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Examining Feedback, Momentum and  
Overreaction in National Equity Markets

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# **Examining Feedback, Momentum and Overreaction in National Equity Markets\***

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## **Abstract**

The matters of asset price feedback, momentum and overreaction are theoretically motivated by a series of papers in the behavioural finance field. These papers propose theoretical conditions and examples pursuant to which traditional pricing rationales are inapplicable and asset prices are influenced by characteristics such as feedback and momentum (Barberis, Shleifer, and Vishny, 1988; De Long et al., 1990a,b; Hong and Stein, 1999). There are few empirical models and results, however, regarding the existence of feedback and reactionary effects, and the manner in which feedback effects are distributed across pricing factors. This paper derives a model entailing both factor-specific feedback and momentum effects, and overreaction. The model is applied to a set of developed equity markets, with results indicating that the majority of the markets exhibit some form of corrective feedback.

## 1. Introduction

This paper considers the significance of price feedback, momentum and overreaction across equity markets. The subject matter is motivated by a series of papers in the behavioural finance field regarding positive feedback and reactionary trading (see, for example, De Long et al., 1990a,b; Daniel, Hirshleifer, and Subrahmanyam, 1998, 2001; Hong and Stein, 1999). The papers relax the assumption of rationality inherent in pricing approaches such as the Arbitrage Pricing Theory (APT) and consider extensions and modifications to rational pricing founded on characteristics injected into financial markets by ostensibly irrational or misinformed agents. There are few empirical frameworks and results, however, regarding the existence of feedback and reactionary effects across national equity markets, and the manner in which feedback effects are distributed across pricing factors.

The methodology adopted in this paper extends the theoretical focus of the feedback, momentum, and overreaction literature by defining and implementing a practical framework for investigating the empirical validity of a wide (but by no means exhaustive) form of common feedback, common momentum and idiosyncratic overreaction. The framework is generally applicable, although the focus of this paper is on national equity markets.

The assessment of market feedback, momentum, and overreaction is undertaken using an augmented rational pricing form. This approach allows for hypotheses that collapse to the rational paradigm in the absence of any feedback or reactionary effects. In the alternative scenario, evidence regarding either positive feedback or overreaction constitutes a rejection of the rational pricing form and the rationale that arbitrageurs sufficiently respond to price momentum and feedback in a corrective manner. In turn, the generalisation of the feedback structure to accommodate various pricing factors recognises the potential that financial agents may digest alternative sources of historical information in a disparate fashion.<sup>1</sup> Consequently, evidence in favour of positive feedback may be limited to a particular set of factors. In this paper, both global and

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<sup>1</sup> The theoretical literature is largely silent in this respect. Empirically, however, the allowance for restricting pricing anomalies to certain subsets of markets is useful given its acknowledgement of the intuitive possibility that a certain subset of markets may exhibit distinct pricing behavior (due, for example, to the greater proportion of speculative trading or lack of arbitrageurs in the subset).

regional factors are considered. Consequently, the extended pricing form is also used to retrieve information regarding the extent to which global, regional or idiosyncratic factors are responsible for deviation from rational pricing.

This paper is structured as follows. Section 2 describes the behavioural framework adopted to examine the market feedback, momentum and reactionary properties. Section 3 presents the empirical model associated with the behavioural framework, and proposes a set of relevant hypothesis tests for the model. The model results are presented and examined in Section 4, with the paper concluding in Section 5.

## 2. A behavioural framework

This section proposes a behavioural framework pursuant to which an empirical model may induce global, regional and country-specific feedback and momentum (if we restrict ourselves to a single market, an alternative country, industry, and asset-specific interpretation is also available). The framework is examined in the context of the relevant theoretically-focused behavioural finance literature and its properties are reviewed in terms of asset feedback and momentum theory. This section also discusses how aspects of feedback and momentum theory are effectively embedded into the adopted model.

Consider the following return structure:

$$E(r_{i,t} | I_{t-1}) = a_i' F_{t|t-1} + b_i' Z_{t|t-1} + c_i u_{i,t}, \quad (1)$$

$$Z_t = f(F_{\{t-1\}}), \quad (2)$$

$$u_{i,t} = r_{i,t-1} - a_i' F_{t-1|t-1} - b_i' Z_{t-1|t-1}, \quad (3)$$

$$F_{\{t-1\}} = F_1, F_2, \dots, F_{t-1}. \quad (4)$$

The first term on the right hand side of Eq. (1) represents the rational approach to pricing whereby expected returns depend on the asset's association with a contemporaneous pervasive component. The pervasive component may be the market portfolio (the conditional CAPM) or some set of common factors which may or may not include the market portfolio (such as the ICAPM or the conditional variant of the equilibrium APT).

The second term is a function of the fundamental information set up to  $F_{t-1}$  and therefore represents the determinants of historic fundamental asset prices. The term provides the opportunity for investors to dynamically incorporate revised or updated

historic fundamental information into the current price of the asset, thereby providing the capacity for fundamental feedback. In the model, investors observe a signal at time  $t-2$ , denoted by  $F_{t-1|t-2}$ , pursuant to which the price of an asset is partially or fully determined. In the next time period, the updated information set  $I_{t-1}$  is used to obtain the revised signal  $F_{t-1|t-1}$ . Any trading activity related to  $F_{t-1|t-1}$  takes place at the end of time  $t-1$  such that the effects of the revised signal are directly reflected in the  $t$ th pricing realization  $r_t$  via the second term in the expected return structure.

Assume that historic fundamental expectations are observed as warranting positive returns over the relevant historical period given knowledge of  $I_{t-1}$ . Assume, further, the existence of a family of positive fundamental feedback traders who extrapolate historic fundamental expectations into the future and adopt market positions in line with the sign of the historic fundamental expectation (viz. trading implied by  $b_i > 0$ ,  $df(F_{\{t-1\}})/dF_{\{t-1\}} > 0$ , for any differentiable function  $f$ ). In accordance with the traditional perspective or its variants (Friedman, 1953; Kyle, 1985) rational speculators counter the effects of mispriced situations thereby inducing an element of mean-reversion in the asset price. In this respect, rational speculators or arbitrageurs take advantage of the positive feedback behaviour by trading against its direction, thereby countering the effects of extrapolative fundamental expectations and guiding the asset price towards its fair value (therefore either  $b_i \rightarrow 0$  or  $df(F_{\{t-1\}})/dF_{\{t-1\}} \rightarrow 0$ ). Consider, however, a subset of rational speculators who seek to maximize utility by acquiring excess stock with the expectation of exploiting any positive feedback property. This antithetic position, relying on the non-negligible demand of positive feedback traders, argues that rational speculators trade in the direction of prices with the object of on-selling to feedback traders (De Long et al., 1990a). Daniel, Hirshleifer, and Subrahmanyam (1988, 2001) also suggest that updated information regarding a historic signal can generate a confidence surge causing prices to rise beyond rational levels.

The third term in the expected return structure represents the asset's pricing error for the immediately preceding pricing period. The term provides the capacity for idiosyncratic revision of the next-period price by reference to the observed pricing error

for the current period. The mispricing term is constructed as the deviation of the observed return from the expected fundamental price of the asset, and therefore operates as a measure of market overreaction or overconfidence. The existence of short-term overreaction, coupled with longer-term mean reversion is advocated in De Bondt and Thaler (1985, 1987). Consistent with the position advocated by Friedman (1953), rational speculators (fully or partially) correct the pricing deviation through contrarian trading such that  $c_i \leq 0$ . Consider, however, a non-negligible set of feedback traders who acquire on positive mispricing and sell in the converse situation. As is the case for the second term on the right hand side of Eq. (1), rational speculators recognising the demand of feedback traders may take excess positions in the direction of the overreaction. The positive feedback traders mentioned in relation to the second term differ, however, from the current set of (idiosyncratic) feedback traders in that their inducement to trade is guided by pricing deviation (or overreaction) rather than any extrapolation of the sign of historical fundamental expectations.

Consider two characteristic sets  $\mathcal{A}, \mathcal{B}$  where  $\mathcal{A}$  defines feedback traders and  $\mathcal{B}$  defines idiosyncratic feedback traders and the intersection of  $\mathcal{A}, \mathcal{B}$  is not necessarily empty. Type  $\mathcal{A}$  traders (viz. a trader belonging to  $\mathcal{A}$ ) take long positions in the asset pursuant to  $b_i' Z_{t|t-1} > 0, b_i > 0$  or short positions in the asset pursuant to  $b_i' Z_{t|t-1} < 0, b_i > 0$ . Type  $\mathcal{B}$  traders contemporaneously adopt short positions given  $c_i u_{i,t} < 0, c_i > 0$  or long positions in the case  $c_i u_{i,t} > 0, c_i > 0$ . Positive fundamental feedback traders belong to  $\mathcal{A}$  and trade in the expected direction of  $Z_t$ , while positive idiosyncratic feedback traders (or noise traders) belong to  $\mathcal{B}$  and trade in the direction of  $u_{i,t}$ . Arbitrageurs or rational speculators are typically assumed to take positions against feedback traders, resulting in  $b_i, c_i \rightarrow 0$ . They may, however, temporarily belong to either the first or second set depending on information asymmetry (for example, the distinction between rational speculators receiving private or public signals of various precision levels), rational heterogeneity in belief systems (choosing between two events neither of which is stochastically dominant given the relevant information set), heterogeneous preferences or some combination thereof. In this respect, some arbitrageurs (or rational

speculators) may behave as positive feedback or noise traders at various times such that the various subsets of the set of arbitrageurs may be associated with different investment schedules.

Contingent on the proposed trading environment, the result  $\tilde{\delta}_{i,t} = b'_i Z_{t|t-1} + c_i u_{i,t} = 0$ ,  $b_i, c_i \neq 0$ , suggests that net arbitrageurs are dynamically correcting the behaviour of positive feedback or noise traders. The result  $\tilde{\delta}_{i,t} > 0$  suggests net buying into information given by  $Z_t$  and  $u_t$  (with the obvious reversal for  $\tilde{\delta}_{i,t} < 0$ ). For  $b_i > 0, c_i > 0$  there is both fundamental feedback and idiosyncratic feedback and at least one of  $Z_t$  or  $u_t$  must be positive for the net buying scenario  $\tilde{\delta}_{i,t} > 0$  to hold. The preceding case is guided by the dominance of feedback traders or the failure of arbitrageurs in executing corrective trades. Arbitrageurs are net buyers, for example, if sufficient numbers of arbitrageurs are failing to adopt corrective positions and are therefore behaving as positive feedback traders or postponing trade until the information set takes a particular character. If  $Z_{t|t-1} > 0$ ,  $u_t < 0$  and  $\tilde{\delta}_{i,t} > 0$ , fundamental feedback outweighs idiosyncratic feedback as a consequence of either  $b_i > c_i$ ,  $|Z_{t|t-1}| > |u_{i,t}|$  or both. In the case of  $|Z_{t|t-1}| > |u_{i,t}|$ , the greater absolute expected value of  $Z_t$  is effectively taken as an indicator of precision of the sign of  $r_{i,t}$  by feedback traders.

The opposing situation  $b_i < 0, c_i < 0$  implies a trading environment where  $\tilde{\delta}_{i,t}$  is dominated by contrarian investment pursuant to the fundamental and idiosyncratic correction implied by the negative terms  $b_i, c_i$ . In the third scenario,  $b_i < 0, c_i > 0$ , there is both fundamental correction and idiosyncratic feedback. Dominance of either contrarian investment or idiosyncratic feedback is determined by the greater absolute of the two relevant terms,  $|b'_i Z_{t|t-1}|$ ,  $|c_i u_{i,t}|$ . Given offsetting terms,  $b'_i Z_{t|t-1} + c_i u_{i,t} = 0$ , contrarian investors (or rational speculators) cancel out the pricing impact of idiosyncratic feedback traders. The final scenario  $b_i > 0, c_i < 0$  also follows the logic of the third scenario subject to fundamental feedback (idiosyncratic correction) in the place of fundamental correction (idiosyncratic feedback).

The model advocates the corrective behaviour postulated by Friedman (1953) if the second and third terms imply mean reversion or behave in an off-setting manner. The former case requires  $b_i = 0$ ,  $c_i < 0$  or, for mild mean reversion,  $c_i < 0$  in a scenario where  $|c_i u_{i,t}|$  dominates  $|b_i' Z_{i,t-1}|$ . In the latter case, the approximate to rational pricing requires a non-zero complement of opposing sign for any significant positive coefficient  $b_i$ ,  $c_i$ . Accordingly, it is expected that a positive  $b_i$  ( $c_i$ ) will result in a negative  $c_i$  ( $b_i$ ) in line with the offsetting behaviour presumed by approximately rational pricing. The offsetting property is similar to the corrective process described in, for example, De Long et al. (1990a) and Hong and Stein (1999) whereby positive short-term autocorrelation induced by overconfidence or overreaction is followed by corrective negative autocorrelation.

In the absence of the idiosyncratic term  $u_{i,t}$ , the pricing effects of overreaction are non-existent and agents do not consider mispricing in their determination of the asset price. Accordingly  $c_i$  measures the sensitivity of the  $i$ th asset to the effects of overreaction or underreaction, and the negligibility of the set of rational speculators and feedback traders who seek to profit from mispriced conditions at the relevant data frequency. An analogous interpretation applies for the fundamental feedback term  $Z_i$ . The fundamental feedback coefficient  $b_i$  is a measure of the  $i$ th asset's sensitivity to updated or revised historic fundamental information. In the context of the proposed market characteristics,  $b_i$  measures the pricing relevance of the set of rational speculators (or contrarian traders) and positive fundamental feedback traders who set market positions in accordance with historic fundamental expectations. A feature of the approach is that the traditional pricing model is nested within the return structure, thereby providing for the joint assessment of the rational pricing hypothesis and any additive qualities of the feedback and overreaction extensions. Empirical support for the traditional approach is contingent on the negligible impact of  $b_i$  and  $c_i$  whereby the pricing effects of feedback or overreaction are irrelevant and the pricing structure collapses to its traditional form.

### 3. An empirical model

Section 3.1 details the model adopted to investigate the momentum, feedback, and reactionary properties of national equity markets. Formal tests of the relevant properties are formulated in the remaining sub-sections.

#### 3.1 Model structure

The returns vector  $r_t$  is treated as a function of  $K$  latent factors

$$r_t = [C(L) \quad \mathbf{I}_N] \begin{bmatrix} \tilde{\mathbf{f}}_t' \\ u_t' \end{bmatrix} = C(L)\tilde{\mathbf{f}}_t + u_t, \quad (5)$$

$$\Psi(L)u_t = G^{1/2}z_t, \quad (6)$$

$$G_{t|t-1} = \text{diag}(\sigma_{1,t|t-1}^2), \quad (7)$$

$$\sigma_{t|t-1}^2 = [\sigma_{1,t|t-1}^2 \quad \sigma_{2,t|t-1}^2 \quad \dots \quad \sigma_{N,t|t-1}^2]', \quad (8)$$

where  $C(L)$  is a polynomial in the lag operator  $L$ ,  $\tilde{\mathbf{f}}_t$  is a  $K$  dimensional vector,  $\Psi(L) = \mathbf{I}_N - \Psi_1 L - \Psi_2 L^2 - \dots - \Psi_d L^d$  for finite  $d$ ,  $\Psi_j = \text{diag}([\psi_{1,j} \quad \psi_{2,j} \quad \dots \quad \psi_{N,j}])$  for  $j = 1, 2, \dots, d$ ,  $z_t$  is a multivariate standard normal vector  $z_t \sim iidMVN(\mathbf{0}_{N,1}, \mathbf{I}_N)$  and  $\mathbf{0}_{a,b}$  is an  $a \times b$  matrix of zeros.

Each factor is constructed as per the following:

$$\phi_k(L)\mathbf{f}_{k,t} = \mu_{k,t} + e_{k,t}, \quad (9)$$

$$e_{k,t} \sim N(0, \sigma_{k,t}^2), \quad (10)$$

where  $\phi_k(L) = 1 - \phi_{k,1}L - \dots - \phi_{k,N(k)}L^{N(k)}$ ,  $\mu_{k,t} = \sum_{m=1}^{M(k)} \mu_{k,m} S_{k,m,t}$  and  $\sigma_{k,t}^2 = \sum_{m=1}^{M(k)} \sigma_{k,m}^2 S_{k,m,t}$  for finite  $N(k), M(k)$ , and  $E(\mathbf{f}_{k,a} u_b) = 0 \forall a, b$ .  $S_{k,m,t}$  is a discrete latent variable taking the value unity if state  $m = m(k)$ ,  $m(k) \in M_k = \{m \in \mathcal{N} : m \leq M(k) \in \mathcal{N}\}$ , and zero otherwise. The probability of state  $m$  prevailing is determined in accordance with the Markovian transition matrix:

$$Pr_k = \begin{bmatrix} P_{1,1,k} & \cdots & P_{M(k),1,k} \\ \vdots & \ddots & \vdots \\ P_{1,M(k),k} & \cdots & P_{M(k),M(k),k} \end{bmatrix},$$

where  $Pr_k' \mathbf{1}_{M(k)} = \mathbf{1}_{M(k)}$  and  $\mathbf{1}_{M(k)}$  is an  $M(k)$ -dimensional column vector of ones, implying that  $P_{m_1, m_2, k}$  represents the unconditional probability of a transition from state  $m_1$  to state  $m_2$  for the  $k$ th factor.

The basic restrictions,  $\phi_k(L) = 1$  and  $\mu_{k,t} = \mu_k$ , imply that the expected value of the global factor is captured entirely by the time-invariant intercept term  $\mu_k$ . The relaxation of the persistence restriction introduces an additional source of information for the construction of expected returns. In this respect,  $\phi_k(L) \neq 1$  recognises the potential for  $N(k)$ -th order effects in  $f_{k,t}$  and provides a capacity for treating common persistence as a relevant pricing construct. Persistent common effects allow for the possibility of serial correlation in equity returns and suggest that the pricing effects of common shocks may be dissipative (assuming that the roots of  $\phi_k(L)$  are outside the unit circle). This approach is contrasted with the restriction  $\phi_k(L) = 1$  and the associated implication that common effects are immediately and fully digested by financial markets. In the case of non-zero  $\phi_{k,j|j \geq 1}$ , the  $k$ th factor is associated with a time-varying expectation and the information retrieved through  $\phi_k(L)$  may be interpreted as capturing a dimension of autocorrelation in the risk-premium. Persistence effects are transmitted to all markets with applicable non-zero loading terms in  $C(L)$ . The extension to a time-varying intercept term  $\mu_{k,t}$  also acknowledges the potential for time variation in the risk-premium. As such, the expected return attributable to the the  $k$ th factor is time varying, even in the case  $\phi_k(L) = 1$  and  $C(L) = C$ , pursuant to its proportionality to the intercept  $\mu_{k,t}$ .

Since  $C(L) = C$  is not imposed a priori, investors are permitted to consider historic fundamental (or pervasive) information in their derivation of prices. The application of

historic fundamental information to future returns may be interpreted as a fundamental feedback or a delayed reaction to (or revised treatment of) fundamental information. Assuming  $\phi_k(L) \neq 1$ , and given the lag operator in  $C(L)$ , lagged effects due to factor  $k$  are transmitted directly via the lag operator  $L$  and indirectly given the  $k$ th factor's autoregressive path.

### 3.2 Pricing scenarios

The pricing structure leads to the following four sets of pricing characteristics or scenarios. In the exposition that follows, it is assumed that  $C(L) = c_1 + c_2L$ , where

$$c_1 = (c_{1,1} \quad \cdots \quad c_{N,1}), \quad c_2 = (c_{1,2} \quad \cdots \quad c_{N,2}).^2$$

*Scenario 1:*  $\psi_i = c_{i,2} = 0$

Lagged values of market  $i$ 's return or the common factors do not provide any explanatory power in determining the time- $t$  excess return for market  $i$ . The current return  $r_{i,t}$  is, therefore, directly unaffected by past information stemming from either the common or idiosyncratic (or country specific) sources stipulated in the model.<sup>3</sup> Note however that  $\Phi(L) \neq 0$  implies that  $\mathbf{f}_t$  exhibits autoregressive properties such that, given  $c_{i,1} \neq 0$ ,  $r_{i,t}$  is not independent with respect to lagged common information. The expectation of  $r_{i,t}$ , conditional on information available at time  $t-1$ , is given by:

$$E(r_{i,t} | \psi_i = c_{i,2} = 0, I_{t-1}) = c_{i,1}' \tilde{\mathbf{f}}_{t-1}. \quad (11)$$

Expected returns are determined by a filtered expectation on the common component. The result  $\psi_i = c_{i,2} = 0$ , conditional on observation of  $\tilde{\mathbf{f}}_t$  at time  $t-1$ , is a special case pursuant to which a result in the spirit of the equilibrium (or exact) APT is obtained. Further, in the case where  $\tilde{\mathbf{f}}_t$  is a single global factor a pricing form following that of the conditional CAPM follows.

<sup>2</sup> Longer memory extensions are straightforward. This assumption is adopted to simplify the exposition.

<sup>3</sup> The terms idiosyncratic and country specific are used interchangeably and refer to the portion of the asset return not attributable to the common component.

*Scenario 2:  $\psi_i \neq 0, c_{i,2} = 0$*

The restrictions  $\psi_i \neq 0, c_{i,2} = 0$  imply that current excess returns are only affected by the contemporaneous common component and any lagged information associated with the idiosyncratic persistence term  $\psi_i$ . Lagged values of the common factor  $\tilde{f}_{t-j}, j \geq 2$ , are irrelevant and  $\tilde{f}_{t-1}$  does not provide any direct explanatory power (i.e. any explanatory power independent of  $\psi_{i,1}$  or  $\Phi$ ). The engagement of the idiosyncratic persistence term implies a rejection of the efficient markets hypothesis (EMH) for market  $i$  given that observed returns provide a significant source of information for future pricing. Given the hypothesized restrictions, the time  $t-1$  expected value of  $r_{i,t}$  is:

$$E(r_{i,t} | c_{i,2} = 0, I_{t-1}) = c_{i,1}' \tilde{f}_{t-1} + \psi_{i,1} (r_{i,t-1} - c_{i,1}' \tilde{f}_{t-1|t-1}). \quad (12)$$

The result is an extension of that given by the first set of restrictions (viz. an extension to the conditional APT result) in that expected returns are not determined solely by market  $i$ 's relationship with the common component. In this respect, the immediate past values pertaining to market  $i$ 's actual return and the common factors are relevant in determining market  $i$ 's expected return. Specifically, the  $i$ th market's expected return is equivalent to that given by the conditional APT adjusted by the weighted deviation of  $r_{i,t-1}$  from its conditional expectation.

*Scenario 3:  $\psi_i = 0, c_{i,2} \neq 0$*

Under this scenario, market  $i$  returns are directly influenced by current and past information in the common factor. Given the set of restrictions, the conditional expectation of  $r_{i,t}$  is determined by current and lagged values of the common factor:

$$E(r_{i,t} | \psi_i = 0, I_{t-1}) = c_{i,1}' \tilde{f}_{t-1} + c_{i,2}' \tilde{f}_{t-2|t-1}. \quad (13)$$

The scenario may be facilitated by market inefficiency resulting in a failure to immediately price common information (alternatively, investors are simply learning new information about some unobserved economic state). In this respect, the hypothesis may be interpreted as the result of investor uncertainty regarding the true value of  $\tilde{f}_{t-1}$  (viz.

uncertainty regarding the true effect of the historic information represented by the common component).

*Scenario 4:*  $\psi_i \neq 0, c_{i,2} \neq 0$

Scenario four is a direct extension of the second and third scenarios. Expected returns follow those for scenario two subject to the addition of an adjustment term. Since  $c_{i,2} \neq 0$ , excess returns are directly influenced by both current and lagged values of the common factors. Market  $i$ 's expected return is given by:

$$E(r_{i,t} | I_{t-1}) = c_{i,1}' \tilde{\mathbf{f}}_{t|t-1} + c_{i,2}' \tilde{\mathbf{f}}_{t-1|t-1} + \psi_{i,1} (r_{i,t-1} - c_{i,1}' \tilde{\mathbf{f}}_{t-1|t-1} - c_{i,2}' \tilde{\mathbf{f}}_{t-2|t-1}). \quad (14)$$

Filtered common information,  $\mathbf{f}_{t-1|t-1}$ , is applied directly in the construction of the expectation while partially smoothed common information,  $\tilde{\mathbf{f}}_{t-2|t-1}$ , is considered in the derivation of the pricing-deviation term. In this scenario, investors are uncertain about the true value of  $\tilde{\mathbf{f}}_{t-1}$  or are unable to allocate resources reflecting all the information represented by the lagged common component. Investor uncertainty regarding historic pricing information  $\tilde{\mathbf{f}}_{t-j}$ , and therefore the fair value of the  $i$ th asset, is also reflected in the deviation term  $r_{i,t-1} - c_{i,1}' \tilde{\mathbf{f}}_{t-1|t-1} - c_{i,2}' \tilde{\mathbf{f}}_{t-2|t-1}$ .

### 3.3 Factor pricing characteristics

Pursuant to the four pricing scenarios, the factor component of expected returns is determined through the expected value  $\mathbf{f}_{t|t-1}$ , the filtered value  $\mathbf{f}_{t-1|t-1}$  and the partially smoothed value  $\mathbf{f}_{t-2|t-1}$ . Since  $\mu_{k,t}$  is a function of the regime-dependent terms  $\mu_{k,m}$ ,  $m = 1, 2, \dots, M(k)$ , any non-zero value for  $\mu_{k,m}$  ensures the pricing relevance of the  $m$ th regime for the  $k$ th common factor as  $\mathbf{f}_{k,t|t-1} \neq 0$  (assuming, of course, sensitivity to the factor is non-zero). Since the factor regimes are identified according to factor volatilities, a test  $\mu_{k,m} \neq 0$  also examines the pricing characteristics of the  $m$ th volatility regime.

The result  $\phi_k \neq 0$  directly imports historical factor prices into the current factor equation. It is clear, therefore, that  $\phi_k \neq 0$  ensures  $\mathbf{f}_{k,t|t-1} \neq 0$  notwithstanding the

relevance of  $\mu_{k,m}$  for any  $m$ . Consequently,  $\phi_k \neq 0$  indicates a pricing relevance for the  $k$ th common factor irrespective of  $\mu_{k,m}$ . In turn, non-zero persistence for the  $k$ th common factor introduces the potential for fundamental momentum since  $\phi_k > 0$  implies that historical factor prices are extrapolated into the future (given positive contemporaneous sensitivity  $c_{i,k,1} > 0$ , where  $c_{i,k,1}$  is the  $i$ th market's sensitivity to factor  $\tilde{f}_{k,t}$ ). The extrapolative effects of  $\phi_k > 0$  also extend to the updated values  $f_{t-1|t-1}$  and  $f_{t-2|t-1}$  through the fundamental feedback term  $c_{i,k,2}$  (given  $c_{i,k,2} > 0$ , where  $c_{i,k,2}$  is the  $i$ th market's sensitivity to factor  $\tilde{f}_{k,t-1}$ ). The momentum induced by  $\phi_k > 0$  should be contrasted with both fundamental feedback momentum  $c_{i,k,2} > 0$  and idiosyncratic momentum  $\psi_{i,1} > 0$  since the former pertains to the fundamental pricing factor itself whereas the latter two forms of momentum are asset-specific.

### ***3.4 Testing for fundamental feedback***

The coefficient  $c_{i,2}$  may be interpreted as a market's delayed pricing reaction to fundamental information. Pursuant to the hypotheses of rational pricing and market efficiency, the pricing effects of new information are determined in an immediate and consistent fashion such that all existing information is reflected in the current asset price. Accordingly, any change in price depends only on (rational) expectations regarding the unobserved future information set such that sensitivity to historical information should not differ significantly from zero. Consequently,  $c_{i,2} = 0$  in the rational, efficient pricing scenario.

The alternative hypothesis of offsetting or momentum-negating historical pricing departs from the restrictive assumptions of strict rationality and efficiency and allows for sensitivity to lagged common information under the condition  $c_{i,2} < 0$ . The negative condition implies that expectations regarding the common information set for the current period are offset in the following period, thereby explicitly negating any fundamental momentum. Under the framework of rational speculators (or contrarian investors) and fundamental feedback traders, the result  $c_{i,2} < 0$  suggests that rational speculators trade

against the sign of existing fundamental information thereby correcting for the behaviour of (fundamental) feedback traders.

In the case  $c_{i,2} > 0$ , however, current period fundamental expectations are carried forward to the next period resulting in fundamental momentum. The case may be reconciled with the notion that a common sign across  $\tilde{f}_{t-1|t-2}$  and its updated equivalent  $\tilde{f}_{t-1|t-1}$  constitutes a confirmation of the time  $t-2$  expectation thereby inducing a fundamental overreaction and resulting in  $c_{i,2} > 0$ . In the same vein, a disparity in the sign of the update  $\tilde{f}_{t-1|t-1}$  relative to  $\tilde{f}_{t-1|t-2}$  implies an incorrect time  $t-2$  expectation regarding the path of the common (or fundamental) information set. In accordance with  $c_{i,2} > 0$ , the mistaken time  $t-2$  expectation results in a time- $t$  pricing correction.

### 3.5 Testing for adjusting expected returns

The expected excess return for market  $i$  is contingent on the value of  $\psi_{i,1}$ .<sup>4</sup> Asset prices are deemed to behave on an adjusting basis if past pricing errors impact on current asset prices. A test of adjusting expected returns may be formulated as:

$$\begin{aligned} H_0 : \psi_{i,1} &= 0, \\ H_1 : \psi_{i,1} &\neq 0. \end{aligned}$$

The conditional expected returns under the two alternative hypotheses are:

$$E(r_{i,t} | H_0, I_{t-1}) = c_{i,1}' \tilde{f}_{t|t-1} + c_{i,2}' \tilde{f}_{t-1|t-1}, \quad (15)$$

$$E(r_{i,t} | H_1, I_{t-1}) = E(r_{i,t} | H_0, I_{t-1}) + \alpha_{i,t} = c_{i,1}' \tilde{f}_{t|t-1} + c_{i,2}' \tilde{f}_{t-1|t-1} + \alpha_{i,t}, \quad (16)$$

$$\begin{aligned} \alpha_{i,t} &= \psi_{i,1} (r_{i,t-1} - E(r_{i,t-1} | H_0, I_{t-1})) \\ &= \psi_{i,1} (r_{i,t-1} - c_{i,1}' \tilde{f}_{t-1|t-1} - c_{i,2}' \tilde{f}_{t-2|t-1}) \\ &= \psi_{i,1} \tilde{\delta}_{i,t}. \end{aligned} \quad (17)$$

Under the null hypothesis, a representative investor determines an asset price as a function of expectations on unobservable factors common to the asset universe. Depending on the values attributed to the common filter set  $\{c_i\}$ , updated information

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<sup>4</sup> The excess return in the context of the discussion regarding  $\psi$  refers to a deviation from fair value, not the return net of some zero-beta or risk-free rate.

regarding the unobservable common factors may also be relevant in determining the asset price. The alternative hypothesis introduces the adjustment term  $\alpha_t$  as an additional pricing consideration. Pursuant to the adjustment term, null-hypothesis pricing is adjusted by the immediately preceding updated pricing error in proportion to  $\psi_i$ . Assuming that  $\tilde{\delta}_{i,t}$  represents information idiosyncratic to the  $i$ th market, a test of the hypothesis  $\psi_{i,1} = 0$  is also a test of a particular form of idiosyncratic pricing for the  $i$ th market.

The excess return for market  $i$  exhibits the additional property of mean reversion or correction in the case  $\psi_{i,1} < 0$ . The case of a negative  $\psi_{i,1}$  implies that the representative investor updates expectations for time- $t$  returns, given the introduction of the time  $t-1$  information set, in a corrective fashion. The adjustment  $\alpha_{i,t} = \psi_{i,1}\tilde{\delta}_{i,t}$  acts as a negative impulse on the  $(i,t)$ th return in the case  $\tilde{\delta}_{i,t} > 0$  (with the converse impulse for a negative  $\tilde{\delta}_{i,t}$ ). Actual time- $t$  returns exceeding the fair value estimate  $E(r_{i,t}|H_0, I_{t-1})$  are treated as inflated, resulting in a downward correction or adjustment for time  $t+1$  pricing. In contrast, returns below the fair value estimate are adjusted upwards in the next period.

#### 4. Estimation and application of the model to a set of national equity markets

A Metropolis-in-Gibbs sampler is used to estimate the model. The estimation process is detailed in Tsiaplias (2008). Pursuant to the sampler, draws from the full posterior  $p(\tilde{\theta}|r_1, r_2, \dots, r_T)$  are used to obtain inferences for the relevant parameters. The full posterior is given by:

$$\begin{aligned} p(\tilde{\theta}|r_1, r_2, \dots, r_T) &= f(F, S, \sigma_\alpha, \sigma_\beta, \sigma_\omega, C, \psi, \phi, \mu, \varpi, P|R) \\ &\propto f(R|F, C, \psi, \sigma_\alpha, \sigma_\beta, \sigma_\omega) f(F|S, \phi, \mu, \varpi) \\ &\quad f(S|P) f(C, \psi, \sigma_\alpha, \sigma_\beta, \sigma_\omega, \phi, \mu, \varpi, P), \end{aligned} \quad (18)$$

where  $R$  is the set of returns,  $F$  is the set of latent common factors,  $S$  is the set of latent common regimes,  $C, \psi, \{\sigma_\alpha, \sigma_\beta, \sigma_\omega\}$  are common factor loadings, idiosyncratic persistence, and GARCH terms respectively,  $\phi, \mu, \varpi$  are the common persistence,

intercept, and volatility terms respectively, and  $P$  is the regime transition matrix. The prior density  $f(C, a, \psi, \sigma_\alpha, \sigma_\beta, \sigma_\sigma, \phi, \gamma, \varpi, P)$  is diffuse.<sup>5</sup>

The model is applied to excess returns data from the set of US-dollar denominated MSCI Developed Country indices.<sup>6</sup> The indices are used to construct weekly returns during the period commencing the first week of Jan. 1980 and ending the second week of Sept. 2004 ( $T = 1289$ ) for the following 18 countries: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Italy, Japan, the Netherlands, Norway, Singapore, Spain, Sweden, Switzerland, the United Kingdom (UK), and the United States (US).

Two common factors are estimated ( $K = 2$ ); a factor common to all markets,  $W_t$ , and a factor common to European markets,  $E_t$ .<sup>7</sup> To ensure identification, unity restrictions are imposed on the contemporaneous German and US loadings, and zero-restrictions are imposed on the European component of  $C(L)$  for the non-European markets in the dataset. Three regimes ( $M(1) = 3$ ) were identified for the global factor and two regimes ( $M(2) = 2$ ) for the European factor; the regimes were identified as the minimal set of regimes required to explain the heteroscedasticity in the factor. In all but two markets, given the incorporation of common volatility shifts, a first-order GARCH process suffices in explaining the residual heteroscedasticity present in the returns data.<sup>8</sup> Extensions to the first-order GARCH process, however, are necessary to satisfactorily accommodate the remaining heteroscedasticity in the Australian ( $P = Q = 2$ ) and Belgian markets ( $P = 2, Q = 1$ ).

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<sup>5</sup>  $N(0,10)$  prior densities are used for the common factor loadings, common factor intercepts, the (common and idiosyncratic) persistence terms and the GARCH terms (re-estimation using a zero-mean normal prior with a variance of 100 had a negligible impact on the results). An inverse-Wishart(1e-05, 1) prior is used for the common factor volatilities, and a Dirichlet( $a$ ) distribution (where  $a$  is an  $M(k)$  vector of ones) for the transition parameters from state  $i$  to state  $1, 2, \dots, M(k)$ .

<sup>6</sup> Excess returns are determined by reference to the 13-week US Treasury Bill.

<sup>7</sup> The extension to  $K$  factors generally is straightforward. Since returns data are obtained from the developed country MSCI indices, the majority of markets in the set are European. Consequently a European factor may be accurately identified.

<sup>8</sup> A preliminary model coupling common regime-dependent volatility (under the single-global factor or joint global and European factor hypotheses) with constant idiosyncratic volatility failed to sufficiently explain the heteroscedasticity observed in any of the markets considered.

#### 4.1 Factor persistence and pricing

In deference to the assumption of independence, the persistence vector  $\phi_w$  suggests significant first-order persistence for the global factor (Table 1). The 95% credible interval for the first element of the (global) persistence term implies significant, positive autocorrelation for the global factor. The results for the second and third elements are insignificant suggesting that only immediately preceding information is relevant in determining the current value of the global factor. Given that the universal factor is unobserved it is difficult to reconcile the first-order correlation in terms of market microstructure. A possible interpretation is that a non-negligible subset of agents determines its market position by extrapolating existing fundamental pricing information. Consider, for example, the agent who places a non-zero weight on extrapolative earnings or macroeconomic forecasts in constructing a portfolio. Accordingly, a portion of the global factor is determined by reference to extrapolated information thereby inducing positive first-order dependence and momentum. Indeed, given  $\phi > 0$  the result  $c_{i,2} < 0$  (pertaining to lagged common information) observed for France, Germany, the Netherlands, Spain, Sweden, Switzerland, the UK, and the US may be interpreted as partially correcting any inherent momentum in the universal factor.

Table 1 Statistics regarding common persistence parameters  $\phi_w, \phi_e$

<i>Variable</i>	<i>Median</i>	<i>Mean</i>	<i>Std. deviation</i>	<i>95% BCI</i>		<i>Pr&gt;0</i>
World (Lag 1)	0.151	0.151	0.047	0.060	0.243	1.000
World (Lag 2)	0.020	0.020	0.033	-0.044	0.084	0.732
World (Lag 3)	0.014	0.014	0.031	-0.047	0.075	0.671
Europe (Lag 1)	0.033	0.034	0.052	-0.068	0.137	0.739
Europe (Lag 2)	0.051	0.051	0.034	-0.016	0.119	0.932
Europe (Lag 3)	0.016	0.017	0.034	-0.051	0.084	0.685

The parameters  $\phi_w, \phi_e$  define the persistence of the global and European factors common to the 18 developed equity markets. The column entitled ‘95% BCI’ provides 95% Bayesian Credible Intervals for the parameters.

In contrast to the significant first-order persistence for the global factor, the persistence term  $\phi_e$  for the European factor does not appear significant. The European result is consistent with the notion of an information efficient orthogonal European

portfolio and suggests that lagged information is not pertinent in determining the current value of the European-specific composite. The evidence for  $\phi_w$ ,  $\phi_e$  tends to the general conclusion that historic information is useful in predicting the time- $t$  set of common factor values only to the extent that it is immediately preceding information of a global character.

Table 2 provides statistics on the regime-dependent intercept terms for the global and European factors. The intercept term  $\mu_{w,t} = \mu_w | S_{w,t} = 1$  is clearly greater than zero suggesting a positive-valued global factor in an environment where the low-volatility regime is expected to prevail. Pursuant to  $S_{w,t} = 1$ , the result  $\mu_w > 0$  also suggests that investors attach a premium to markets during periods of low global volatility (see, also, Bae, Kim, and Nelson, 2007). The global intercept terms during periods of medium and high (global) volatility are negative and suggest a relationship between greater volatility, and negative returns. The intercept term for the medium-volatility regime,  $\mu_w | S_{w,t} = 2$ , is significant at the 10% level while the intercept associated with high levels of volatility lacks significance at any conventional level. The marginal probability of the low-volatility regime (associated with a positive global intercept) exceeds those of the medium and high volatility regimes (associated with negative and extremely negative intercept terms respectively) by factors of approximately two and ninety-six respectively.

Table 2 Statistics for the regime-dependent intercept terms  $\mu$

<i>Variable</i>	<i>Median</i>	<i>Mean</i>	<i>Std. deviation</i>	<i>95% BCI</i>		<i>Pr&gt;0</i>	<i>Marginal pr (regime j)</i>
World (regime 1)	0.189	0.189	0.052	0.088	0.292	1.000	0.672
World (regime 2)	-0.232	-0.237	0.146	-0.541	0.034	0.044	0.321
World (regime 3)	-2.212	-2.232	2.503	-7.231	2.802	0.173	0.007
Europe (regime 1)	0.068	0.070	0.074	-0.072	0.219	0.830	0.725
Europe (regime 2)	-0.154	-0.171	0.232	-0.682	0.238	0.224	0.275

The  $\mu$  parameters are the intercepts for the global and European factors. The global intercepts are assigned to the three volatility regimes identified for the global factor, while the European intercepts are assigned to the two volatility regimes identified for the European factor. The column entitled '95% BCI' provides 95% Bayesian Credible Intervals.

Given that the low-volatility regime prevails approximately two-thirds of the time for the global factor, the implication is that investors with exposure to globally-sensitive

markets expect a positive return for two of every three periods. Investors are also willing to accept exposure to medium levels of volatility (viz. potentially negative returns) approximately one out of every three periods. The lack of significance for the intercept term associated with high (or extreme) global volatility implies that the construction of the expected value of the global factor is generally unaffected by the extreme probability associated with the high volatility regime. The result suggests that investors refrain from attaching any significance to the impact of the high volatility regime in constructing the expected value of the global factor and, consequently, the expected return of markets associated with the global factor.

The intercept terms for the low and high European volatility regimes are positive and negative respectively. Given the failure to observe any significant persistence in the European factor, as per  $\phi_e(L) = 0$ , the form for the European factor collapses to  $E_t = \mu_{e,t} + e_t$ , where  $\mu_{e,t}$  constitutes the expected value of the European factor and  $e_t$  is the innovation. Although the signs of the state-contingent expectations are consistent with those observed for the intercept terms in the equivalent global cases, neither term differs significantly from zero. Accordingly,  $\mu_{e,t}$  is deemed irrelevant in the construction of the expected value of the European factor. Since neither of the null hypotheses  $\mu_e = 0$  or  $\phi_e(L) = 0$  is rejected, the expected value of the European factor is zero and prices for European markets appear to be determined solely by reference to the global factor.

#### ***4.2 Fundamental feedback***

Table 3 presents each market's sensitivity to lagged common information. The 95% global factor credible intervals for France, Germany, Netherlands, Singapore, Spain, Sweden, Switzerland, the US and the UK are concentrated away from zero implying significant sensitivity to updated fundamental information. Significant sensitivity to updated historical common information for the US, UK, French and German markets suggests that a non-negligible portion of international equity trade depends on updated or revised interpretations of historic information. All but one of the nine markets for which sensitivity is significant at the 5% level exhibits offsetting behaviour while Singapore reacts in an extrapolative manner. The offsetting behaviour suggests that rational

speculators or arbitrageurs operating in the relevant markets respond to fundamental feedback mispricing in a timelier (relative to the positive fundamental feedback scenario) fashion.

Table 3 Statistics regarding  $c_{:,2}$  parameters representing sensitivity to lagged common information

<i>Variable</i>	<i>Median</i>	<i>Mean</i>	<i>Std. deviation</i>	<i>95% BCI</i>		<i>Pr&gt;0</i>
Austria (world)	-0.036	-0.036	0.038	-0.110	0.037	0.171
Austria (Europe)	0.031	0.032	0.039	-0.045	0.109	0.788
Belgium (world)	-0.054	-0.054	0.037	-0.128	0.019	0.072
Belgium (Europe)	-0.079	-0.079	0.036	-0.151	-0.008	0.015
Denmark (world)	0.002	0.002	0.040	-0.077	0.080	0.521
Denmark (Europe)	-0.053	-0.053	0.039	-0.130	0.024	0.089
France (world)	-0.179	-0.179	0.039	-0.256	-0.104	0.000
France (Europe)	-0.127	-0.127	0.035	-0.195	-0.059	0.000
Germany (world)	-0.122	-0.122	0.041	-0.202	-0.043	0.001
Germany (Europe)	-0.075	-0.075	0.036	-0.146	-0.005	0.018
Italy (world)	-0.070	-0.070	0.043	-0.155	0.013	0.049
Italy (Europe)	-0.051	-0.051	0.040	-0.129	0.028	0.103
Netherlands (world)	-0.140	-0.140	0.037	-0.213	-0.069	0.000
Netherlands (Europe)	-0.095	-0.096	0.031	-0.156	-0.036	0.001
Norway (world)	0.072	0.072	0.049	-0.025	0.168	0.926
Norway (Europe)	0.005	0.005	0.047	-0.087	0.097	0.542
Spain (world)	-0.091	-0.091	0.045	-0.179	-0.003	0.021
Spain (Europe)	-0.051	-0.051	0.041	-0.133	0.030	0.108
Sweden (world)	-0.124	-0.125	0.052	-0.226	-0.024	0.008
Sweden (Europe)	-0.068	-0.068	0.047	-0.160	0.024	0.072
Switzerland (world)	-0.089	-0.089	0.035	-0.158	-0.020	0.006
Switzerland (Europe)	-0.085	-0.085	0.032	-0.147	-0.023	0.004
UK (world)	-0.163	-0.163	0.038	-0.237	-0.088	0.000
UK (Europe)	-0.079	-0.079	0.032	-0.142	-0.016	0.007
Canada	-0.034	-0.034	0.038	-0.110	0.041	0.190
US	-0.221	-0.221	0.036	-0.290	-0.150	0.000
Australia	0.056	0.055	0.046	-0.034	0.145	0.886
Hong Kong	0.073	0.073	0.061	-0.046	0.191	0.885
Japan	-0.013	-0.014	0.051	-0.114	0.085	0.398
Singapore	0.147	0.146	0.054	0.040	0.251	0.996

Table 3 provides descriptive statistics for the coefficients assigned to the lagged common factors. There are two sets of coefficients for European markets (a coefficient describing sensitivity to lagged global information, and a coefficient describing sensitivity to lagged European information). The column entitled '95% BCI' provides 95% Bayesian credible intervals.

In contrast, Singapore exhibits positive sensitivity to revised historic expectations suggesting a less timely response by rational speculators such that the mispricing induced by fundamental feedback trading persists for a greater period of time. Three of the remaining nine markets, Australia, Hong Kong and Norway, react in an extrapolative

manner while Austria, Belgium, Canada, Italy, and Japan exhibit an offsetting response. An alternative interpretation for the results, along the lines of De Long (1990b), is that the short-term behaviour of rational speculators in the set of (larger) markets is offsetting (or momentum-negating), and consistent with Friedman's treatise (1953), while rational speculators in the second (generally smaller) set of markets trade in the direction of revised or updated fundamental information to exploit inefficiencies stemming from persistent mispricing induced by fundamental feedback.

In terms of the European markets, the UK, France, Germany, Netherlands and Switzerland exhibit a significant offsetting response for both the universal and European factors. Spain and Sweden appear to exhibit sensitivity to revised information only for the universal factor while the converse appears to hold for Belgium. The Belgian response to historical European information is also offsetting implying that no European market exhibits a significant positive response to historical European information.

#### ***4.3 Adjusting and corrective returns***

The probabilities associated with approximate Wald statistics for three sets of restrictions are presented in Table 4. The first column returns the results of tests for the restrictions given by  $\theta_0 : \psi_{i,1} = c_{i,2} = 0$  (corresponding to scenario 1 in Section 3.2). The second and third columns present probabilities associated with tests for the null hypotheses given by the restrictions  $\theta_1 : c_{i,2} = 0$  and  $\theta_2 : \psi_{i,1} = 0$  (corresponding to pricing scenarios 2 and 3). The fourth column of Table 4 returns the probabilities associated with the hypothesis  $\psi > 0$ .

The probabilities in Table 4 suggest that the null hypothesis  $\psi_{i,1} = 0$  is rejected at the 5% level for five of the eighteen markets (Australia, France, the Netherlands, the UK and the US). The credible interval for Spain is borderline significant at the 10% level. Importantly, the adjustment term activated in accordance with  $\psi_{i,1} \neq 0$  is non-negligible for two of the world's major markets (the UK and the US). The null hypothesis given by  $\theta_2 : c_{i,2} = 0$  is rejected at the 5% level for Germany, Singapore, Sweden, and Switzerland, in addition to four of the five markets for which the hypothesis  $\psi_{i,1} = 0$  is rejected at the

5% level (Australia constituting the exception). The hypothesis  $\theta_2 : c_{i,2} = 0$  is also rejected at the 10% level for Belgium and Spain.

Table 4 Probabilities regarding parameter restrictions  $\theta_0$ ,  $\theta_1$ ,  $\theta_2$

<i>Market</i>	$\theta_0 : \psi_{i,1} = c_{i,2} = 0$	$\theta_1 : c_{i,2} = 0$	$\theta_2 : \psi_{i,1} = 0$	$\Pr(\psi_{i,1} > 0)$
Austria	0.739	0.860	0.605	0.930
Belgium	0.919	0.741	0.944	0.129
Denmark	0.607	0.715	0.596	0.142
France	1.000	0.978	1.000	0.012
Germany	0.985	0.628	0.994	0.186
Italy	0.759	0.119	0.877	0.560
Netherlands	1.000	1.000	1.000	0.000
Norway	0.521	0.505	0.653	0.248
Spain	0.956	0.898	0.922	0.951
Sweden	0.980	0.846	0.980	0.077
Switzerland	0.987	0.041	0.995	0.478
UK	1.000	0.970	1.000	0.015
Canada	0.330	0.078	0.622	0.540
US	1.000	1.000	1.000	0.000
Australia	0.984	0.990	0.774	0.005
Hong Kong	0.586	0.405	0.771	0.296
Japan	0.216	0.477	0.211	0.737
Singapore	0.976	0.354	0.993	0.677

Table 4 provides the probabilities associated with the null hypotheses  $\theta_0$ ,  $\theta_1$ , and  $\theta_2$ . The hypotheses are differentiated on the basis of sensitivity to the lagged adjustment term,  $r_{i,t-1} - E(r_{i,t-1} | I_{t-1})$  (described by  $\psi_{i,1}$ ) and sensitivity to lagged common information (described by  $c_{i,2}$ ). The probabilities are based on Wald statistics constructed using mean and variance-covariance levels determined using the sample output. The final column determines the probability that  $\psi_{i,1}$  is greater than zero (the probability is obtained directly from the out of the sampler used to estimate the model).

The rejection of  $\theta_0 : \psi_{i,1} = c_{i,2} = 0$ ,  $\theta_1 : \psi_{i,1} = 0$ , and  $\theta_2 : c_{i,2} = 0$  suggests that expected returns for France, the Netherlands, the UK, and the US are approximated by the non-linear form:

$$E(r_{i,t} | I_{t-1}) = c_{i,1}' \mathbf{f}_{t|t-1} + c_{i,2}' \mathbf{f}_{t-1|t-1} + \alpha_{i,t}. \quad (19)$$

According to Eq. (19), the representative investor pricing market  $i$  considers both updated common information and past pricing errors in setting the market price. In considering updated common information, the representative investor ensures that the entire set of common information (given the adopted common structure) as at time  $t-1$  is accounted

for in determining the expected return. The representative investor also adjusts the time- $t$  expectation for market  $i$  in accordance with past pricing errors. The adjustment takes the form of a pricing correction in the case of a negative country specific persistence term,  $\psi_{i,1} < 0$ . From the probabilities in Table 4, the one-sided hypothesis  $\psi_{i,1} > 0$  is rejected at the 5% level in favour of  $\psi_{i,1} < 0$  for all four relevant markets. In the Australian case, the evidence similarly suggests  $\psi_{i,1} < 0$  although  $c_{i,2} = 0$ .

The hypothesis  $\theta_0$  is also rejected at the 5% level for Spain suggesting that at least one of the  $\psi_{i,1}, c_{i,2}$  terms is non-zero. In this respect,  $\psi_{i,1}$  is concentrated in the positive space such that corresponding pricing result is antithetic to that of the other five markets for which  $\psi_{i,1} < 0$  since the pricing error adjustment exhibits a memory (or temporary overreaction) effect rather than adopting a corrective character. In this respect, given that  $\psi_i < 1$ , the dampened time  $t-1$  pricing error is carried forward to the time- $t$  pricing result. The null  $c_{i,2} = 0$  is also rejected at the 10% level for Spain. In contrast to  $\psi_{i,1}$ , fundamental feedback in the Spanish case appears to be negative (viz. offsetting or momentum-negating) and restricted to the global factor.

Significant positive feedback is, however, observed for Singapore where the common feedback term  $c_{i,2}$  is clearly in the positive range (the null hypothesis  $\psi_{i,1} = 0$  cannot be rejected). According to the Singapore result, updated global information is carried forward to the next period return whereas any country specific excess is deemed irrelevant. The result may be interpreted as the converse of the Australian result in terms of the prevalence of only one component of the common versus country specific lagged effect structure. Further, the Singapore result may be contrasted with the results for Sweden, Switzerland and Germany since  $c_{i,2}$  is in the negative range for the aforementioned European markets notwithstanding the similarly negligible (country specific) persistence observed for all four markets.

#### *4.3.1 Interpretation for France, the Netherlands, the UK, and the US*

The set of results  $\psi_{i,1} < 0$ ,  $c_{i,2} < 0$  suggests a corrective reaction to both pricing error and fundamental feedback for the four relevant markets. In this case of  $\psi_{i,1} < 0$ , a

corrective response is observed in the following (weekly) period. This result is consistent with the hypothesised presence of rational speculators or arbitrageurs who, predominantly, engage in the rapid correction of trading patterns induced by idiosyncratic (or country specific) feedback or noise traders.

Negative sensitivity to the lagged common factors,  $c_{i,2} < 0$ , implies a contrarian response to updated common information. The significant impact of the feedback term suggests the non-negligible presence of feedback or noise traders for France, the Netherlands, the UK, and the US. Given the presence of positive first-order persistence in the global factor,  $\phi_w(1) > 0$ , the negative sensitivity term  $c_{i,2}$  also operates as an offset to the extrapolative (or momentum inducing) pricing behaviour observed for the global factor.

The jointly corrective behaviour inherent in the result  $\psi_{i,1} < 0$ ,  $c_{i,2} < 0$  also leads to the conclusion that rational speculators generally trade away the effects of idiosyncratic pricing or fundamental momentum such that the rational pricing form  $E(r_{i,t} | I_{t-1}) = c_{i,1}' \mathbf{f}_{t|t-1} = c_{i,1,1} W_{t|t-1} + c_{i,2,1} E_{t|t-1}$  appears to be a reasonable approximation of longer-term (for example, monthly) returns for the four markets. The rational pricing form is obtained exactly in the case where both feedback terms are considered irrelevant by investors. Given the additional result  $E_{t|t-1} \approx 0$ , the presence of the European factor does not appear to provide any additional pricing information relative to the global factor model for the developed European equity markets.

Daniel, Hirshleifer, and Subrahmanyam (1988, 2001) and De Long et al. (1990a,b) propose models whereby momentum in asset returns is driven by overconfidence or by the actions of feedback traders. De Long et al. (1990a,b) also suggest that rational speculators may engage in feedback trading given the reasonable anticipation of future feedback trading. The results for the US, UK, France, and the Netherlands admit the possibility of mispricing but suggest that mispricing effects are generally offset on a week by week reference period. Any momentum for the relevant markets, including momentum of the form given in the aforementioned papers, is therefore most likely to be observed in higher (than weekly) frequency data. Hong, Lim and Stein (2000) propose a relationship between momentum and the rate of diffusion of private information, while

Miller (1977) and Scheinkman and Xiong (2003) suggest that momentum levels may be related to the presence of short-sale constraints limiting the capacity for offsetting over-optimistic trading. In this respect, possible explanations for the failure to observe any momentum in larger markets such as the US, the UK and France may reasonably be given by the rapid diffusion of information and/or the existence of sufficient short-sale capacity.

#### *4.3.2 Interpretation for the remaining markets*

The null hypothesis given by  $\theta_0$  is rejected at the 5% level for Australia, Germany, Singapore, Spain, Sweden and Switzerland, and at the 10% level for Belgium. Exact rational pricing is, therefore, rejected for 10 of the 18 developed markets at the 5% level and 11 of the 18 markets at the 10% level. The mispricing term  $\psi_i$  is concentrated below zero for Australia (at the 5% level), Germany, and Sweden, and above zero for Spain (at the 10% level). It appears, then, that mispricing persists beyond the weekly period for the latter market in contrast to the inter-period corrective reaction applicable to the former markets (and the US, UK, France and the Netherlands).

The joint-test of sensitivity to updated common information  $\theta_2$  is rejected at the 5% level for Germany, Singapore, Sweden, and Switzerland and at the 10% level for Belgium and Spain. The latter two markets exhibit significant (at the 5% level) marginal sensitivity to updated information corresponding to the European and global sources respectively. The sensitivity term is in the positive direction for Singapore whereas the converse holds true for Belgium, Germany, Spain, Sweden and Switzerland. Given both sets of results, the strongest evidence of persistent mispricing at the weekly frequency over the period of study is found for Singapore, where significant (at the 5% level) overreaction to revised fundamental information is observed. Weaker evidence of overreaction, grounded in persistent idiosyncratic mispricing, is available for the Spanish and Austrian cases. In contrast to the evidence suggesting an offsetting (or momentum-negating) response to historical information for the US, UK, France, and the Netherlands, the opposing scenario of significant overreaction to both fundamental and idiosyncratic sources of historical information is not observed for any market.

Expected returns for eight of the developed markets do not appear to exhibit a pricing error adjustment, or sensitivity to lagged common information.<sup>9</sup> The implication resulting from  $\psi_{i,1} = 0$  is that expected returns are not derived subject to an adjustment based on past prediction errors. Prediction errors, resulting in either mean-reverting expectations or short-term idiosyncratic memory, are therefore an irrelevant consideration in determining expected returns. A second implication stemming from  $c_{i,2} = 0$  is that updated information attributable to the set of time  $t-1$  common factors impacts negligibly on time  $t$  pricing. In the context of the speculative trader framework, the non-negligible presence of feedback traders may be deduced for Australia, France, Germany, the Netherlands, Singapore, Spain, Sweden, Switzerland, the UK, and the United States.<sup>10</sup> It would, therefore, appear that expected returns for the minority eight markets failing to reject  $\theta_0$  at the 5% level follow:

$$E\left(r_{i,t} \mid \psi_{i,1} = c_{i,2} = 0, I_{t-1}\right) = c_{i,1} \mathbf{f}_{t|t-1} = c_{i,1,1} W_{t|t-1} + c_{i,2,1} E_{t|t-1}. \quad (20)$$

The failure to reject  $\psi_{i,1} = c_{i,2} = 0$ , in tandem with the adopted factor structure, suggests the prima facie general validity of the more general (conditional) APT or ICAPM pricing theories for the relevant markets. In the ICAPM sense,  $W_t$  is representative of an observed world portfolio while  $E_t$  may be interpreted as a hedge for European investors. Pursuant to the result  $\phi_e(L) = 1$ , the conditional expectation of the European factor-portfolio is zero and provides no incremental explanatory capacity in terms of expected returns. The expected return for the  $i$ th market therefore collapses to:

$$E\left(r_{i,t} \mid \psi_{i,1} = c_{i,2} = 0, I_{t-1}\right) = c_{i,1,1} W_{t|t-1}. \quad (21)$$

Consequently, expected returns for the specified markets (Austria, Belgium, Canada, Denmark, Hong Kong, Italy, Japan and Norway) are determined solely by reference to

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<sup>9</sup> Austria, Belgium, Canada, Denmark, Hong Kong, Italy, Japan and Norway fail to reject  $\theta_0$  at the 5% level. Belgium sensitivity to lagged European information, however, is significant at the 5% level and its hypothesis  $\theta_0$  is rejected at the 10% level.

<sup>10</sup> The presence of fundamental or idiosyncratic feedback is deemed to be non-negligible if the null hypothesis  $\theta_0$  is rejected at the conventional 5% level.

the expected value of the global factor in a manner consistent with the spirit of the world CAPM.

## **5. Conclusion**

A number of scenarios were proposed whereby equity market prices are derived according to the conditional variants of the multi-beta CAPM or exact APT approaches subject to the possibility of common or idiosyncratic adjustments to the rational pricing forms. The adjustments are capable of introducing feedback or corrective behaviour into market prices.

Evidence of feedback or reactionary effects is available for ten of the eighteen markets at the weekly frequency. France, the Netherlands, the UK, and the US constitute special cases since they exhibit non-zero sensitivity to both updated common and idiosyncratic information; the sensitivity is corrective in both cases suggesting that any price momentum is traded away rather than exacerbated.

In contrast, the converse characteristics of positive common feedback and idiosyncratic overreaction do not appear jointly present in any market. Evidence regarding the significant presence of positive common feedback is, however, available for Singapore while weaker evidence of idiosyncratic overreaction is available for Spain.

The evidence negating the existence of positive common feedback or idiosyncratic overreaction is significant given the preponderance of momentum-driven trading strategies. In this respect, the capacity to exploit positive momentum or overreaction appears to be traded away in weekly terms. As such, the results effectively limit the validity of the positive feedback or momentum hypotheses to higher than weekly return frequencies for major securities in developed markets or developed index-based assets.

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