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Abstract

When measuring poverty with counting measures, it is worth inquiring into the conditions prompting poverty reduction which not only reduce the average poverty score further but also decrease deprivation inequality among the poor, thereby emphasizing improvements among the poorest of the poor. For comparisons of cross-sectional datasets of the same society in different periods of time (i.e. an anonymous assessment), the literature offers a second-order dominance condition based on reverse generalized Lorenz curves, whose fulfillment ensures that multidimensional poverty decreases along with a reduction in deprivation inequality for a broad family of inequality-sensitive counting poverty measures. However, the condition holds for a predetermined vector of weights for the poverty dimensions. In this paper we refine this second-order condition in order to obtain necessary and sufficient conditions whose fulfillment ensures that multidimensional poverty reduction is robust to a broad array of weighting vectors and inequality-sensitive poverty measures. We illustrate these methods with an application to multidimensional poverty in Peru.

JEL classification: I32

Keywords: Pro-poorest poverty reduction, multidimensional poverty, reverse generalized Lorenz curve

1 Introduction

The “pro-poor" nature of income or per-capita GDP growth has received much attention from both academics and policymakers for the last couple of decades. While a straightforward notion regards income growth to be “pro-poor" when the poor’s incomes rise, a more interesting notion declares growth to be “pro-poor" when the income of the poorest grows faster than the income of the less poor. Whenever income grows monotonically faster at lower initial quantiles, “pro-poor" growth reduces inequality according to a broad family of Lorenz-consistent measures. The related literature on pro-poor concepts, dominance conditions and indices is vast (see e.g. Deutsch and Silber, 2011, for a review).

Now the “pro-poor" growth literature has traditionally worked with one continuous variable. However, recently there has been an interest in connecting the “pro-poor" growth concepts with non-monetary measures of well-being, and multidimensional poverty indices in particular. For instance, Berenger and Bresson (2012) provide dominance conditions to probe the “pro-poorness" of growth when well-being is measured jointly by continuous and discrete variables. Ben Haj Kacem (2013) measures the “pro-poorness" of growth in income when the initial conditioning situation is not income itself but a non-monetary multidimensional index of poverty or well-being. Boccanfuso et al. (2009) apply the now traditional “pro-poor" growth toolkit to assess changes in the individual scores of a non-monetary poverty composite index, where the weights are determined by multiple correspondence analysis (MCA). Since they use a vast number of indicators, their scores can take several values, thereby mimicking a continuous variable.

In this paper we pose a related question in the context of *multidimensional poverty counting measures*: What are the conditions under which a poverty reduction experience is more “pro-poorest" than another one? In other words, under which conditions does poverty reduction not only reduce the average poverty score further but also decrease deprivation inequality among the poor more? In order to answer these questions we first address the most common anonymous assessment which compares cross-sectional datasets of the same society in different periods of time. In this context, Boccanfuso et al. (2009) have already shown a way in which the “pro-poor” measurement toolkit for continuous variables can be applied to the case of non-monetary deprivations if a composite index is constructed based on them, using data reduction techniques (e.g. MCA). However, in many empirical applications, the number of indicators may not be large enough, so that the number of values that the individual deprivation score can take is quite limited, for a given set of weights and deprivation lines. Hence, in such situations, an anonymous assessment of “pro-poor” growth linking initial-period and final-period quantiles is not really feasible.

Instead we propose the use of a second-order dominance condition based on *reverse generalized Lorenz curves (RGL)*, which in turn are built using a counting poverty index developed by Alkire and Foster (2011) and called the "Adjusted head-count ratio". The condition's fulfillment ensures that multidimensional poverty decreases along with a reduction in deprivation inequality for a broad family of inequality-sensitive poverty measures.

The RGL curve is the counting-measure equivalent of the TIP curve (Jenkins and Lambert, 1997), and its associated second-order dominance condition was originally proved by Lasso de la Vega (2010) and Chakravarty and Zoli (2009, 2012).[1] However, when the condition holds, the poverty comparison is robust to the choice of several poverty indices and multidimensional poverty cut-offs, *but only for a particular vector of deprivation weights*. That is, should the weights change, the second-order condition may not hold. Hence our main contribution in this paper is to refine the second-order dominance condition based on RGL curves, in order to obtain necessary and sufficient conditions whose fulfillment ensures that multidimensional poverty reduction is robust, not only to different multidimensional poverty cut-offs and inequality-sensitive poverty indices, but also to a broad array of weighting vectors. Firstly, we derive a condition whose fulfillment is both necessary and sufficient to ensure second-order dominance *for every conceivable weighting vector*. Secondly, we derive a batch of useful conditions whose fulfillment is necessary, but insufficient, to ensure the same second-order dominance for every conceivable weighting vector. While limited by their insufficiency, these latter conditions are much easier to test. Their failure to hold immediately rules out the full robustness of second-order dominance to any choice of weights. [2]

We illustrate the anonymous conditions using cross-sections from the Peruvian National Household Surveys corresponding to 2002 and 2013. During this period, Peru experienced a commodity boom, which translated into high GDP growth rates, from 4% in 2003 to 8.9% in 2007, and a steady decrease in monetary poverty headcounts, from 58.7% in 2004 to 42.4% in 2007. However, between 2008 and 2013, Peru's economic performance was affected by the world economic situation: GDP growth fell from 9.8% in 2008 to 0.9% in 2009, and then stabilizing around 7% between 2010 and 2012. Notwithstanding this fluctuation, monetary poverty levels kept decreasing steadily, from 37.3 to 27.8%. But how did the Peruvian population fare in terms of non-monetary multidimensional poverty? We measure

[1] There is also a first-order condition (which implies the second-order one) developed by Lasso de la Vega (2010) and Alkire and Foster (2011).

[2] As shown by Grimm (2007), we can also perform a non-anonymous assessment of pro-poorest poverty reduction, when we have a panel dataset. We pursue this line of inquiry in a companion paper (Gallegos and Yalonetzky, 2014).

non-monetary poverty with wellbeing indicators corresponding to four dimensions: household education, dwelling material infrastructure, access to services, and vulnerability related to household dependency burden.

We compare RGL curves between 2002 and 2013 for the whole country, for urban and rural areas, and for each of the 25 Peruvian departments (provinces) and autonomous territories. We find that the observed egalitarian reduction of our measure of poverty, between 2002 and 2013, is robust to different choices of poverty functions and poverty-identification cut-offs for national, urban and rural samples, as well as in 22 out of the 25 departments. Then we test whether the robust cases of egalitarian poverty reduction are also fully robust to every conceivable weight using two of our necessary conditions. Remarkably, we find that the identified situations of robust egalitarian poverty reduction in the national, urban and rural samples are not robust to different weighting choices.

The rest of the paper proceeds as follows: The next section presents the "pro-poorest" poverty-reduction conditions for the case of fixed weights. First, it introduces the family of counting poverty measures for which the conditions are relevant and applicable, then it shows the conditions for the anonymous case. The third section develops the refinements whose fulfillment enable the second-order condition to hold over a range of deprivation weights. The fourth section briefly explains the statistical tests. The fifth section provides the empirical illustration on multidimensional poverty reduction in Peru. Finally, the paper concludes with some remarks.

2 Pro-poorest poverty reduction with counting measures

2.1 Inequality-sensitive poverty measures

Consider N individuals and $D > 1$ indicators of wellbeing. x_{nd} stands for the level of attainment by individual n on indicator d. If $x_{nd} < z_d$, where z_d is a deprivation line for indicator d (i.e. an element from a D-dimensional vector of deprivation lines, Z), then we say that individual n is deprived in indicator d. In order to account for the breadth of deprivations, counting measures rely on individual deprivation scores which produce a weighted count of deprivations. The weights are elements of a vector $W := (w_1, w_2, ..., w_D)$ such that $w_d > 0 \ \wedge \ \sum_{d=1}^{D} w_d = 1$.

The deprivation score for individual n is: $c_n \equiv \sum_{d=1}^{D} w_d \mathbb{I}(x_{nd} < z_d)$, where $\mathbb{I}$ is the indicator function. [3] Following Alkire and Foster (2011) we can also identify those

[3] Taking the value of 1 if the argument in parenthesis is true, otherwise it is equal to 0.

multidimensionally poor with a flexible counting approach that compares each c_n against a multidimensional cut-off $k \in [0,1] \subset \mathbb{R}_+$, so that person n is poor if and only if: $c_n \geq k$.

Our analysis focuses on a family of social poverty counting measures that are symmetric across individuals, additively decomposable (hence also subgroup consistent), scale invariant and population-replication invariant. If $p_n : c_n \times k \to [0,1] \in \mathbb{R}_+$ is the individual poverty measure, and $P : [0,1]^N \to [0,1]$ is the social poverty measure then our family is the following:

$$P = \frac{1}{N} \sum_{n=1}^{N} p_n \tag{1}$$

Our conditions of pro-poorest poverty reduction will also be useful for a broader family of subgroup consistent measures: $Q = H(P)$ as long as $H()$ is a strictly increasing, continuous function. For the sake of subgroup consistency, the weights must be set exogenously. Additionally we want P to fulfill the following key properties:

Axiom 1. *Focus (FOC): P should not be affected by changes in the deprivation score of a non-poor person as long as for this person it is always the case that: $c_n < k$.*

Axiom 2. *Monotonicity (MON): P should increase whenever c_n increases and n is poor.*

Axiom 3. *Progressive deprivation transfer (PROG): A rank-preserving transfer of a deprivation from a poorer individual to a less poor individual, such that both are deemed poor, should decrease P.*

In relation to the latter axiom, there are different approaches to capture sensitivity to deprivation inequality in the literature, although most of the approaches are virtually equivalent.[4] Axiom PROG is critical to the assessment of "pro-poorest" poverty reduction, as it forces social poverty indices to be sensitive to the distribution of deprivation across the poor, and to prioritize the wellbeing of the most jointly deprived among them.

In order to fulfill the above key properties, we narrow down the family of social poverty indices by rendering the functional form of p_n less implicit:

$$P = \frac{1}{N} \sum_{n=1}^{N} \mathbb{I}(c_n \geq k) g(c_n), \tag{2}$$

where $\mathbb{I}(c_n \geq k)$ is the Alkire-Foster poverty identification function that also secures the fulfillment of FOC; and $g : c_n \to [0,1]$, such that: $g(0) = 0$, $g(1) = 1$, $g' > 0$

[4]For a comparative review of these approaches see Silber and Yalonetzky (2013). A different framework is provided by Alkire and Seth (2014).

and $g'' > 0$. The function g captures the intensity of poverty, which is positively related to the number of deprivations in the counting approach. Several examples of g have been proposed by Chakravarty and D'Ambrosio (2006).

Finally, we introduce four counting poverty statistics that play key roles in the dominance conditions. Firstly, the multidimensional poverty headcount, i.e. the proportion of people whose score is at least as high as k, the multidimensional poverty cut-off:

$$H(k) = \frac{1}{N}\sum_{n=1}^{N}\mathbb{I}(c_n \geq k). \tag{3}$$

Secondly, we rely on the adjusted headcount ratio proposed by Alkire and Foster (2011) in order to construct the RGL curves. It is basically a particular case of P in which $g(c_n) = c_n$:

$$M(k) = \frac{1}{N}\sum_{n=1}^{N}\mathbb{I}(c_n \geq k)c_n \tag{4}$$

Thirdly, we consider the *uncensored* deprivation headcount ratio, which measures the proportion of people deprived in variable/dimension d *irrespective* of their overall poverty status:

$$U_d \equiv \frac{1}{N}\sum_{n=1}^{N}\mathbb{I}(x_{nd} < z_d). \tag{5}$$

The statistics in (5) are crucial to condition 5 below. Fourthly, we define the *sub-dimensional intersection headcounts*, which measure the proportion of people who are *only* deprived in a particular subset of variables/dimensions. For example subdimensional intersection headcount for people who are deprived only in dimensions i and j would be:

$$H_{i,j} \equiv \frac{1}{N}\sum_{n=1}^{N}\mathbb{I}(x_{ni} < z_i \wedge x_{nj} < z_j \wedge x_{nd} > z_d \forall d \neq \{i,j\}). \tag{6}$$

The statistics in (6) are crucial to our condition 3 below. Note that, naturally: $H_{1,2,\ldots,D} = H(1)$. That is, if we choose all variables, then the sub-dimensional intersection headcount is basically the intersection headcount, i.e. the proportion of people who are deprived in each and every possible dimension.

2.2 The anonymous case

In the counting approach, there is only one vector of possible values of c_n for each particular choice of deprivation lines and weights. Moreover it is easy to show that the maximum number of possible values is given by: $\sum_{i=0}^{D}\binom{D}{i} = 2^D$. In the particular, but common, case of equal weights ($w_d = \frac{1}{D}$), the number of possible values is much smaller: $D+1$. Hence the distribution of c_n in the sample is bound to be discrete, as there will be several individuals for every value of c_n. The vector of possible values is defined as: $V := (v_1, v_2, ..., v_l)$, where $\max l = 2^D$, $v_i < v_{i+j}$, $v_1 = 0$ and $v_l = 1$.

In this subsection we show that for an assessment of inequality-reducing poverty reduction in the anonymous case it is necessary and sufficient to compare the adjusted headcount ratios at the beginning and at the end of the time period.

Let P^A and $H^A(k)$ refer, respectively, to the social poverty index and the multidimensional headcount of population A. The following first-order condition enables us to assert whether a poverty reduction experience is robust to any counting poverty index satisfying the basic properties of FOC and MON (i.e. including those satisfying PROG, but not restricted to them):

Condition 1. *$P^A < P^B$ for all P in (2) satisfying FOC and MON, if and only if* $H^A(k) \leq H^B(k) \quad \forall k \in [0, v_2, ..., 1] \quad \wedge \exists k | H^A(k) < H^B(k)$.

Proof. See Lasso de la Vega (2010). □

If A stands for final period, and B stands for initial period, then whenever condition 1 is fulfilled, any experience of poverty reduction is robust to any choice of poverty function satisfying FOC and MON, *for every relevant value of* k.

Now let $M^A(k)$ refer to the adjusted headcount ratio, hence the RGL curve, of population A. The following second-order condition enables us to assert whether a poverty reduction experience has been "pro-poorest" according to any inequality-sensitive poverty index:

Condition 2. *$P^A < P^B$ for all P in (2) satisfying FOC, MON and PROG, if and only if* $M^A(k) \leq M^B(k) \quad \forall k \in [0, v_2, ..., 1] \quad \wedge \exists k | M^A(k) < M^B(k)$.

Proof. See Lasso de la Vega (2010) and Chakravarty and Zoli (2009). □

If A stands for final period, and B stands for initial period, then whenever condition 2 is fulfilled, any experience of poverty reduction occurs alongside decreasing inequality among the poor, as measured by indices satisfying PROG, *for every relevant value of* k. The following remark links condition 1 to 2:

Remark 1. *If* $H^A(k) \leq H^B(k) \quad \forall k \in [0,1] \quad \wedge \exists k | H^A(k) < H^B(k)$ *then* $M^A(k) \leq M^B(k) \quad \forall k \in [0, v_2, ..., 1] \quad \wedge \exists k | M^A(k) < M^B(k)$.

Proof. See Alkire and Foster (2011, Theorem 2). □

Finally, conditions 1 and 2 can also be restricted to apply only to a subset of relevant k values, ruling out the lowest ones below a minimum k: k_{min}. In order to proceed this way, we construct censored deprivation scores such that: $c_n = 0$ whenever $c_n < k_{min}$. Then conditions 1 and 2 apply only to those P which rule out poverty identification approaches with $k < k_{min}$.

3 The case of variable deprivation weights

When condition 2 is fulfilled we can assert that poverty decreased or increased consistently, in the sense that the result is robust to different choices of inequality-sensitive social poverty indices and counting identification approaches. A similar conclusion is warranted for condition 1 in relation to any social poverty index of the form (2) (and its monotonic transformations) satisfying FOC and MON. However these conditions only hold for specific choices of deprivation lines and dimensional weights. With alternative selections of the latter two parameter sets, the conditions would need to be tested again, in principle.

In this section we show that some extra conditions, complementary refinements of condition 2, can help us determine whether a second-order dominance relationship based on the adjusted headcount-ratios would be robust to a vast array of deprivation weighting choices. We present, firstly, the necessary and sufficient condition whose fulfillment guarantees the robustness of condition 2 to a *any possible choice* of strictly positive weights. Then we present a set of conditions whose fulfillment is necessary (but insufficient) to guarantee the robustness of condition 2 to *any possible choice* of strictly positive weights. The latter's advantage reside in their easier implementation for testing purposes vis-a-vis the necessary and sufficient condition.

3.1 Necessary and sufficient condition

Condition 3 is both necessary and sufficient to ensure the fulfilment of condition 2 for any choice of weighting vectors:

Condition 3. $M^A(k) \leq M^B(k) \;\; \forall k \in [0, v_2, ..., 1] \;\; \wedge \exists k | M^A(k) < M^B(k)$ *for all possible weighting vectors W if and only if no sub-dimensional intersection headcount in A is higher than in B and there exists at least one sub-dimensional intersection headcount whose value is strictly lower in A relative to B.*

Proof. First note that: $M(k) = \frac{1}{N}\sum_{n=1}^{N} \mathbb{I}(c_n \geq k) c_n$, and $c_n = \sum_{d=1}^{D} w_d \mathbb{I}(x_{nd} < z_d)$. Then we can express the adjusted headcount ratio as a weighted sum of censored deprivation headcounts, i.e. the proportions of people both deemed poor and deprived in one particular variable: $M(k) = \sum_{d=1}^{D} w_d Q_d$ where $Q_d(k) = \frac{1}{N}\sum_{n=1}^{N} \mathbb{I}(c_n \geq k)\mathbb{I}(x_{nd} < z_d)$. So we get:

$$M^A(k) - M^B(k) = \sum_{d=1}^{D} w_d[Q_d(k)^A - Q_d(k)^B]. \tag{7}$$

The next step is to show that all censored deprivation headcounts, $H_d(k)$, can be expressed as sums of sub-dimensional intersection headcounts:

$$Q_d(k) = \frac{1}{N}\sum_{n=1}^{N} \mathbb{I}(c_n \geq k)[\mathbb{I}(x_{nd} < z_d \wedge x_{ni} > z_i \forall i \neq d) + \mathbb{I}(x_{nd} < z_d \wedge x_{nj} < z_j \wedge x_{ni} > z_i \forall i \neq \{d, j\}) + \dots \\ + \mathbb{I}(x_{nd} < z_d \forall d)] \tag{8}$$

It should be straightforward to note that the right-hand side of (8) is a sum of sub-dimensional intersection headcounts. For example, given a choice of Z, W and k, if $x_{nd} < z_d \wedge x_{ni} > z_i \forall i \neq d$ is incompatible with $c_n \geq k$ (i.e. none who is only deprived in d would be deemed poor) then H_d will not appear on the right-hand side of (8); whereas if, say, $x_{nd} < z_d \wedge x_{nj} < z_j \wedge x_{ni} > z_i \forall i \neq \{d, j\}$ is compatible with $c_n \geq k$ then $H_d(k)$ will be a function of $H_{d,j}$ along with other sub-dimensional intersection headcounts involving equal or higher total deprivation counts (e.g. $H_{d,j,l}$, $H(1)$, etc.).

Hence we can write (7) in terms of sums of sub-dimensional intersection headcounts:

$$M^A(k) - M^B(k) = \sum_{d=1}^{D} w_d[S_d(k)^A - S_d(k)^B]. \tag{9}$$

Where $S_d(k)$ is the sum of all sub-dimensional intersection headcounts that: (1) involve a deprivation in d, and (2) are compatible with $c_n \geq k$ in the sense that all people included in those headcounts are also deemed poor.

Now from (9) it is clear that if all sub-dimensional intersection headcounts in A are either equal or lower than their counterparts in B (with at least one strictly lower) then $S_d(k)^A \leq S_d(k)^B \ \forall d$ and $\exists d | S_d(k)^A < S_d(k)^B$. Then this implies $M^A(k) \leq M^B(k) \ \ \forall k \ \ \wedge \exists k | M^A(k) < M^B(k)$ for all possible W. This proves the sufficiency part of condition 3.

The necessity of having equal or lower sub-dimensional intersection headcounts in A relative to B can be established by considering different weighting vectors and poverty cut-offs (k). For example, if we choose $k = 1$ then $H(1)^A > H(1)^B$ will lead to $M^A(1) > M^B(1)$, hence $H(1)^A \leq H(1)^B$ is a necessary condition (given that $k = 1$ is an admissible option for the poverty cut-off). Likewise, if we choose weights subject to $[w_i + w_j] \to 1$, then $S_i(k)^A - S_i(k)^B$ and $S_j(k)^A - S_j(k)^B$ will have to be non-positive.

If we raise k from 0 to 1, given that $[w_i + w_j] \to 1$, then we can show that each element of $S_i(k)^A$ and of $S_j(k)^A$ cannot be higher than its respective counterpart in B. With similar reasonings we can prove the necessity part of condition 3: if $M^A(k) \le M^B(k) \quad \forall k \quad \wedge \exists k | M^A(k) < M^B(k)$ for all possible W then it must be the case that all sub-dimensional intersection headcounts in A are either equal or lower than their counterparts in B.

□

3.2 General necessary conditions

Testing condition 3 requires computing and comparing $2^D - 1$ pairs of statistics (e.g. 255 pairs if $D = 8$). However there is a much smaller set of useful necessary conditions whose violations implies that condition 2 is not robust to any possible choice of W. The first of these necessary condition is the following:

Condition 4. *If* $M^A(k) \le M^B(k) \quad \forall k \in [0, v_2, ..., 1] \quad \wedge \exists k | M^A(k) < M^B(k)$ *for all possible weighting vectors* W *then:* $H^A(1) \le H^B(1)$.

Proof. Whenever a person is deprived in every dimension, then $c_n = 1$ irrespective of the weighting vector. Therefore, in those cases: $M^A(1) = H^A(1)$. Since $M^A(k) \le M^B(k) \quad \forall k \in [0, v_2, ..., 1]$ naturally implies that $M^A(1) \le M^B(1)$, it must then be the case that: $H^A(1) \le H^B(1)$. □

Condition 4 states that if condition 2 holds for every possible weighting vector W favouring population A over B then it must be the case that the percentage of people deprived in every dimension in A (i.e. following an intersection approach to poverty identification) cannot be higher than the percentage of people from B in the same situation. This is a simple but powerful condition. It basically means that without even computing the adjusted headcount ratios, we can compute just $H(1)$, and if we get $H^A(1) > H^B(1)$, then we can rule out the possibility of finding any vector W whatsoever with which A dominates B according to condition 2.

However, condition 4 is not sufficient to ensure dominance of A over B whenever $H^A(1) \le H^B(1)$ (since, e.g. it could be the case that $M^A(k) > M^B(k)$ for some $k < 1$).

The second necessary condition is:

Condition 5. *If* $M^A(k) \le M^B(k) \quad \forall k \in [0, v_2, ..., 1] \quad \wedge \exists k | M^A(k) < M^B(k)$ *for all possible weighting vectors* W *then:* $U_d^A \le U_d^B \ \forall d \in [1, 2, ..., D]$.

Proof. Condition 5 can be derived from part of the proof for condition 3. However the following is an alternative, more straightforward proof:

First note that: $M(0) = \frac{1}{N} \sum_{n=1}^{N} c_n$, and $c_n = \sum_{d=1}^{D} w_d \mathbb{I}(x_{nd} < z_d)$. Then it is easy to deduce that: $M(0) = \sum_{d=1}^{D} w_d U_d$ and so:

$$M^A(0) - M^B(0) = \sum_{d=1}^{D} w_d[U_d^A - U_d^B]. \tag{10}$$

Now add the fact that: if $M^A(k) \leq M^B(k) \quad \forall k \in [0,1]$ then it must be the case that: $M^A(0) \leq M^B(0)$.

Finally we need to prove that: $M^A(0) \leq M^B(0)$ for any vector W if and only if $U_d^A \leq U_d^B \ \forall d \in [1,2,...,D]$:

1. $H_d^A \leq H_d^B \ \forall d \in [1,2,...,D]$ implies $M^A(0) \leq M^B(0)$. This follows from direct inspection of (10). If $w_d > 0$ then the right-hand side being non-positive renders the left-hand side non-positive as well.

2. $M^A(0) \leq M^B(0)$ for any vector W implies $U_d^A \leq U_d^B \ \forall d \in [1,2,...,D]$. If $\exists d | H_d^A > H_d^B$ then we could find a vector W (especially with $w_d \to 1$)such that $M^A(0) > M^B(0)$. □

Condition 5 states that if condition 2 holds for every possible weighting vector W favouring population A over B then it must be the case that all the uncensored deprivation headcount ratios in A cannot be higher than their respective counterparts from B. This is, again, a simple but powerful condition. Without computing the adjusted headcount ratios, if we get just one variable d for which $U_d^A > U_d^B$, then we can rule out the possibility that A dominates B for every conceivable weighting vector W, according to condition 2.

However, condition 5 is not sufficient to ensure dominance of A over B whenever $H_d^A(0) \leq H_d^B(0) \ \forall d \in [1,2,...,D]$ (since, e.g. it could be the case that $M^A(k) > M^B(k)$ for some $k > 0$).

Finally, while both conditions are necessary, they are not jointly sufficient to guarantee condition 2. But together they can provide further useful conclusions in the form of the following sufficient condition:

Condition 6. *If $H^A(1) > H^B(1)$ and $\exists d | H_d^A < H_d^B$ (or if $H^A(1) < H^B(1)$ and $\exists d | H_d^A > H_d^B$) then there will be at least one weighting vector W such that A and B cannot be ordered according to the stochastic-dominance criterion embedded in condition 2.*

Even though condition 6 is not necessary to ensure the violation of condition 2, its fulfillment guarantees that there is at least one vector W preventing either A dominating B or the other way around. Hence it provides a straightforward test of "curve-crossing" (where the curves' coordinates are either $(k, M(k))$, or $(H(k), M(k))$ as in Lasso de la Vega (2010)).

3.2.1 Extensions to different deprivation lines

The main results throughout this paper consider one fixed choice of Z. However the necessary conditions 4 and 5 are actually valid for any vector Z. Hence we can consolidate all the results from the previous section in the following set of necessary conditions:

Condition 7. *If* $M^A(k) \leq M^B(k) \quad \forall k \in [0, v_2, ..., 1] \quad \wedge \exists k|M^A(k) < M^B(k)$ *for all possible weighting vectors* W *and all possible deprivation-line vectors* Z *then: (1)* $H^A(1) \leq H^B(1)$ *for all all deprivation-line vectors* Z*; and (2)* $U_d^A \leq U_d^B \; \forall d \in [1, 2, ..., D]$ *for all possible deprivation-line vectors* Z.

4 Statistical inference

4.1 Test of condition 2

In the empirical illustration, we first test condition 2 for a particular choice of W (equal weights). While different tests are possible, we implement a convenient one in which a null hypothesis of $M^A(k) \leq M^B(k)$ for every relevant value of k (i.e. values that the score c_n can take given a set of weights and deprivation lines) is set against an alternative whereby $\exists k|M^A(k) > M^B(k)$. Basically if we do not reject the null we can state that either A dominates B or the two distributions perfectly overlap, which implies that poverty in A can never be above B for the given choice of weights and deprivation lines, and for any poverty function considered in condition 2. Alternatively, rejecting in favour of the alternative means that "A does not dominate B", i.e. either A is dominated by B or the two curves of adjusted headcount ratios cross (which in turn implies that the poverty comparison is sensitive to the choice of poverty functions and/or k, even for a given set of weights and deprivation lines).

In practice, we can have a joint intersection null hypothesis: $Ho : M^A(k) = M^B(k) \; \forall k \in [0, v_2, ..., 1]$ against a union alternative $Ha : \exists k|M^A(k) > M^B(k)$. For that purpose, and considering that A and B are independently distributed, we construct the following statistics:

$$T(k) = \frac{M^A(k) - M^B(k)}{\sqrt{\frac{\sigma^2_{M^A}(k)}{N^A} + \frac{\sigma^2_{M^B}(k)}{N^B}}}, \tag{11}$$

where:

$$\sigma^2_{M^A}(k) \equiv \frac{1}{N^A}\sum_{n=1}^{N^A}[c_n]^2 \mathbb{I}(c_n \geq k) - [M^A(k)]^2 \tag{12}$$

Then we test $Ho : T(k) = 0$ against $Ha : T(k) > 0$ for every relevant value of k. Given the requirements of condition 2, we conclude that A does not dominates B in terms of condition 2 if there is at least one k for which $T(k) > T_\alpha$, where T_α

is the right-tail critical value for a one-tailed "z-test" corresponding to a level of significance α. Since we test multiple comparisons, the actual size of the whole test is not α. Under reasonable assumptions, it can be shown that it is $\beta = \sum_{i=1}^{l}[l - i + 1]\alpha^i(-1)^{i-1}$. We choose $\alpha = 0.01$, so that $\beta \approx 0.05$.

4.2 Test of condition 4

The testing procedure relies on the formula in (11) but now we have only one comparison based on $T(1)$. Plus we note that in the case of $k = 1$ the formula for the variance simplifies to:

$$\sigma^2_{M^A}(1) = H(1)[1 - H(1)] \tag{13}$$

Then we test $Ho : T(1) = 0$ against $Ho : T(1) > 0$, using standard critical values for a one-tailed "z-test". If we reject the null then we conclude that A does not dominate B, irrespective of W.

4.3 Test of condition 5

The testing procedure is exactly the same as the one used for condition 2 but now we construct the following statistics:

$$T_d = \frac{U_d^A - U_d^B}{\sqrt{\frac{\sigma^2_{U_d^A}}{N^A} + \frac{\sigma^2_{U_d^B}}{N^B}}}, \tag{14}$$

where:

$$\sigma^2_{U_d^A} \equiv U_d^A[1 - U_d^A] \tag{15}$$

If we reject the null then we conclude that it is not true that A dominates B for every conceivable weighting vector W.

4.4 Test of condition 3

The testing procedure is exactly the same as the one used for condition 2 but now we construct statistics for each sub-dimensional intersection headcount, similar to those in (11) and (14). For example, in the case of $H_{i,j}$ we have:

$$T_{i,j} = \frac{H_{i,j}^A - H_{i,j}^B}{\sqrt{\frac{\sigma^2_{H_{i,j}^A}}{N^A} + \frac{\sigma^2_{H_{i,j}^B}}{N^B}}}, \tag{16}$$

where:

$$\sigma^2_{H^A_{i,j}} \equiv H^A_{i,j}[1 - H^A_{i,j}] \quad (17)$$

If we reject the null then we conclude that it is not true that A dominates B for every conceivable weighting vector W.

5 Empirical illustration: Multidimensional poverty in Peru

5.1 Background and data

As mentioned, Peru experienced a commodity boom between 2003 and 2007, which translated into high GDP growth rates, from 4 % in 2003 to 8.9 % in 2007, and a steady decrease in monetary poverty headcounts, from 58.7 % in 2004 to 42.4 % in 2007. However, between 2008 and 2013, Peru's economic performance was affected by the world economic situation: GDP growth fell from 9.8 % in 2008 to 0.9 % in 2009, and then stabilized at around 7 % between 2010 and 2013. Notwithstanding this fluctuation, monetary poverty levels kept decreasing steadily, from 37.3 to 27.8 %. How did the Peruvian population fare in terms of non-monetary multidimensional poverty?

For this anonymous assessment of robust egalitarian poverty reduction with counting measures, we use the Peruvian National Household Surveys (ENAHO) of 2002 and 2013. Our multidimensional poverty measure relies on four dimensions, and on the household as the unit of analysis. Firstly, household education, comprising two indicators: (1) school delay, which is equal to one if there is a household member in school age who is delayed by at least one year, and (2) incomplete adult primary, which is equal to one if the household head or his/her partner has not completed primary education. The household is considered deprived in education if any of these indicators takes the value of one.

The second dimension considers two indicators on infrastructure dwelling conditions: (i) overcrowding, which takes the value of one if the ratio of the number of household members to the number of rooms in the house is larger than three; and (ii) inadequate construction materials, which takes the value of one if the walls are made of straw or other (almost certainly inferior) material, if the walls are made of stone and mud or wood combined with soil floor, or if the house was constructed at an improvised location inadequate for human inhabitation. The household is deprived in living conditions if any of the above indicators takes the value of one.

The third dimension is access to services. The household is deemed deprived in this dimension if any of the following indicators takes the value of one: (i) lack of electricity for lighting, (ii) lack of access to piped water, (iii) lack of access to sewerage or septic tank, and (iv) lack of access to a telephone landline. The fourth dimension is household vulnerability to dependency burdens. The household is deprived or vulnerable if household members who are younger than 14 or older than 64 are three times or more as numerous as those members who are between 14 and 64 years old (i.e. in working age).

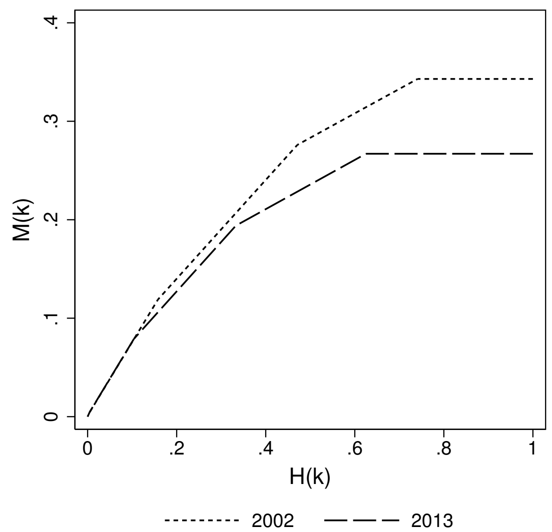
We weigh each dimension equally. Therefore the household score can take only any of the following five values: $(0, 0.25, 0.5, 0.75, 1)$.

5.2 Results

5.2.1 Point estimates

Figure 1 shows the reversed generalized Lorenz (RGL) curves for Peru in 2002 and 2013, based on the measurement choices of the previous subsection. The coordinates for the four kink points[5] in the figure represent combinations of the multidimensional headcount $H(k)$ (horizontal axis) and the adjusted headcount ratio $M(k)$ (vertical axis) for the same relevant value of k (1, 0.75, 0.5, 0.25, from the origin outward in our case). For 2002 these values are: (0.004,0.004), (0.158,0.119), (0.471,0.276), (0.741,0.343), (1,0.343). Meanwhile for 2013 they are: (0.005,0.005),

Figure 1 – Reverse generalized Lorenz curves of deprivation counts. Peru, 2002-2013.



[5]One of them is barely visible as it stands very close to the origin.

(0.104,0.079), (0.333,0.194), (0.625,0.267), (1, 0.267). The curve for 2013 is never above that of 2002, except for when $k = 1$. Hence condition 2 is not fulfilled. Based exclusively on the point estimates, we cannot conclude that, given a particular choice of deprivation lines and dimensional weights, multidimensional poverty in Peru decreased between 2002 and 2013, *along with a reduction in deprivation inequality among the poor*, for a broad family of inequality-sensitive poverty indices (at least those in (2)) and for any relevant choice of the poverty cut-off k.

Figure 2 shows the experiences of urban and rural areas between 2002 and 2013. Again, in both cases, the 2013 curve is below that of 2002, except for a reversal close to the origin, when $k = 1$. Hence, based only on the point estimates, we cannot confirm that both regions experienced poverty reduction accompanied by lower inequality among the poor for any inequality-sensitive index and choice of k.

Figure 3 shows the RGL curves for the five rainforest Peruvian departments (the dotted lines are for 2002, and the dashed lines for 2013). Except for the cases of San Martin and Ucayali, condition 2 is fulfilled, pointing to "pro-poorest" poverty reduction (dashed RGL lines representing 2013 appear always below dotted lines representing 2002). By contrast, in San Martin and Ucayali, the RGL curve is higher in 2013 for $k = 1$, meaning that the most severe forms of poverty (with $k = 1$) increased between 2002 and 2013.

Figure 2 – Reverse generalized Lorenz curves of deprivation counts. Urban and rural Peru, 2002-2013.

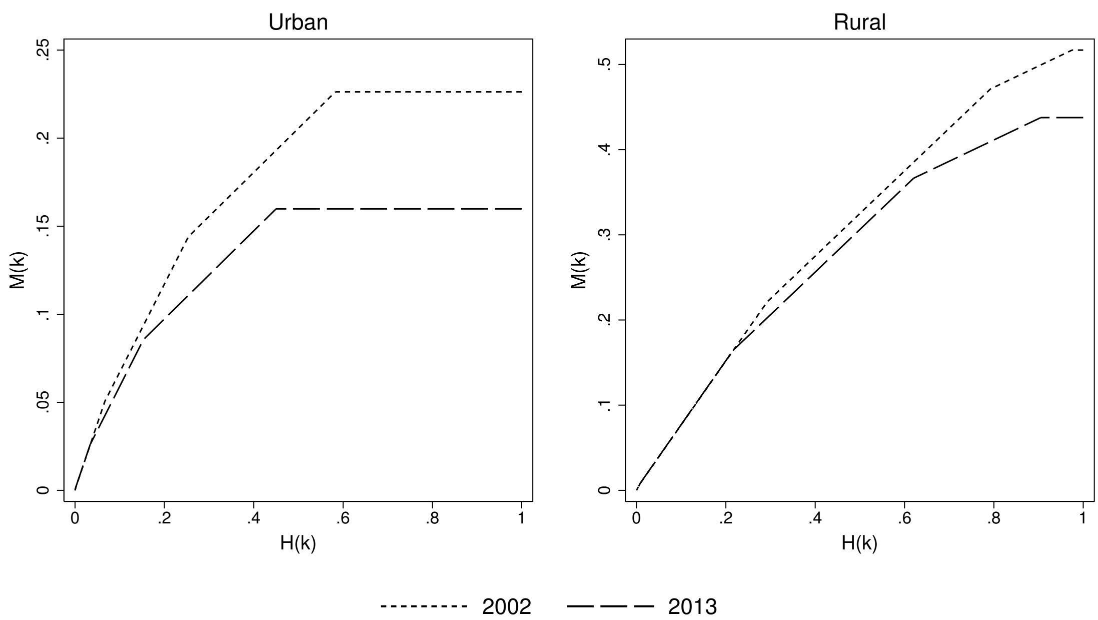


Figure 3 – Reverse generalized Lorenz curves of deprivation counts. Peruvian rainforest provinces, 2002-2013.

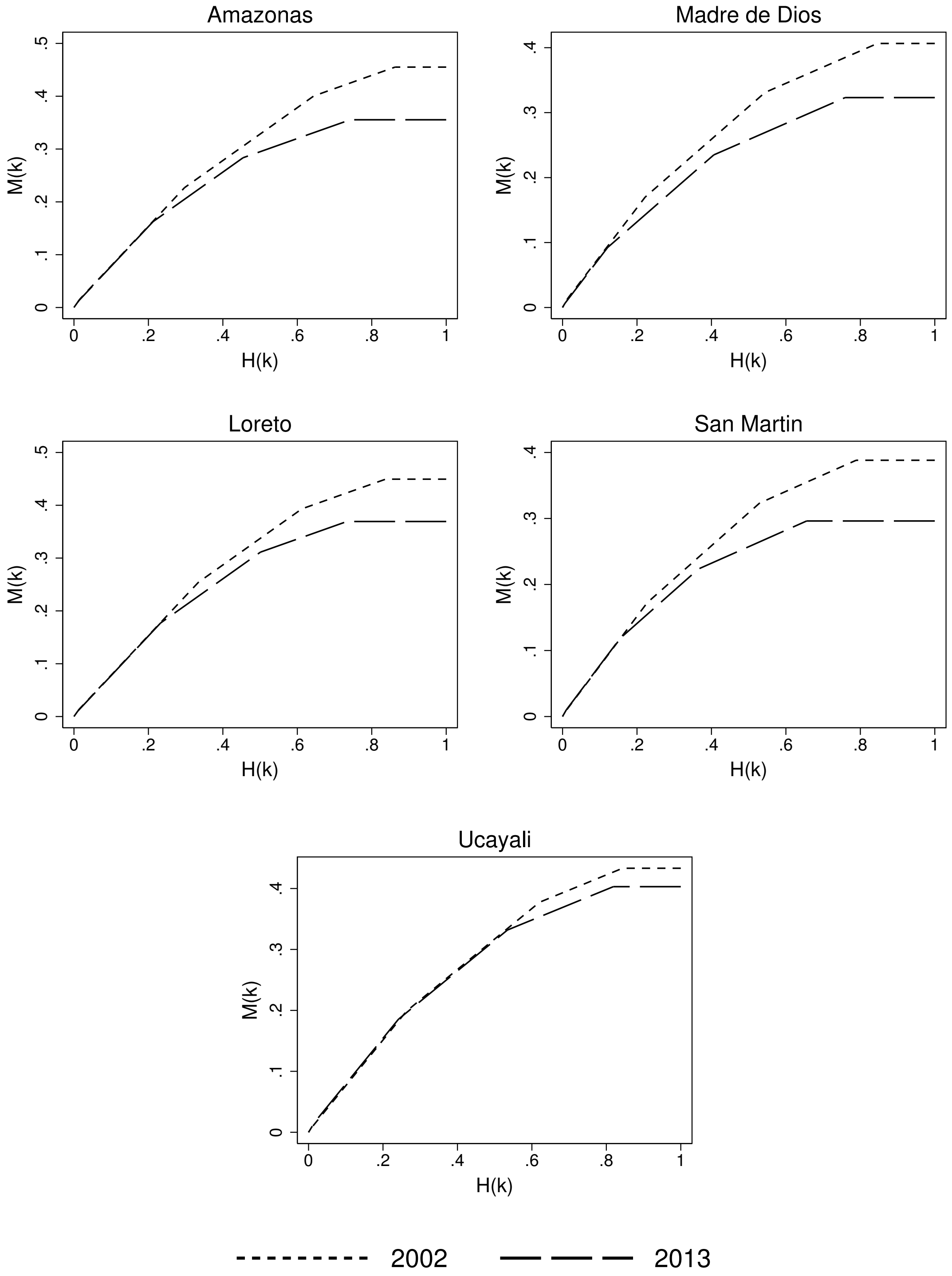


Figure 4 shows the RGL curves for the four southernmost Peruvian departments, comprising both coastal and highland regions. In the case of Arequipa the two curves cross once very close to the origin. Then the 2013 curve appears always below the 2002 curve. This means that the assessment of inequality and poverty reduction depends on measurement choices. For example, with the intersection approach ($k = 1$), poverty actually increased in Arequipa during the period, whereas for less stringent identification approaches ($k < 1$) the conclusion depends on the choice of both k and individual poverty functions. By contrast, Moquegua, enjoying the benefits of a thriving mining industry spread across a small population, saw a robust reduction of poverty accompanied by a decrease in inequality among the poor. Puno's case is similar to Arequipa's: the curves cross once at the highest levels of k rendering the poverty assessment inconclusive. Likewise, the most severe forms of poverty (with $k = 1$) in Puno increased between 2002 and 2013. Finally, Tacna's situation mimicks Puno's and Arequipa's: curve-crossing at the highest levels of k. However, unlike Puno and Arequipa, Tacna's curves are closer to the origin in both years, reflecting lower (robust) poverty levels in both years. Tacna benefits both from its mining industry and its active border with Chile.

Figure 5 shows the RGL curves for five south-central Peruvian departments, also comprising both coastal and highland regions. In the cases of landlocked Cusco and mainly coastal Ica their respective pairs of curves cross at the highest levels of k so that poverty reduction is not robust. Again, with an intersection approach, both departments actually saw increases in the most severe forms of poverty. By contrast, the other three departments did experience fully robust poverty reduction with lower deprivation inequality among the poor.

Figure 6 shows the RGL curves for five central Peruvian departments (including coastal and highland regions) and the coastal, city-sized, autonomous Callao province. All cases show a robust poverty reduction accompanied by inequality reduction, with the exceptions of the landlocked Junin and Huanuco. For the two latter departments, there is, again, a curve crossing at the highest levels of k, just as in the previous situations of curve-crossing encountered so far.

Figure 7 shows the RGL curves for the five northern Peruvian Departments along the coast and the highlands. Except for Cajamarca, all departments feature curve-crossing at the highest levels of k, just as in the previous cases of crossing above. Hence the conclusion of poverty reduction with lower inequality between 2002 and 2013 is not robust to all choices of k or functional forms. For instance, with an intersection approach, this extreme form of poverty actually increased throughout the region (except for Cajamarca).

Figure 4 – Reverse generalized Lorenz curves of deprivation counts. Peruvian southern coastal and highland provinces, 2002-2013.

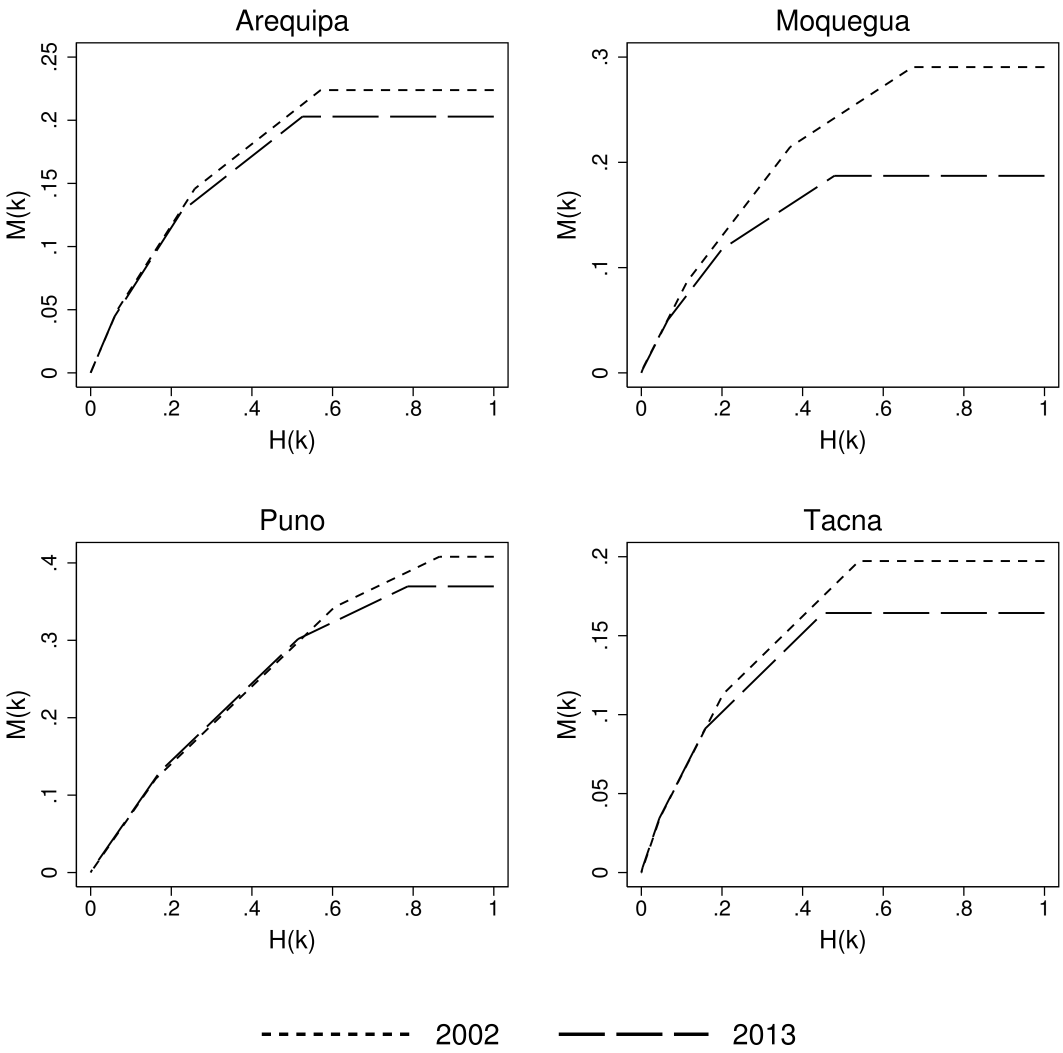


Figure 5 – Reverse generalized Lorenz curves of deprivation counts. Peruvian south-central coastal and highland provinces, 2002-2013.

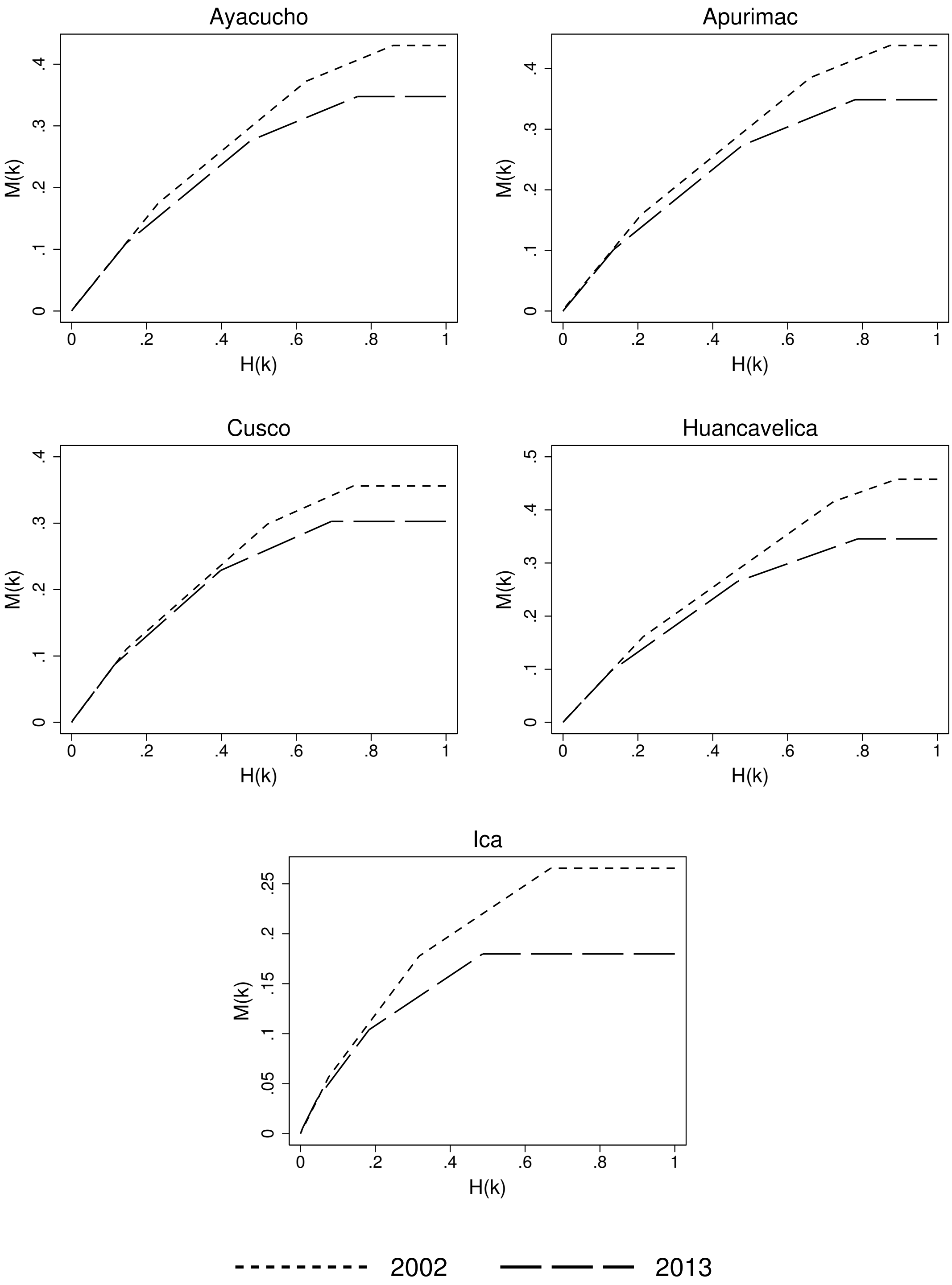


Figure 6 – Reverse generalized Lorenz curves of deprivation counts. Peruvian central coastal and highland provinces, 2002-2013.

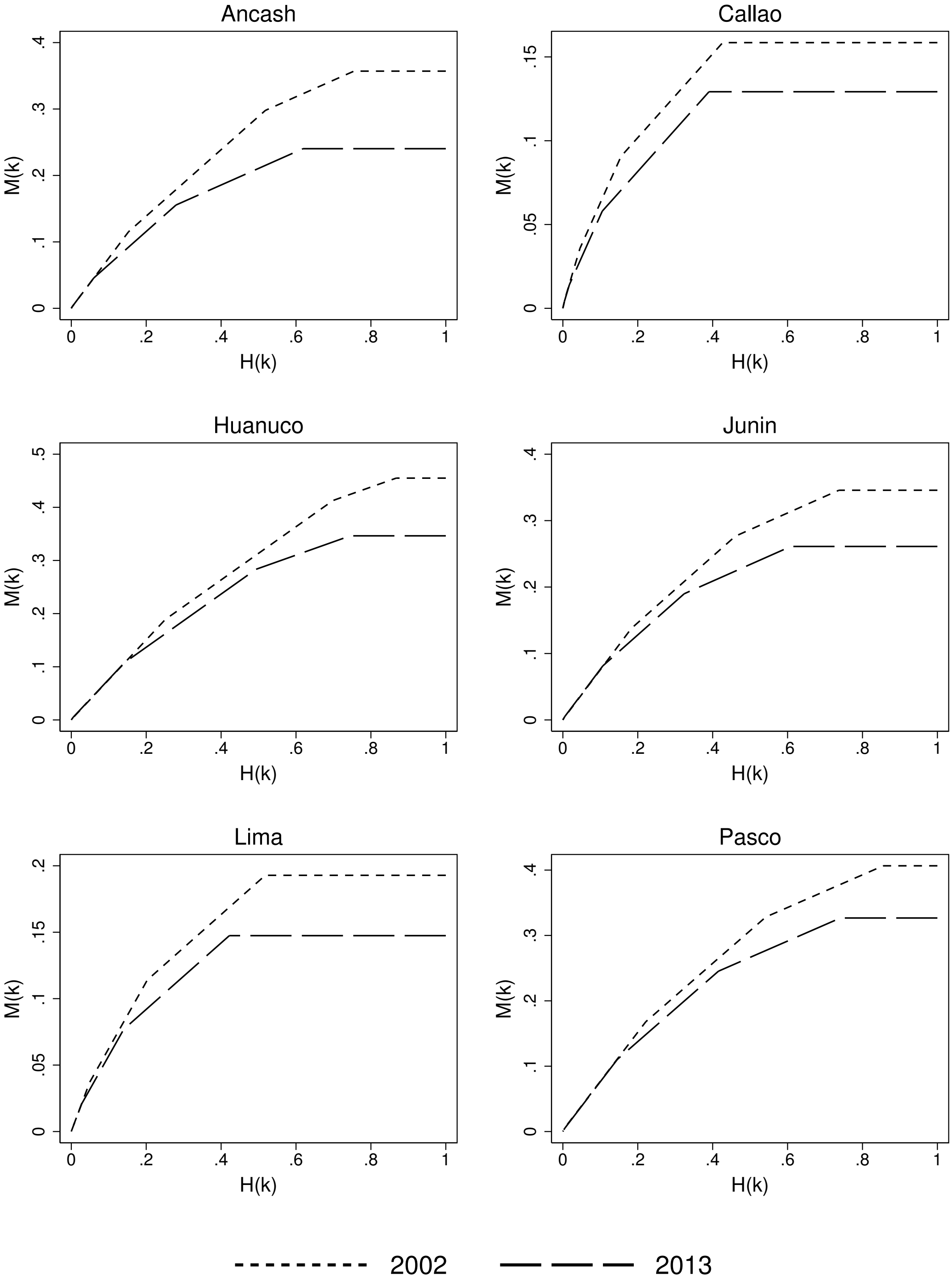
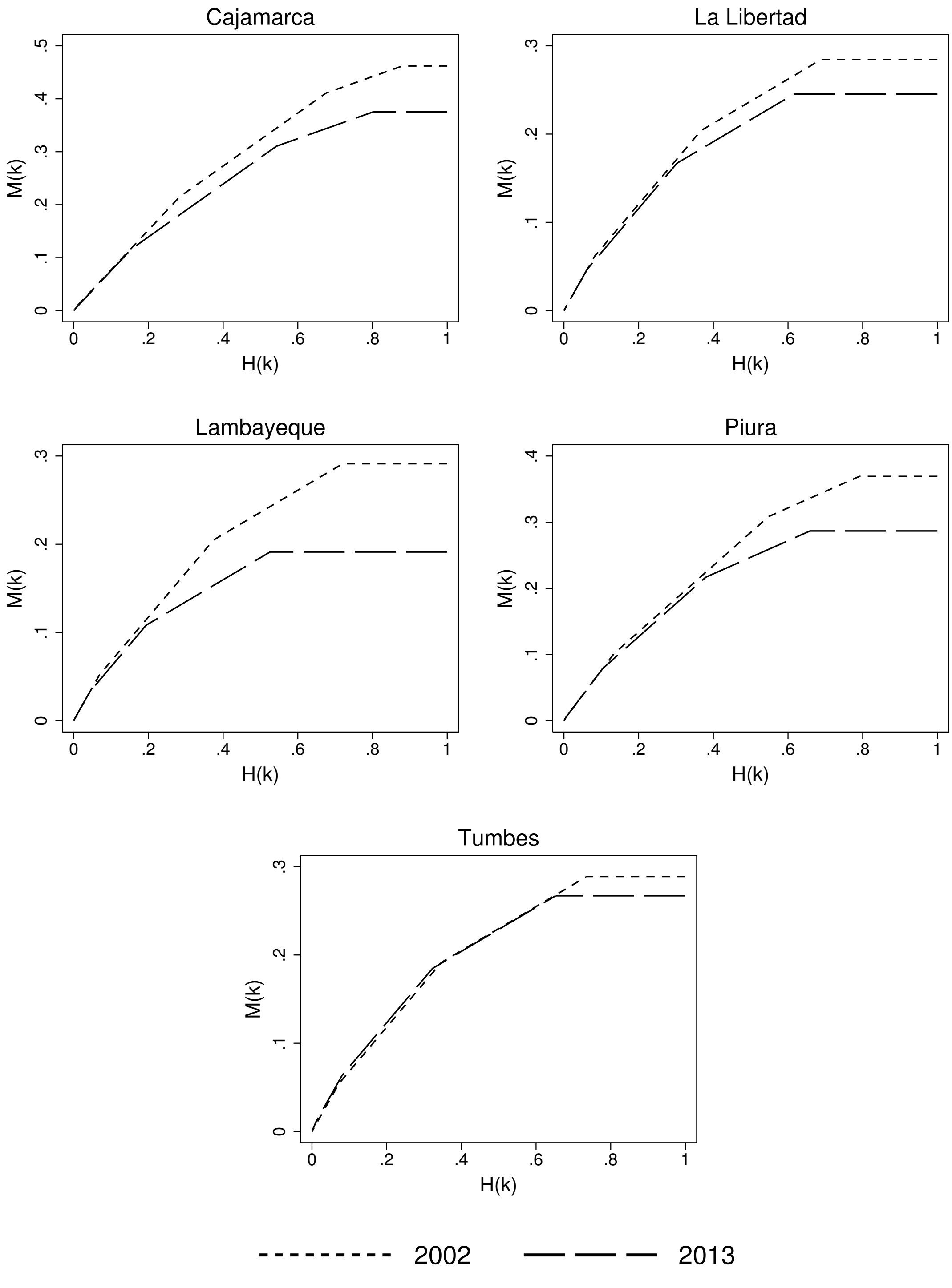


Figure 7 – Reverse generalized Lorenz curves of deprivation counts. Peruvian northern coastal and highland provinces, 2002-2013.



5.2.2 Test results for condition 2

Table 1 shows the $T(k)$ statistics for relevant values of k, as required for the test of condition 2, which we apply to the national, urban, and rural samples. In the three cases we reject the null hypothesis, using the aforementioned 5% overall significance level, in favour of the alternative whereby poverty robustly decreased in an egalitarian manner between 2002 and 2013.[6]

Table 1 – $Ho: M^{2002}(k) = M^{2013}(k) \forall k$ **versus** $Ha: \exists k | M^{2002}(k) > M^{2013}(k)$

k	National	Urban	Rural
0.25	31.862	24.784	25.758
0.5	29.742	20.580	25.158
0.75	16.519	11.881	11.565
1	-0.763	-0.261	-0.954

Table 2 shows the $T(k)$ statistics for the test of condition 2, involving the rainforest provinces. With them we uphold the alternative hypothesis of robust egalitarian poverty reduction, except in the case of Ucayali, for which we have evidence of statistically significant curve-crossing.

Table 2 – $Ho: M^{2002}(k) = M^{2013}(k) \forall k$ **versus** $Ha: \exists k | M^{2002}(k) > M^{2013}(k)$

k	Amazonas	Loreto	Madre de Dios	San Martin	Ucayali
0.25	7.454	5.987	5.549	7.103	2.189
0.5	7.296	5.181	5.188	6.610	2.719
0.75	3.788	4.609	4.435	3.711	0.741
1	0.581	0.619	1.328	-0.613	-2.268

Table 3 shows the $T(k)$ statistics for the test of condition 2, involving the southern provinces. We reject the null hypothesis in all cases, except for Arequipa. We conclude robust egalitarian poverty reduction for Tacna and Moquegua. By contrast, we find evidence of significant curve-crossing in the case of Puno.

Table 3 – $Ho: M^{2002}(k) = M^{2013}(k) \forall k$ **versus** $Ha: \exists k | M^{2002}(k) > M^{2013}(k)$

k	Puno	Arequipa	Tacna	Moquegua
0.25	3.391	2.002	2.901	8.019
0.5	2.950	1.576	1.874	6.844
0.75	-0.678	0.658	0.045	3.137
1	-2.456	-1.000	-1.735	0.724

Table 4 shows the $T(k)$ statistics for the test of condition 2, involving the southern-central provinces. For the whole region we reject the null hypothesis

[6] The signs of $T(1)$ are actually in contradiction to the other sings, yet in none of the samples $T(1)$ is significantly different from zero.

in favour of the alternative of robust egalitarian poverty reduction between 2002 and 2013.

Table 4 – $Ho : M^{2002}(k) = M^{2013}(k)\forall k$ **versus** $Ha : \exists k|M^{2002}(k) > M^{2013}(k)$

k	Cusco	Ayacucho	Apurimac	Huancavelica	Ica
0.25	4.498	7.220	7.008	9.864	8.307
0.5	5.096	6.885	7.098	10.742	6.446
0.75	1.906	4.742	3.742	4.565	2.683
1	-0.664	0.700	2.245	0.853	-1.865

Table 5 shows the $T(k)$ statistics for the test of condition 2, involving the central provinces. Again, for the whole region we reject the null hypothesis in favour of the alternative of robust egalitarian poverty reduction between 2002 and 2013.

Table 5 – $Ho : M^{2002}(k) = M^{2013}(k)\forall k$ **versus** $Ha : \exists k|M^{2002}(k) > M^{2013}(k)$

k	Pasco	Huanuco	Callao	Junin	Lima	Ancash
0.25	5.728	9.340	2.181	7.202	7.907	10.421
0.5	4.818	9.347	2.467	6.441	6.313	11.074
0.75	3.503	5.682	2.243	4.635	4.098	6.398
1	0.543	-0.881	0.422	-1.018	0.610	-0.068

Table 6 shows the $T(k)$ statistics for the test of condition 2, involving the northern provinces. We reject the null hypothesis in favour of the alternative of robust egalitarian poverty reduction, except in the case of Tumbes.

Table 6 – $Ho : M^{2002}(k) = M^{2013}(k)\forall k$ **versus** $Ha : \exists k|M^{2002}(k) > M^{2013}(k)$

k	Tumbes	La Libertad	Cajamarca	Piura	Lambayeque
0.25	1.587	3.586	7.901	7.838	9.827
0.5	0.430	2.995	7.459	7.449	8.328
0.75	-0.769	1.665	7.038	2.147	2.384
1	-3.181	-1.000	1.718	-0.355	-0.555

5.2.3 Test results for condition 4

Table 7 shows the $T(1)$ statistics for the test of condition 4, which we apply to the national, urban, and rural samples, as well as to the provinces. Results vary widely. With a 1% significance level (for simple one-tailed tests), we cannot reject the null hypothesis in the national, urban, and rural samples, nor in the case of most provinces. At that level of significance we only reject in favour of an alternative hypothesis of higher intersection poverty (i.e. $H(1)$) in 2013 for the cases of Puno and Tumbes. By contrast, we never reject in favour of an alternative of higher intersection poverty in 2002.

Table 7 – $Ha : H^{2002}(1) = H^{2013}(1)$ **versus** $Ha : H^{2002}(1) > H^{2013}(1)$

Department	T(1)	Department	T(1)
National	-0.763	Junin	-1.018
Urban	-0.261	La Libertad	-1.000
Rural	-0.954	Lambayeque	-0.555
Amazonas	0.581	Lima	0.610
Ancash	-0.068	Loreto	0.619
Apurimac	2.245	Madre de Dios	1.328
Arequipa	-1.000	Moquegua	0.724
Ayacucho	0.700	Pasco	0.543
Cajamarca	1.718	Piura	-0.355
Callao	0.422	Puno	-2.456
Cusco	-0.664	San Martin	-0.613
Huancavelica	0.853	Tacna	-1.735
Huanuco	-0.881	Tumbes	-3.181
Ica	-1.865	Ucayali	-2.268

5.2.4 Test results for condition 5

Table 8 shows the T_d statistics for each of the four dimensions involved in the test of condition 5, which we apply to the national, urban, and rural samples. We find strong evidence against the fulfillment of condition 5 in favour of any of the two years. On one hand, clearly the uncensored deprivation headcounts for education, dwelling characteristics and access to services significantly decreased between 2002 and 2013 (as is apparent from the size of their respective T_d statistics). On the other hand, the uncensored deprivation headcount for dependency burden significantly *increased* during the same period. In other words, we can conclude that the experience of egalitarian poverty reduction, documented earlier for the national, urban, and rural samples, is not robust to any choice of weighting vectors. For example, given the particular test results in Table 8, we could select a weighting vector heavily tilted toward the dependency burden dimension such that, together with other parameter and functional form choices, it would yield an increase in poverty between 2002 and 2013, among the national, urban and rural samples. This would contrast with the robust egalitarian poverty reduction that we found above with a different set of weights.

Table 8 – $Ho : U_d^{2002} = U_d^{2013} \forall d$ **versus** $Ha : \exists d | U_d^{2002} > U_d^{2013}$

U_d	National	Urban	Rural
Education	22.818	20.187	11.268
Dwelling	20.701	12.661	16.193
Services	37.414	24.917	42.225
Burden	-18.082	-11.423	-14.786

6 Concluding remarks

This paper claimed that the second-order dominance condition of Lasso de la Vega (2010) and Chakravarty and Zoli (2009) is well-suited to ascertain the presence of robust egalitarian poverty reduction when poverty is measured using the counting approach, and comparing two cross-sectional samples. When the condition holds we can conclude that poverty decreased or increased consistently, in the sense that the result is robust to different choices of inequality-sensitive social poverty indices and counting identification approaches. However these conditions only hold for specific choices of deprivation lines and dimensional weights. With alternative selections of the latter two parameter sets, the conditions would need to be tested again.

Therefore in this paper we sought to refine the existing conditions by figuring out situations under which they would work for any choice of strictly positive weights. We found the fundamental condition whose fulfillment is both necessary and sufficient to ensure that second-order propositions work for any conceivable weighting vector with positive elements. However, since this condition may be cumbersome to implement when the number of variables is large, we also derived two useful conditions whose fulfillment is necessary, but insufficient, for robust second-order comparisons using any possible weighting vector. While these conditions are insufficient, they are fewer in number, and much easier to compute. When they are not met we can immediately rule out the robustness of second-order dominance in poverty reduction to any choice of weights.

Above and beyond the conditions derived in this paper, it is also possible to derive sets of necessary and sufficient conditions which guarantee robust egalitarian poverty comparisons for a subset of weights, as well as for broader sets of weights (e.g. admitting zero values). Likewise further useful necessary conditions are derivable if we opt to restrict the set of admissible weighting vectors, or the domain of k cut-offs, or both jointly. Some examples are available upon request.

Measuring poverty with indicators capturing dimensions of education, dwelling infrastructure, service access and dependency burdens, the empirical illustration showed that 22 out of 25 Peruvian provinces experienced robust poverty reduction between 2002 and 2013, accompanied by lower deprivation inequality among the poor. Similar results turned up for national, urban, and rural samples. However these findings relied on a fixed choice of deprivation lines and weights. When we probed the national, urban, and rural cases further with our necessity conditions, we found that egalitarian poverty reduction was not robust to different weighting choices.

Finally, the empirical illustration highlighted the importance of the intersection headcounts, i.e. the proportions of people deprived in every dimension. Often the differences between the intersection poverty headcounts of two samples were not statistically significant. However, as it turned out in many comparisons, the adjusted headcount ratio of one sample would be consistently above another's for every poverty identification cut-off, except for the intersection approach ($k = 1$). Further studies based on a greater number of dimensions should help gauge the pervasiveness of this apparent relative difficulty in finding egalitarian poverty comparisons that remain robust when moving toward more stringent poverty identification criteria.

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