevail is generally quite short, is robust and does not change qualitatively when the prior on the expected durations of the zero lower bound policy is altered so as to be informative or whether yield curve data are used in estimation.

6.3 The Smets Wouters (2007) Model

Finally, we show that the estimation methods developed in this paper are feasible in larger DSGE models, such as those typically used at central banks. The analysis, however, goes beyond this and is of some independent interest because there are more observable variables used in estimation, more frictions incorporated into the model, and monetary policy responds to a model-consistent measure of the output gap. The Smets and Wouters [2007] model, which we utilise, is an influential medium-scale DSGE model. The model features nominal rigidities, namely Calvo pricing, in both intermediate goods prices and wages, as well as inflation indexation. It also includes other features, such as external habits in consumption and adjustment costs on the rate of change of investment. A summary of the main equations of the model is provided in Appendix C. Following Smets and Wouters [2007], seven observed variables were used in estimation: the log difference of real GDP, consumption, investment (all per capita) and wages, the log level of hours worked, inflation and the federal funds rate.[21]

The estimated expected durations obtained from the Smets and Wouters [2007] model are akin to those from Ireland [2004] in that they are short, with the mode typically 1 quarter (Table 4).[22] However, the mode does not increase when calendar-based forward guidance was introduced. To investigate this further, we re-estimated Smets and Wouters [2007], but constrained the observed variables during the zero lower bound to inflation and output growth alone, to make it comparable to the observables in Ireland [2004]. The mode of the expected durations obtained in this case generally increases in the same quarters as those from the Ireland [2004] model, albeit more sharply.[23] This suggests that the additional data used in estimation, such as the labor market data, are at least partially responsible for altering the estimates. The different model structure also matters, as the distribution of the expected duration in all quarters has more mass at longer durations, and consequently the mean expected duration is longer (compare Figures 7 and 4).

[21]The data are defined further in Appendix C.

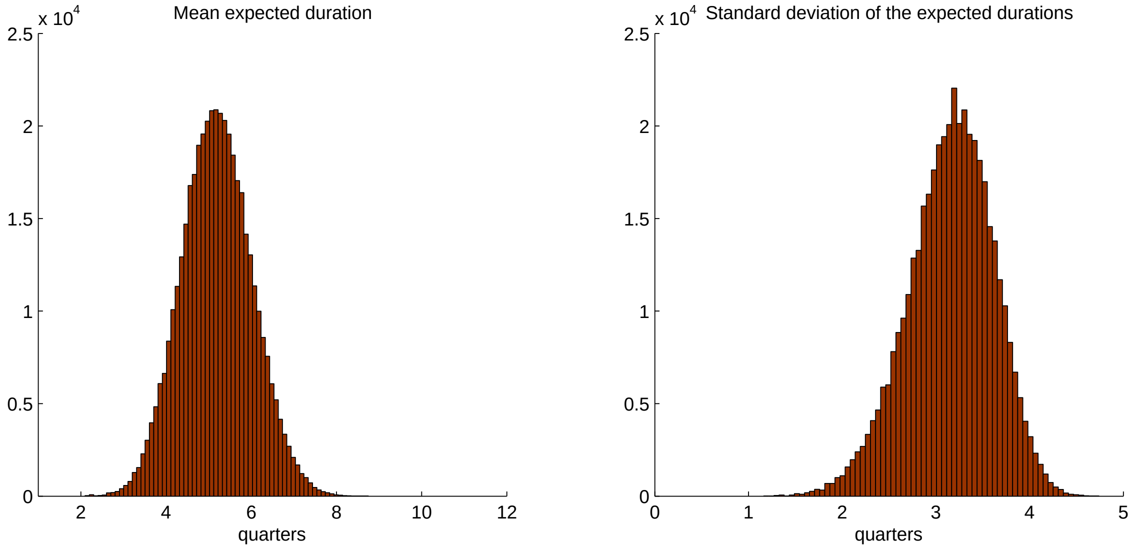
[22]The posteriors of the structural parameters and variances of the shocks are reported in Appendix C.

[23]These additional results are available on request.

Table 4: Estimated Expected Duration of the Zero Lower Bound From the Smets Wouters (2007) Model

	Expected Duration (qtrs)		
Quarter	Mode	Mean	Std. Dev.
2009Q1	2	5.46	3.18
2009Q2	1	5.45	3.29
2009Q3	1	6.54	3.54
2009Q4	2	4.73	2.89
2010Q1	1	4.75	3.11
2010Q2	1	5.84	3.43
2010Q3	1	4.37	2.78
2010Q4	1	5.38	3.33
2011Q1	1	4.72	2.84
2011Q2	1	4.77	3.02
2011Q3	1	4.86	3.07
2011Q4	1	5.27	3.37
2012Q1	1	4.83	3.05
2012Q2	1	4.99	3.13
2012Q3	2	4.97	3.11
2012Q4	1	6.28	3.37
2013Q1	1	4.90	3.07
2013Q2	1	5.31	3.32
2013Q3	1	5.27	3.26
2013Q4	1	5.19	3.17

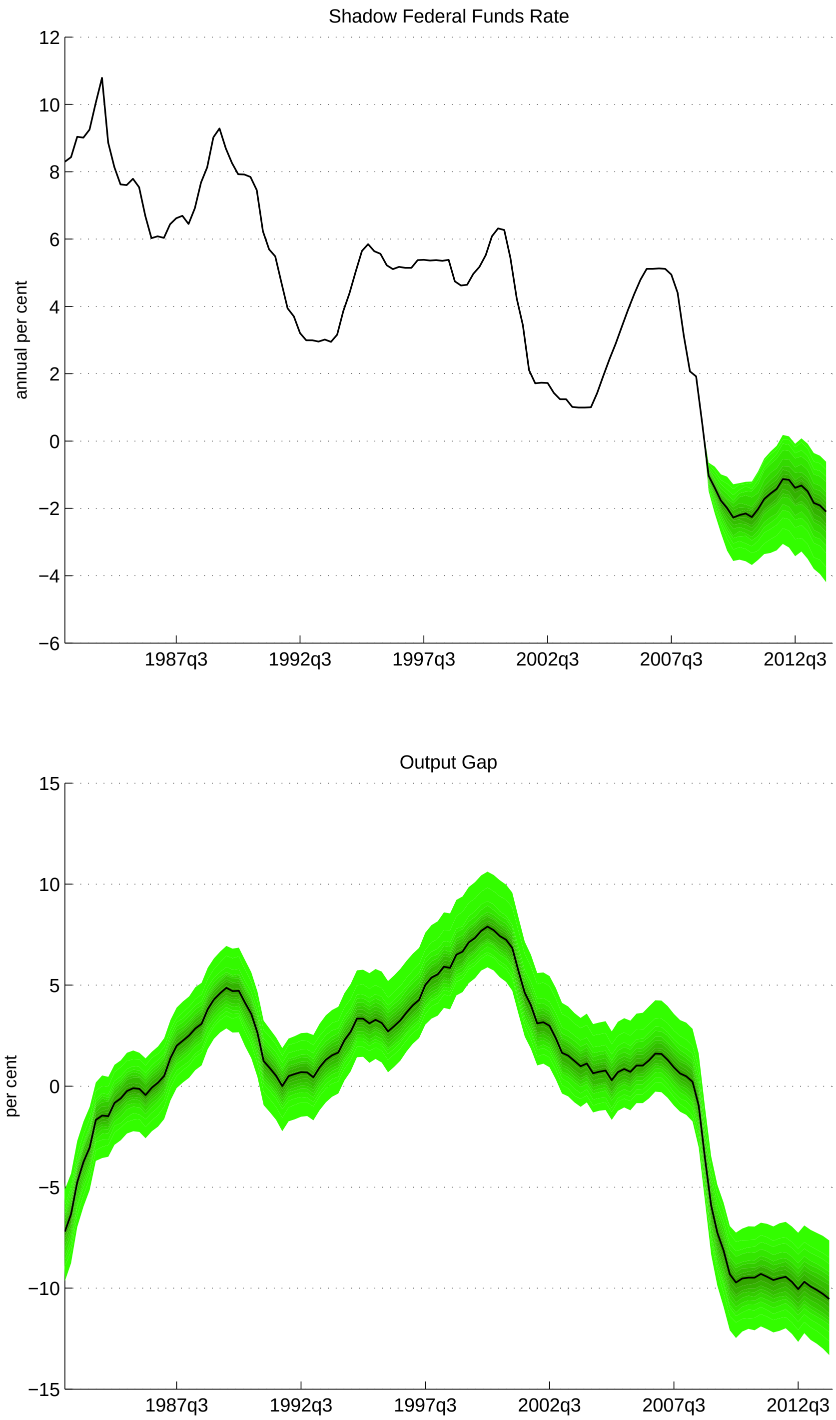
Figure 7: Posterior Distributions: Smets-Wouters Model



Notably, the shadow rate obtained from the Smets and Wouters [2007] model (Figure 8) is negative throughout the zero lower bound period, which is in contrast to the brief period in 2009 suggested by our baseline results (Figure 5). This difference mainly reflects the fact that the estimated Taylor rule in the Smets and Wouters [2007] model includes a sizable reaction to the *level* of the flexible-price output gap (see Appendix C for the posterior estimates of ψ_2, the coefficient on the output gap). This measure of the output gap for the United States has been substantially negative - around 10 per cent - during the zero lower bound period as Figure 8 shows. A policy rule that responds to growth rates only, and not to the level of the output gap, would lead to positive shadow rates in the Smets and Wouters [2007] model.[24] Although we estimate an increase in the shadow rate over 2011, akin to our baseline results, it is less pronounced. Overall, the negative shadow rate, the longer average expected duration, and the lack of a clear pick up in the expected duration in mid 2011 suggest that changes in the expected duration in the Smets and Wouters [2007] model could reflect changes in expectations of how long the zero lower bound constraint will bind in the future rather than necessarily reflecting the impact of forward guidance.

[24]Another factor may be that the Smets and Wouters [2007] model also includes hours worked as an observable variable, which has been subdued following the financial crisis, reflecting the so-called "jobless recovery".

Figure 8: Shadow Federal Funds Rate and Output Gap: Smets-Wouters Model



7 Conclusions

The Great Recession has had important ramifications for the implementation of monetary policy. Facing the zero lower bound on nominal interest rates, many central banks, including the Federal Reserve, have engaged in other policies such as forward guidance in an attempt to stimulate the economy. The contribution of this paper is to demonstrate how a New Keynesian model can be estimated for an economy in which forward guidance has been implemented.

Our approach builds on Kulish and Pagan [2012], which developed a method for solving DSGE models with structural change that yields a solution as a time-varying VAR process, a form which is convenient for estimation. The solution of Kulish and Pagan [2012] also accommodates private agents' expectations about structural change to differ from a policymaker's announcement. In this paper, we cast this approach in a Bayesian framework and in particular show how the expected duration of the zero interest rate policy can be estimated so as to shed light on the effectiveness of forward guidance about the future path of the policy rate. Further, we demonstrate how information from the bond market, in addition to the macroeconomic variables which are typically used in the estimation of DSGE models, may be used to inform these estimates of the expected durations of the policy.

We apply our approach to a variant of the New Keynesian model of Ireland [2004], jointly estimating the structural parameters and the expected durations. We find that with an uninformative prior on the expected durations typically the zero interest rate policy is expected to persist for an average of 3 more quarters during the zero lower bound regime. These results are qualitatively unchanged if an informative prior on the expected durations is used or additional data from the yield curve data are included. These results are broadly in line with those obtained obtained from several other approaches, such as examining the forecasts of professional forecasters Swanson and Williams [2012] or affine term structure models Bauer and Rudebusch [2013]. The estimates of the average expected duration from the larger Smets and Wouters [2007] model were moderately longer and typically around 5 quarters.

For our benchmark model, the shift to calendar-based forward guidance by the FOMC in mid 2011 is estimated to have been associated with a marked increase in the expected duration for which the zero interest rate policy will be maintained. Notably, this increase in expected duration occurs even though the shadow Federal Funds rate also increases, suggesting that the Federal Reserve's forward guidance is best characterized as a commitment to follow more expansionary policy rather than the zero lower bound constraint holding for longer due to shocks hitting the economy. Using the Smets and Wouters [2007] model, the interpretation is less clear, as the shadow rate is estimated to be persistently negative, with changes in the expected duration possibly reflecting revisions to expectations about how long the zero lower bound will bind.

In summary, we propose a method of estimating DSGE models when the policy rate is at the zero lower bound during part of the sample and policymakers are conducting forward guidance about the future path of policy. Our method takes the zero lower bound explicitly into account and allows inferences to be made about the expected duration that such a policy will be maintained.

Appendix A: Kalman Filter with Forward Guidance

The zero lower bound eliminates variation in the one-step ahead prediction error of the federal funds rate. This lack of variation translates into a singularity in the Kalman filter equations unless the federal funds rate is removed as an observable variable in the zero lower bound sub-sample. We therefore drop the federal funds rate, but not longer maturity rates, as an observed variable during the period when the zero lower bound binds. The implementation is similar to Harvey [1989]'s approach for handling missing observations.

For the sub-sample prior to the zero lower bound there are n_z observable variables and the observation equation is Equation (19):

$$z_t = Hy_t + v_t,$$

where H is a $n_z \times n$ matrix that links observables to the state, y_t, and, v_t is a vector of measurement errors such that $v_t \sim \mathcal{N}(0, V)$. The zero lower bound binds from 2009:1 until the end of our sample. During the zero lower bound, the observation equation becomes

$$\bar{z}_t = \bar{H}y_t + \bar{v}_t,$$

where W is a $(n_z - 1) \times n_z$ matrix, in our case $[\mathbf{0}_{3\times 1}, I_{3\times 3}]$. We define $\bar{z}_t \equiv Wz_t$, $\bar{H} \equiv WH$ and $\bar{v}_t \equiv Wv_t$. Pre-multiplying by W removes the first observable variable, the policy rate in our case and changes the dimensions of the matrices in the Kalman Filter equations. The state equation during the zero lower bound sub-sample is given by Equation (14):

$$y_t = Q_t y_{t-1} + G_t \varepsilon_t.$$

Taken together, Equations (19) or 20 and Equations (12) and (14) as applicable constitute a state-space system to which the Kalman filter can be used to construct the likelihood function.

Denoting forecasts with a superscript $\hat{}$, given $\hat{y}_{t|t-1}$ and $\Sigma_{t|t-1}$ (the variance-covariance matrix of prediction error of $y_{t|t-1}$), the forecast of the observed variables becomes $\hat{\bar{z}}_{t|t-1} = \bar{H}\hat{y}_{t|t-1}$, and their forecast error, $u_t \equiv \bar{z}_t - W\hat{\bar{z}}_{t|t-1} = \bar{H}(y_t - \hat{y}_{t|t-1}) + \bar{v}_t$, with variance-covariance matrix $\hat{H}\Sigma_{t|t-1}\hat{H}' + V$.

The updated values for the model variables are then given by

$$\hat{y}_{t|t} = \hat{y}_{t|t-1} + \Sigma_{t|t-1}\bar{H}' \left(\bar{H}\Sigma_{t|t-1}\bar{H}' + \bar{V}\right)^{-1} + u_t,$$

where $\bar{V} \equiv WV$.

The forecast values of the model variables are

$$\hat{y}_{t+1|t} = Q_{t+1}y_t + \bar{K}_t u_t,$$

where $\bar{K}_t \equiv Q_{t+1}\Sigma_{t|t-1}\bar{H}' \left(\bar{H}\Sigma_{t|t-1}\bar{H}' + \bar{V}\right)^{-1}$ is the Kalman gain matrix.

The recursion for $\Sigma_{t+1|t}$ is

$$\begin{aligned}\Sigma_{t+1|t} &= G_{t+1}\Omega G'_{t+1} + \\ &\quad Q_{t+1}\left(\Sigma_{t|t-1}\Sigma_{t|t-1}\bar{H}' \left(\bar{H}\Sigma_{t|t-1}\bar{H}' + \bar{V}\right)^{-1} \bar{H}\Sigma_{t|t-1}\right) Q'_{t+1}.\end{aligned}$$

Finally, the log-likelihood function for 2009:1 onwards, $\bar{\mathcal{L}}$ is

$$\begin{aligned}\bar{\mathcal{L}} &= -\left(\frac{n_z L}{2}\right)\ln(2\pi) - \frac{1}{2}\sum_t \ln\det\left(\bar{H}\Sigma_{t|t-1}\bar{H}' + \bar{V}\right) \\ &\quad -\frac{1}{2}\sum_t u_t'\left(\bar{H}\Sigma_{t|t-1}\bar{H}' + \bar{V}\right)^{-1} u_t\end{aligned}$$

where L is the number of quarters of the zero lower bound sub-sample. For the period prior to the zero lower bound the construction of the likelihood, $\mathcal{L}$, is standard; see Kulish and Pagan [2012]. The log-likelihood for the entire sample, then is $\mathcal{L} + \bar{\mathcal{L}}$.

Appendix B: Additional Results

B.1 Alternative Estimates of the Structural Parameters

Table 5 shows moments from the posterior of the structural parameters when an informative prior is used on the expected duration or when yield curve data are included as observed variables in estimation.

Table 5: Structural Parameters Posterior Estimates†

	Baseline Model				Informative Prior				Yield Curve Data			
			90 percent				90 percent				90 percent	
Parameter	Mode	Mean	HPD interval		Mode	Mean	HPD interval		Mode	Mean	HPD interval	
Variances of the innovations to the shocks												
σ_r^2	0.25	0.26	0.19	0.34	0.25	0.26	0.19	0.35	0.26	0.27	0.20	0.35
σ_a^2	61.42	73.26	35.47	129.20	41.76	54.55	25.88	101.73	187.63	197.12	125.87	291.97
σ_e^2	0.20	0.21	0.15	0.29	0.19	0.21	0.15	0.28	0.25	0.27	0.20	0.36
σ_z^2	4.11	4.43	3.24	5.89	4.30	4.58	3.39	6.00	3.86	4.17	3.00	5.63
σ_η^2	–	–	–	–	–	–	–	–	0.04	0.04	0.03	0.05
σ_2^2	–	–	–	–	–	–	–	–	0.02	0.02	0.02	0.03
σ_4^2	–	–	–	–	–	–	–	–	0.02	0.02	0.02	0.03
σ_8^2	–	–	–	–	–	–	–	–	0.03	0.03	0.03	0.04
Structural Parameters												
ρ_r	0.76	0.77	0.72	0.81	0.76	0.76	0.71	0.81	0.80	0.79	0.75	0.83
ρ_π	0.54	0.54	0.45	0.63	0.51	0.53	0.45	0.62	0.60	0.59	0.51	0.68
ρ_g	0.16	0.16	0.12	0.21	0.16	0.16	0.12	0.21	0.13	0.13	0.09	0.17
α	0.03	0.07	0.01	0.16	0.04	0.08	0.01	0.18	0.03	0.07	0.01	0.15
ρ_a	0.97	0.97	0.96	0.98	0.97	0.96	0.95	0.98	0.98	0.98	0.98	0.99
ρ_e	0.17	0.22	0.09	0.39	0.19	0.22	0.09	0.38	0.44	0.45	0.33	0.56
g	0.44	0.44	0.35	0.53	0.45	0.45	0.37	0.54	0.51	0.51	0.43	0.60
π	0.59	0.58	0.50	0.66	0.58	0.58	0.50	0.66	0.59	0.59	0.52	0.65
c_2	–	–	–	–	–	–	–	–	-0.20	-0.19	-0.59	0.20
c_4	–	–	–	–	–	–	–	–	0.12	0.15	-0.26	0.55
c_8	–	–	–	–	–	–	–	–	0.99	0.96	0.52	1.40
ρ_s	–	–	–	–	–	–	–	–	0.77	0.77	0.66	0.88

† Moments multiplied by 10^{-5}. HPD denotes highest probability density.

Appendix C: The Smets and Wouters (2007) Model

Summary of the Equations

This summary of the equations of the Smets and Wouters [2007] model draws on Pagan and Robinson [2014]. We provide the equations for the sticky price and wage economy; the equations for the flexible price economy (as monetary policy reacts to the the output gap defined using the flex-price level of output), the shock processes and some steady-state parameters are available from their code.

Technology in the Smets and Wouters [2007] model includes a deterministic trend, and consequently it is necessary to de-trend some of the variables so as a steady state exists (these generally are denoted in lower case). The variables in he model are: w_t real wages (faced by firms); w_t^h the real wages received by households; u_t capacity utilisation; r_t^K rental rate on capital, k_t^s the flow of capital services; k_t the capital stock; i_t investment; q_t Tobin's Q; c_t consumption; mc_t real marginal costs; π_t goods inflation; l_t labor; r_t the policy rate; y_t output. The flexible-price level of output is denoted with a superscript f.

There are seven shocks: a_t technology; b_t risk premium; g_t government expenditure; μ_t investment-specific technology; ε_t^r monetary policy; λ_t^p price mark-up; λ_t^w wage mark-up. These shocks follow AR(1) processes, although the mark-up shocks are ARMA(1,1), and government spending also reacts to the innovation in the technology shock.

The parameters include: δ is the rate of depreciation; $(\phi_w - 1)$ and $(\phi_p - 1)$ are the mark ups for labour and goods prices; ϵ_w and ϵ_p are the curvature of the Kimball aggregator for wages and prices respectively; φ governs investment adjustment costs; σ_c governs the intertemporal elasticity of substitution; h is the degree of external habits; σ_l the elasticity of labour supply; ξ_w and ξ_p are the Calvo parameters and ι_w and ι_p the indexation parameters for wages and goods prices; α determines the share of capital services in production; ρ_R and $\psi_i,\ i \in [1, \ldots 3]$, are parameters in the Taylor rule; γ the rate of deterministic technology growth. Additionally $\bar{\beta} \equiv \beta\gamma^{-\sigma_c}$. The structural equations of the model are:

Goods Phillips curve

$$\hat{\pi}_t = \frac{1}{1+\bar{\beta}\gamma\iota_p}\left(\bar{\beta}\gamma E_t\left(\hat{\pi}_{t+1}\right) + \iota_p\hat{\pi}_{t-1} + \frac{\left(1-\xi_p\right)\left(1-\bar{\beta}\gamma\xi_p\right)}{\xi_p\left(\left(\phi_p-1\right)\epsilon_p+1\right)}\hat{mc}_t\right) + \hat{\lambda}_{p,t}$$

Real marginal costs

$$\hat{mc}_t = \alpha\hat{r}_t^K + (1-\alpha)\,\hat{w}_t - \hat{a}_t$$

Wage Phillips curve

$$\begin{aligned}\hat{w}_t \quad = \quad & \tfrac{1}{1+\bar{\beta}\gamma}\hat{w}_{t-1} + \tfrac{\bar{\beta}\gamma}{1+\bar{\beta}\gamma}E_t\left(\hat{w}_{t+1}\right) + \tfrac{\iota_w}{1+\bar{\beta}\gamma}\hat{\pi}_{t-1} - \tfrac{1+\bar{\beta}\gamma\iota_w}{1+\bar{\beta}\gamma}\hat{\pi}_t \quad + \quad \tfrac{\bar{\beta}\gamma}{1+\bar{\beta}\gamma}E_t\left(\hat{\pi}_{t+1}\right) \quad + \\ & \tfrac{\left(1-\xi_w\right)\left(1-\bar{\beta}\gamma\xi_w\right)}{\left(\left(1+\bar{\beta}\gamma\right)\xi_w\right)\left(\left(\phi_w-1\right)\epsilon_w+1\right)}\left(\sigma_l\hat{l}_t + \tfrac{1}{1-\frac{h}{\gamma}}\hat{c}_t - \tfrac{\frac{h}{\gamma}}{1-\frac{h}{\gamma}}\hat{c}_{t-1} - \hat{w}_t\right) \quad + \quad \hat{\lambda}_{w,t}\end{aligned}$$

Return on capital

$$\hat{r}_t^K = \hat{w}_t + \hat{l}_t - \hat{k}_t^s$$

Tobin's Q

$$\hat{q}_t = -\left(\hat{r}_t - E_t\left(\hat{\pi}_{t+1}\right)\right) + \frac{\sigma_c(1+\frac{h}{\gamma})}{(1-\frac{h}{\gamma})}\hat{b}_t + \frac{\bar{R^K}}{\bar{R^K}+(1-\delta)}E_t\hat{r}^K_{t+1} + \frac{1-\delta}{\bar{R^K}+(1-\delta)}E_t\left(\hat{q}_{t+1}\right)$$

Investment

$$\hat{i}_t = \frac{1}{1+\bar{\beta}\gamma}\left(\hat{i}_{t-1} + \bar{\beta}\gamma E_t\left(\hat{i}_{t+1}\right) + \frac{1}{\gamma^2\varphi}\hat{q}_t\right) + \hat{\mu}_t$$

Evolution of the capital stock

$$\hat{k}_t = \left(1-\frac{\bar{I}}{K}\right)\hat{k}_{t-1} + \frac{\bar{I}}{K}\hat{i}_t + \frac{\bar{I}}{K}\left(\gamma^2\varphi\right)\hat{\mu}_t.$$

Capacity utilisation

$$\hat{u}_t = \frac{1-\psi}{\psi}\hat{r}^K_t$$

Capital services

$$\hat{k}^s_t = \hat{k}_{t-1} + \hat{u}_t$$

Euler equation for consumption

$$\hat{c}_t = \frac{\frac{h}{\gamma}}{1+\frac{h}{\gamma}}\hat{c}_{t-1} + \frac{1}{1+\frac{h}{\gamma}}E_t\left(\hat{c}_{t+1}\right) + \frac{(\sigma_c-1)\frac{\bar{W^h}L}{C}}{\sigma_c\left(1+\frac{h}{\gamma}\right)}\left(\hat{l}_t - E_t\left(\hat{l}_{t+1}\right)\right) - \frac{1-\frac{h}{\gamma}}{\sigma_c\left(1+\frac{h}{\gamma}\right)}\left(\hat{r}_t - E_t\left(\hat{\pi}_{t+1}\right)\right) + \hat{b}_t$$

Accounting identity

$$\hat{y}_t = \frac{\bar{C}}{Y}\hat{c}_t + \frac{\bar{I}}{Y}\hat{i}_t + \hat{g}_t + \bar{R^K}\frac{\bar{K}}{Y}\hat{u}_t$$

Supply

$$\hat{y}_t = \phi_p\left(\alpha\hat{k}^s_t + (1-\alpha)\hat{l}_t + \hat{a}_t\right)$$

Taylor rule

$$\hat{r}_t = \rho_R\hat{r}_{t-1} + (1-\rho_R)\left(\psi_1\hat{\pi}_t + \psi_2\left(\hat{y}_t - \hat{y}^f_t\right)\right) + \psi_3\left(\hat{y}_t - \hat{y}_{t-1} - \left(\hat{y}^f_t - \hat{y}^f_{t-1}\right)\right) + \hat{\varepsilon}^r_t$$

Posterior Estimates

The priors used for the shock variances and the structural parameters are as in Smets and Wouters [2007]. In addition to those defined above several further parameters are estimated, namely: the standard deviations of the innovations, to the shocks (σ); the autoregressive terms of the shocks (ρ); the response of government expenditure to the technology shock (ρ_{ga}) and the constants in the measurement equations for labour $\bar{l}$ and inflation $\bar{\pi}$.

Table 6: Posterior Estimates of the Smets Wouters (2007) Model†

Parameter	Posterior Mode	Posterior Mean	90 per cent C. I.	
Standard Deviations of the Innovations to the Shocks				
σ_a	0.41	0.41	0.37	0.46
σ_b	0.05	0.05	0.04	0.07
σ_g	0.38	0.39	0.35	0.43
σ_{qs}	0.32	0.33	0.25	0.42
σ_m	0.11	0.12	0.10	0.13
σ^p	0.10	0.10	0.09	0.12
σ^w	0.41	0.42	0.36	0.48
Parameters of the Shock Processes				
ρ_a	0.95	0.94	0.90	0.98
ρ_b	0.95	0.95	0.92	0.97
ρ_g	0.97	0.96	0.94	0.99
ρ_l	0.66	0.65	0.53	0.77
ρ_r	0.45	0.45	0.31	0.61
ρ_p	0.78	0.73	0.53	0.88
ρ_w	0.98	0.97	0.94	0.99
μ_p	0.70	0.64	0.40	0.83
μ_w	0.91	0.86	0.74	0.95
ρ_{ga}	0.39	0.40	0.26	0.54
Structural Parameters				
φ	6.29	6.41	4.82	8.16
σ_c	0.91	0.92	0.76	1.10
h	0.62	0.61	0.52	0.69
ξ_w	0.67	0.65	0.49	0.79
σ_l	1.46	1.60	0.79	2.61
ξ_p	0.91	0.90	0.87	0.92
ι_w	0.46	0.46	0.22	0.70
ι_p	0.14	0.17	0.07	0.30
ψ	0.79	0.77	0.63	0.89
ϕ_p	1.39	1.39	1.27	1.52
ψ_1	1.97	1.95	1.59	2.28
ρ_r	0.84	0.83	0.80	0.87
ψ_2	0.10	0.12	0.07	0.18
ψ_3	0.11	0.12	0.08	0.16
$\bar{\pi}$	0.67	0.67	0.53	0.82
$\beta^{-1} - 1$	0.16	0.18	0.08	0.30
$\bar{l}$	0.17	0.31	-1.24	1.79
$\gamma - 1$	0.47	0.46	0.41	0.50
α	0.16	0.16	0.13	0.19

† Moments multiplied by 10^{-2}.

The data used in estimation closely follow Smets and Wouters [2007] and were mostly obtained from the FRED database of the Federal Reserve of St Louis. The consumption and investment measures were both constructed by deflating nominal measures (personal consumption expenditure, PCEC, and fixed private investment, FPI) by the GDP deflator (GDPDEF) and the civilian non-institutional population (CNP16OV). Hours worked was constructed by multiplying average weekly non-farm hours worked (PRS85006023) by the civilian population (CE16OV) and then normalizing by CNP16OV. The other series used were: real GDP (GDPC96), the Federal Funds rate (FEDFUNDS), nonfarm business hourly compensation (PRS85006103 from the BLS web site) deflated by the GDP deflator, which also was the price series used to construct the inflation measure.

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