THE UNIVERSITY OF MELBOURNE

FACULTY OF BUSINESS & ECONOMICS

Melbourne Institute Working Paper Series

Working Paper No. 5/15

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Melbourne Institute Working Paper No. 5/15

ISSN 1328-4991 (Print)

ISSN 1447-5863 (Online)

ISBN 978-0-7340-4373-3

January 2015

* This study uses unit record data from the Household, Income and Labour Dynamics in Australia (HILDA) Survey. The HILDA Survey project was initiated and is funded by the Australian Government Department of Social Services and is managed by the Melbourne Institute of Applied Economic and Social Research (at the University of Melbourne). We also acknowledge the use of PanelWhiz. Corresponding author: <s.tsiaplias@unimelb.edu.au>.

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Abstract

This paper relates indicators of household financial stress to household income and expenditure with the objective of identifying household stress thresholds and comparable equivalence scales. A model is proposed whereby households try to absorb income shocks or shifts by shrinking consumption or engaging in consumption substitution, and is applied to data from the Household, Income and Labour Dynamics in Australia Survey. Households have different capacities to absorb a shock or shift, with those unable to absorb a shock or shift being financially stressed. The distribution of household financial stress thresholds is estimated, and is found to be dependent on heterogeneity in both welfare sensitivity and discretionary consumption stickiness. We also examine the equivalence scales implied by the model and derive the distortion associated with the assumption of homogeneity. We find that this assumption has a distortionary impact and results in significant over- or under-estimation of equivalence scales depending on household type and size.

JEL classification: D10, I31, I32, D30

Keywords: Household financial stress, heterogeneity, equivalence scales, household welfare, minimum expenditure

1 Introduction

The aim of this paper is to identify and estimate household financial stress thresholds taking into account household-specific consumption preferences and sensitivities, derive equivalence scales based on these estimates and then show how accounting for heterogeneity is important in a welfare context.

Household equivalence scales are used to specify the level of income a household requires to achieve a benchmark welfare level taking into account the household's particular needs and characteristics. Numerous studies have attempted to compute household equivalence scales by reference to the household's perceived minimum expenditure requirements (Goedhart et. al. 1977; van Praag et al. 1982; van der Gaag and Smolensky, 1982). While this approach accounts for household specific features of the welfare function, it is open to biases and judgment calls stemming from the requirement that households attach a monetary value to a notional yardstick: being the amount they require to 'survive' or 'get by'. We avoid this problem by identifying minimum expenditure by reference to the presence or absence of financial stress.

Our methodology relates indicators of household financial stress to household income and expenditure with the objective of identifying the household's stress threshold. The stress threshold is the point at which household income is insufficient to meet household expenditure requirements. To identify required household expenditure, a model is proposed whereby households face income and expenditure shocks or shifts, and attempt to absorb these shocks or shifts by shrinking consumption or engaging in consumption substitution. A household becomes financially stressed when it is unable to absorb an income shock or shift without falling below its expenditure requirement or when it encounters an unexpected expense that results in a breach of its budget condition.

We pay particular attention to examining the extent to which heterogeneity in welfare sensitivity and discretionary consumption preferences impact on the household's stress threshold. Specifically, we allow each household to have its own level of sensitivity to an income or expenditure shock or shift (Goedhart et al., 1977). Two ostensibly similar households may produce a markedly different stress response following a change in income. Furthermore, discretionary consumption is divided into permanent and time-varying components, with the level of consumption stickiness allowed to be household specific. We therefore avoid the assumption of a representative household or the assumption that two households, because of some commonality in size or income, engage in the same - or even a similar - type of discretionary consumption.

There are numerous reasons to suspect that welfare sensitivity changes significantly across households. Differences in somewhat amorphous concepts such as household resilience will induce heterogeneity in welfare sensitivity, as will the household's capacity to purchase insurance against risks that impede its capacity to consume or the household's relative ability to engage in consumption substitution (Aiyagari,1994; Blundell et al. 2008; Lusardi et al. 2011). The presence of incomplete insurance markets may lead to greater welfare sensitivity for some households. In terms of the household's ability to substitute goods and services, factors such as information asymmetries or the household's opportunity cost of time may induce heterogeneity in welfare sensitivity. In this respect, Aguiar and Hurst (2005), for example, provide evidence that food expenditure declines in both retirement and unemployment, yet in retirement substitution can take place that leaves consumption largely unaltered. Notwithstanding the strong basis for heterogeneous welfare sensitivity, there is little research on the impact of such heterogeneity on household stress levels.

Heterogeneity in the stickiness of discretionary consumption, and its relevance for household stress, can be motivated in a number of ways. The presence of children, for example, may result in the household requiring a greater level of contingent, time-varying discretionary consumption. In this case, the household's decreased capacity to smooth its consumption may render it relatively more susceptible to financial stress (Clarida, 1991). More generally, the presence of consumption lifecycle effects, and consumption habits will impact the relative magnitude of the permanent component of the household's discretionary consumption (Browning and Crossley, 2001; Gourinhas and Parker, 2002; Fernandez-Villaverde and Krueger, 2007; Attanasio and Weber, 1995). Habit effects, in particular, will almost invariably be household specific although the importance of such heterogeneity for household financial stress is largely unknown.

Since heterogeneity is likely to be important in determining a household's stress level, we seek to identify the household financial stress threshold allowing for heterogeneity in welfare sensitivity and discretionary consumption preferences. Our model is applied to data from the Household, Income and Labour Dynamics in Australia (HILDA) survey which provides a rich dataset describing household income, expenditure and well-being for a representative cross-section of Australian households. The main contribution is summarised in an estimated distribution of financial stress thresholds spanning the set of households represented in the Survey.

The estimated distribution exhibits severe positive skewness (with the mode being less than the median, which is in turn less than the mean), and significant kurtosis in the right tail of the distribution. Analysis based on the distribution yields a number of key results.

First, the distribution is distinct from that of household income and shows that the financial stress threshold is poorly represented using measures based solely on household income. Second, there are significant differences in the stress threshold across different family sizes In particular, we provide evidence that the dispersion of stress thresholds declines with family size, indicating that the household's required level of expenditure, as opposed to its total expenditure on goods and services, tends to become more homogeneous as family size increases. Third, our estimates show significant heterogeneity in the stickiness of discretionary consumption and in welfare sensitivity. Importantly, the degree of heterogeneity revealed in this paper is not adequately captured through common categories such as household size, household income and household type (e.g. couple, lone parent, couple with children).

Finally, we examine the equivalence scales arising from our model. We derive the distortion in the equivalence scales when there is a failure to account for heterogeneity, and estimate these distortions across a range of household types and sizes. We show that accounting for both forms of heterogeneity produces equivalence scales that are substantially different to those where homogeneity is assumed, thereby providing a measure of the welfare impact of the household specific consumption preferences and tastes that Fisher (1987) highlights as relevant to any welfare comparison. The differences are particularly pronounced for couples and larger sized households, with the assumption of homogeneity resulting in under-compensation for the former and over-compensation for the latter household type.

The paper is organised as follows. Section 2 presents the framework for obtaining the household stress threshold. Section 3 relates the stress threshold to the measurement of equivalence scales and derives the distortions stemming from the assumption of homogeneity. Section 4 discusses the data used to estimate to the model and the estimated results. The following two sections discuss the results. First we learn abouth the general characteristics of the thresholds, and how they differ with respect to factors such as income, family size, and age of the primary wage earner. Second, we examine the household equivalence rates stemming from our model, and show the magnitude of the welfare distortions associated with the assumption of homogeneity. Concluding remarks are in section 7.

2 Modelling the household stress threshold

A household is considered to be financially stressed if its residual income is insufficient to fund unexpected expenditure requirements or support minimal variable consumption. These concepts are operationalised by separating out the fixed and discretionary components of expenditure.

Household income h_t^y in period t comprises income stemming from employment, government pensions and benefits, and capital income. The household also has a period t cost function $h_t^c(q)$ in state $q \in \mathbb{N}$. Household costs at time t can be further divided into fixed costs v_{ot} and variable costs v_{1t}

$$h_t^c(q) = v_{0t} + S_q(v_{1t}). \tag{1}$$

where S is a non-negative shrinkage function applied to variable costs. The shrinkage function depends on q and provides households with the capacity to make state-contingent consumption decisions. The shrinkage function allows a household to both reduce its level of consumption or to re-arrange its expenditure by, for example, engaging in substitution (such as preparing a greater proportion of food at home) or accessing state-contingent credit.[1]

Costs are characterised as fixed or variable based on the capacity of the household to reduce or adjust the quantum of the cost in the short term. Fixed costs cannot be reduced in period t, and are further divided into costs that depend only on t and costs that are both state and time dependent

$$v_{0t} = \overline{v_{0t}} + \varepsilon_{0t}(q).$$

In this setting, housing accommodation is treated as a known, fixed cost and is subsumed into $\overline{v_{0t}}$.[2] On the other hand, $\varepsilon_{0t}(q)$ includes state-dependent costs that a household must incur such as necessary medical treatment.[3] Since these costs are state-dependent, the household does not have a priori knowledge of their occurrence. Once incurred, however, $\varepsilon_{0t}(q)$ is a known, fixed amount.

In contrast, variable costs v_{1t} may be adjusted in the short-term and always depend on q. Variable costs may be durable or non-durable (see, further, Aguiar and Hurst, 2009) and include costs such as groceries and entertainment, but may also include variable outlays such as depositing funds into a savings account. To facilitate interpretation, it is assumed that $v_{1t} = \sup_q S_q(v_{1t})$ such that $S = 1$ in some optimal state q. In any other state $q' \neq q$, households curtail their variable spending such that $S_{q'}(v_{1t}) \leq S_q(v_{1t})$.

Now define the household residual income r_t as the difference between household income and known, fixed costs. Residual income is allocated to variable costs $S_q(v_{1t})$ or to $\varepsilon_{0t}(q)$

$$r_t \equiv h_t^y - \overline{v_{0t}} = S_q(v_{1t}) + \varepsilon_{0t}(q). \tag{2}$$

Equation (2) is the household's budget constraint across all states and times.

The household undertakes its expenditure and allocation decision at time t by reference to an unobserved utility function $U(S_q(v_{1t}), v_{0t}, h_t^y)$ such that the household's utility is determined by its state-dependent shrinkage of discretionary consumption, its known and unanticipated fixed costs $v_{0t} = \overline{v_{0t}} + \varepsilon_{0t}(q)$, and its income. Household utility declines when the household is required

[1] As such, the primary purpose of the shrinkage function is to identify expenditure rather than consumption (see, further, Aguiar and Hurst, 2009).

[2] Our treatment of housing accommodation expenditure as being markedly different to $\varepsilon_{0t}(q)$ or v_{1t} is also consistent with research indicating a different consumption lifecycle for housing costs; while the consumption of non-housing goods is typically hump-shaped over the lifecycle, housing consumption appears to increase monotonically before flattening out (Fernandez-Villaverde and Krueger, 2006; Yang, 2008).

[3] Technically, $\varepsilon_{0t}(q)$ also includes windfall gains such that there is a small, but non-zero, probability of a negative $\varepsilon_{0t}(q)$.

to impose additional constraints on variable spend such that $S'(v_{1t}) < S(v_{1t}) \Longrightarrow U' < U$, where U', U is household utility with variable spend given by $S'(v_{1t})$ and $S(v_{1t})$ respectively.

The utility function also specifies a necessary amount of variable spend v_{1t}^* that the household requires, and is discontinuous at that point which we denote k_t

$$k_t(q) = v_{1t}^* + \varepsilon_{0t}(q). \tag{3}$$

In accordance with (3), following the observation of state q, the minimal outlay required by a household at time t is determined by the sum of its minimal variable allocation v_{1t}^* and its state-contingent cost $\varepsilon_{0t}(q)$.

The household's goal is to maximize its utility function $U(S_q(v_{1t}), v_{0t}, h_t^y)$ subject to its budget constraint (2) and the condition that the difference between the household's actual expenditure $S_q(v_{1t}) + \varepsilon_{0t}(q)$ and its minimal requirement $k_t(q)$ is non-negative

$$S_q(v_{1t}) + \varepsilon_{0t}(q) \geq k_t(q) \tag{4}$$
$$\Longrightarrow k_t(q) \leq r_t. \tag{5}$$

Equation (4) is the model's fundamental equation and is used to determine whether a household is in a state of stress. Pursuant to this equation, to satisfy its objective function the household must find a state-contingent shrinkage function $S_q(\cdot)$ that enables it to fund state-contingent, fixed outlays $\varepsilon_{0t}(q)$. The household must also ensure that its remaining income after accounting for $\varepsilon_{0t}(q)$, being $r_t - \varepsilon_{0t}(q)$, allows it to achieve a level of variable consumption $S_q(v_{1t})$ that is at least as great as v_{1t}^*. A household that is unable to fund its unexpected requirements $\varepsilon_{0t}(q)$ or its minimal variable consumption v_{1t}^* breaches the condition $k_t(q) \leq r_t$ and is identified as stressed.

Define the indicator variable m_{it} as equal to 1 when household i is stressed in period t and 0 otherwise. The indicator is constructed using

$$m_{it} = I(k_{it} > r_{it}) \tag{6}$$

where $I(\cdot)$ is a binary indicator taking on the value unity if $k_{it} > r_{it}$.[4]

The household observes its own stress threshold k_{it} and therefore knows the quantum of $\varepsilon_{0it}(q)$ and the minimal amount of variable outlay v_{1it}^* that it requires. These values are given by

$$v_{1it}^* = \gamma_{0i} + x_{it}'\gamma \tag{7}$$
$$\varepsilon_{0it} = z_{it}'\beta \tag{8}$$

where γ_{0i} is household i's time-invariant variable outlay requirement, x_{it} is a set of covariates or instruments used to estimate time-variation in household i's mininum variable outlay, and z_{it} is a set of covariates used to estimate the unexpected component of household i's fixed cost v_{0it}. Stickiness in necessary variable expenditure is measurable by reference to the relative magnitude of permanent variable expenditure $\gamma_{0i}/x_{it}'\gamma$. The household's stress threshold is therefore

$$\widehat{k}_{it} = v_{1it}^* + \varepsilon_{0it} = \gamma_{0i} + x_{it}'\gamma + z_{it}'\beta. \tag{9}$$

Household i's welfare sensitivity is η_i such that the econometrician's observation of the household's stress threshold by reference to its response m_{it} is

$$k_{it} = \gamma_{0i} + x_{it}'\gamma + z_{it}'\beta + u_{it}. \tag{10}$$
$$u_{it} \sim N\left(0, \eta_i^2\right). \tag{11}$$

[4]For notational convenience, the stress threshold's dependence on the state q is no longer explicitly denoted.

Heterogeneity in welfare sensitivity represents factors such as the household's capacity to insure against permanent and transitory shocks (Blundell et al., 2008). In the context of the model specified here, η_i may differ by reference to the household's capacity to insure against shifts or temporary changes in household income h_t^y, anticipated fixed costs $\overline{v_{0t}}$, the nature of its consumption shrinkage or substitution function S_q and unanticipated fixed costs ε_{0it}. The econometrician's estimate of household i's probability of being stressed is then

$$\Phi\left(\frac{\gamma_{0i} + x_{it}'\gamma + z_{it}'\beta - r_{it}}{\eta_i}\right)$$

where $\Phi(\cdot)$ is the standard normal distribution function.

Each household is characterised by the set of observables $\{m_i, X_i, r_i\}$ for $i = 1, 2, ..., N$. X_i is comprised of x_{it}, z_{it} for $t = t_{0i}, t_{0i}+1, ..., T_i$. Given (6), (10) and (11), the ith household's contribution to the likelihood function L is

$$\begin{aligned} & L_i\left(m_i|\beta, \gamma, \gamma_{0i}, \eta_i, X_i\right) \\ = & \prod_{t=t_{0i}}^{T_i} \Phi\left(\frac{\gamma_{0i} + x_{it}'\gamma + z_{it}'\beta - r_{it}}{\eta_i}\right)^{m_{it}} \left(1 - \Phi\left(\frac{\gamma_{0i} + x_{it}'\gamma + z_{it}'\beta - r_{it}}{\eta_i}\right)\right)^{1-m_{it}} \end{aligned}$$

with the model's overall likelihood function given by

$$L\left(m|\beta, \gamma, \gamma_0, \eta, X\right) = \prod_{i=1}^{N} L_i\left(m_i|\beta, \gamma, \gamma_{0i}, \eta_i, X_i\right).$$

We adopt a Bayesian approach to learning about k_{it} and the household's welfare sensitivity and specify the following prior distributions for $\widehat{k}_{it}$ and η_i^2

$$\widehat{k}_{it} \sim N\left(\mu_0, \sigma_0^2\right) \quad (12)$$

$$\eta_i^2 \sim IG\left(n_0, s_0\right) \quad (13)$$

where $N(\cdot)$ is the normal density with location and scale parameters μ_0, σ_0, and IG is the Inverse-Gamma density with shape n_0 and scale s_0.

Accordingly, the posterior density of the parameter set is

$$p\left(\beta, \gamma, \gamma_{0i}, \eta_i|\, m_i, X_i, r_i\right) \propto L\left(m|\beta, \gamma, \gamma_0, \eta, X\right) \pi\left(k, \eta\right)$$

where $\pi(k, \eta)$ is the prior density based on (12) and (13).

3 Stress thresholds and welfare

A household's probability of financial stress is based on the extent to which its income is sufficient to meet household expenditure requirements, which depend on its stress threshold and its accommodation expenditure. The threshold k_{it} is the minimal outlay (outside of accommodation expenditure) required by a household at time t and is determined as the sum of its minimal variable (or 'shrinkable') expenditure v_{1t}^* and its state-contingent expenditure $\varepsilon_{0t}(q)$. In our setting, it is natural to also consider the probability of stress as the basis for household i's period t welfare function

$$V_{it} = 1 - \Phi\left(\frac{\widehat{k}_{it} - r_{it}}{\widehat{\eta}_i}\right) = 1 - \Phi\left(\frac{\widehat{\gamma}_{0i} + x'_{it}\widehat{\gamma} + z'_{it}\widehat{\beta} - (h^y_{it} - \overline{v_{0it}})}{\widehat{\eta}_i}\right). \tag{14}$$

This welfare function is intrinsically tied to household well-being, has a well-defined meaning, and is determined by a set of estimated parameters. Household i's welfare may change over time according to its residual income r_{it}, its ability to substitute and shrink consumption $\gamma_{0i} + x'_{it}\gamma$, and pursuant to its unanticipated 'fortunes' $z'_{it}\beta$. Welfare may be sticky, however, with the level of stickiness depending on the household's fixed expenditure γ_{0i} and its welfare sensitivity η_i. Accordingly, the welfare function avoids the assumption that household preferences may be accounted for by some set of standard demographic characteristics, and is consistent with Fisher's (1987) assertion that any two households on the same indifference curve cannot be treated as being equally well-off without accounting for their tastes and preferences.

It can be ascertained that the welfare function (14) has the following properties:

Proposition 1 *Household i's welfare is defined and bounded between 0 and 1 if its welfare sensitivity η_i is a strictly positive, bounded real number and its mimimum requirement $\widehat{k}_{it}$ is a bounded, real number.*[5]

Definition 1 *Household i is balanced if its requirements are equal to its residual income, $\widehat{k}_{it} = r_{it}$, whereby its welfare level, if defined, will be 0.5.*

Corollary 1 *If the household's welfare function is defined its welfare level will approach unity in household income h^y_{it}.*

Corollary 2 *A fixed 'shrinkable' consumption γ_{0i} satisfying $\frac{\gamma_{0i}}{|x'_{it}\widehat{\gamma}|+|z'_{it}\widehat{\beta}|+h^y_{it}} \longrightarrow k_1, \frac{\gamma_{0i}}{\eta_i} \longrightarrow k_2$ where k_1, k_2 are large positive numbers will result in a welfare level of 0 since the permanent component of the household's minimal level of discretionary expenditure greatly exceeds its household income and welfare sensitivity, and the impact of any state-dependent changes to the stress threshold.*

Corollary 3 *As η_i approaches zero, household i's welfare function becomes discrete and degenerates to the value 0.5 and the two polar extremes 0,1. The household's welfare range becomes {0, 0.5, 1} and any deviation from the balanced outcome renders the household completely 'satisfied' or completely 'unsatisfied'.*

Corollary 4 *As η_i approaches some arbitrarily large number k, household welfare is always 0.5.*

Corollary 5 *If only the outcome $V_{it} = 0.5$ and household income h^y_{it} are observed, it is not possible to distinguish between a balanced household and a household with an arbitrarily large welfare (in)sensitivity η_i.*

Using (14) we can calculate the equivalent income for household type j, being the income required to ensure that household type j exhibits the same level of welfare $V(\cdot)$ as the benchmark household. The computation of equivalent income is generally represented as the following problem

$$\min_{h^y_j} \left\| V\left(h^y_*, X_*\right) - V\left(h^y_j, X_j\right) \right\| \tag{15}$$

[5] Since household income is always a bounded real number.

where (h_*^y, X_*) represents the income and characteristics of the benchmark household. The solution $\overline{h}_j^y$ to the problem (15) yields the income required by household type j to achieve the benchmark welfare level $\overline{V} = V(h_*^y, X_*)$. The resulting equivalence scale is expressed as the ratio $\overline{h}_j^y / h_*^y$.

Approaches to deriving $\overline{h}_j^y$ include, but are not limited to, setting $V(\cdot)$ equal to the log-normal distribution $\Lambda(h^y; X)$, adopting Stone–Geary utility (in the context of estimating welfare using a Linear Expenditure System), or using translog utility across a vector of goods that the household consumes (van Praag, 1968; Goedhart et. al. 1977; van Praag et al. 1982; van der Gaag and Smolensky, 1982). Researchers have also log-linearised an unknown function $\widetilde{h}^y = f(h^y, X)$ to yield a linear equation which can be solved for $\overline{h}_j^y$ (Danziger, 1984; Phipps and Garner, 1994; van Praag et al., 1980). Nelson (1992; 1993) provides a detailed review of methods for estimating equivalence scales.

The welfare function $V(\cdot)$ can be generalised to include a measure σ that reflects welfare sensitivity, pursuant to which we obtain $V\left(h_j^y, X_j, \sigma\right)$. For example, assuming log-normal welfare $\Lambda(h^y; X)$, the σ coefficient expresses the household's sensitivity to the difference $h^y - E(h^y)$. In solving for $\overline{h}_j^y$, it is generally assumed that σ is a common, exogenous parameter such that all households exhibit the same level of welfare sensitivity (see, for example, van Praag et al., 1980; Goedhart et al., 1977). In terms of our model, welfare sensitivity is identified by the stress threshold error $u_{it} \sim N\left(0, \eta_i^2\right)$. The posterior mode of the density of η_i, denoted $\overline{\eta}$, is equivalent to σ with the implication that welfare dependence depends on uncertainty regarding expenditure v_{1t}^* and ε_{0it}.

3.1 Welfare distortions and the assumption of homogeneity

Pursuant to the above discussion, it is apparent that the general assumption of common welfare sensitivity requires the non-trivial acceptance that all households exhibit the same level of sensitivity to a change in their stress threshold or household income. Pursuant to corollaries 3-5, the assumption of common welfare sensitivity in the existing literature also implicitly rules out situations where households have large values of η_i or where household welfare degenerates in accordance with corollary 3. As such, the existing literature assumes that a household with welfare level 0.5 is necessarily a balanced household rather than a household with large welfare (in)sensitivity. In so far as these assumptions are inconsistent with the data, reported equivalence rates may be significantly distorted. We use our model to measure the extent of this distortion.

To illustrate the distortion, assume $\left(\eta_j, \gamma_{0j}\right)$ is the welfare sensitivity, permanent minimum discretionary consumption pair for group j, η^*, γ_0^* is the equivalent pair for the reference group and, for convenience, accommodation expenditures $\overline{v_{0j}}, \overline{v_{0*}}$ are zero. Solving the problem (15) taking into account $\left(\eta_j, \gamma_{0j}\right)$ yields the equivalent income $\overline{h}_j^y$ and associated equivalence scale $\frac{h_j^y}{h_*^y}$

$$\overline{h}_j^y = \gamma_{0j} + \widetilde{c}_j + \frac{\eta_j}{\eta^*}\left(h_*^y - \gamma_0^* - \widetilde{c}_*\right) = \widehat{k}_j + \frac{\eta_j}{\eta^*}\left(h_*^y - \widehat{k}^*\right)$$

$$\frac{\overline{h}_j^y}{h_*^y} = \frac{\eta_j}{\eta^*} + \frac{\widehat{k}_j - \frac{\eta_j}{\eta^*}\widehat{k}^*}{h_*^y}$$

where $\widehat{k}_j$ is group j's stress threshold, $\widehat{k}^*$ is the reference group's stress threshold and $\widetilde{c} = x'\gamma + z'\beta$. The values $\overline{h}_j^y$, h_*^y are specified on a per-person basis. The equivalent income for group j is, therefore, a linear function of the reference group's 'excess' income $h_*^y - \widehat{k}^*$ (viz. income net of the stress threshold) with an intercept given by its stress threshold $\widehat{k}_j$ and a slope parameter that depends on the ratio of welfare sensitivities $\frac{\eta_j}{\eta^*}$.

The household level equivalent income is $\overline{h}_j^{y,total} = fs_j \times \overline{h}_j^y$, where fs_j is household size for group j, with the analogous equivalence scale derived as the following straightforward adjustment of $\frac{\overline{h}_j^y}{h_*^y}$

$$\frac{\overline{h}_j^{y,total}}{h_*^{y,total}} = \frac{fs_j}{fs_*} \times \frac{\overline{h}_j^y}{h_*^y}.$$

If η_j happens to equal η^* then equivalent income, denoted as $\overline{h}_j^y | \left(\eta_j = \eta^*\right)$, is given by household income for the reference group h_*^y adjusted by the difference in the stress thresholds for the two groups

$$\overline{h}_j^y | \left(\eta_j = \eta^*\right) = h_*^y + \left(\widehat{k}_j - \widehat{k}^*\right).$$

such that the equivalence scale simplifies to

$$\frac{\overline{h}_j^y}{h_*^y} | \left(\eta_j = \eta^*\right) = 1 + \frac{\widehat{k}_j - \widehat{k}^*}{h_*^y}.$$

Conversely, if we assume $\gamma_{0j} = \gamma_0^*$ then we obtain

$$\overline{h}_j^y | \left(\gamma_{0j} = \gamma_0^*\right) = \widehat{k}_j + \left(\gamma_0^* - \gamma_{0j}\right) + \frac{\eta_j}{\eta^*}\left(h_*^y - \widehat{k}^*\right)$$

$$\frac{\overline{h}_j^y}{h_*^y} | \left(\gamma_{0j} = \gamma_0^*\right) = \frac{\eta_j}{\eta^*} + \frac{1}{h_*^y}\left(\widehat{k}_j - \frac{\eta_j}{\eta^*}\widehat{k}^*\right) + \frac{1}{h_*^y}\left[\gamma_0^* - \gamma_{0j}\right].$$

It is straightforward to show that the distortion stemming from the assumption that group j's welfare sensitivity is η^* is non-zero unless $\eta_j = \eta^*$, with the bias in the equivalence scale being a linear function of $\widehat{k}^*$ with intercept $\left(\frac{\eta_j}{\eta^*} - 1\right)$ and slope parameter $\left(\frac{1-\eta_j/\eta^*}{h_*^y}\right)$.

$$\overline{h}_j^y - \overline{h}_j^y | \left(\eta_j = \eta^*\right) = \left(\frac{\eta_j}{\eta^*} - 1\right)\left(h_*^y - \widehat{k}^*\right) \tag{16}$$

$$\frac{\overline{h}_j^y}{h_*^y} - \frac{\overline{h}_j^y}{h_*^y} | \left(\eta_j = \eta^*\right) = \left(\frac{\eta_j}{\eta^*} - 1\right) + \left(\frac{1-\eta_j/\eta^*}{h_*^y}\right)\widehat{k}^* \tag{17}$$

where $\overline{h}_j^y - \overline{h}_j^y | \left(\eta_j = \eta^*\right)$ is the difference between unconstrained equivalent income $\overline{h}_j^y$ and equivalent income conditional on the assumption $\eta_j = \eta^*$, $\overline{h}_j^y | \left(\eta_j = \eta^*\right)$. Equation (17) provides the analogous difference in terms of equivalence scales.

Since the bias stemming from the adoption of $\eta_j = \eta^*$ lies in the interval $\left(\widehat{k}^* - h_*^y, \infty\right)$, $\overline{h}_j^y | \left(\eta_j = \eta^*\right)$ may constitute either an under- or -over- estimate of the household income required to equate group j's welfare with that of the reference group. As the interval is unbounded on the right, however, there is a greater risk that $\frac{\overline{h}_j^y}{h_*^y} | \left(\eta_j = \eta^*\right)$ understates the additional compensation required to equivalise welfare for group j. Figure 1 shows the bias in the equivalence scale when $\frac{\eta_j}{\eta^*} = 2$ for a reference household with $h_*^y = 75,000$. At $\widehat{k}^* = 50,000$, for example, the assumption $\left(\eta_j = \eta^*\right)$ understates the scale by 1/3, with household type j requiring an additional $1/3 \times h_*^y = \$25,000$ to equivalise welfare in the presence of sensitivity differences.

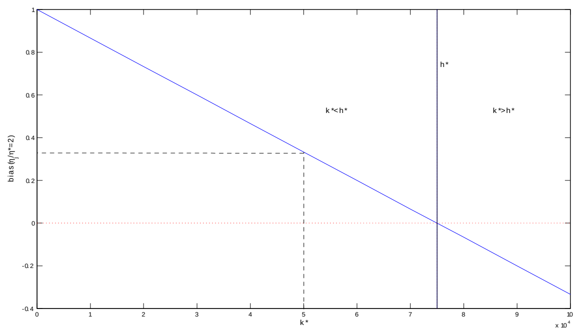

Figure 1: Bias in equivalence scale as k^* changes for household type j with $\frac{\eta_j}{\eta^*} = 2$

The bias stemming from the erroneous assumption $\gamma_{0j} = \gamma_0^*$ is an affine function of the simple difference $\gamma_{0j} - \gamma_0^*$.

$$\overline{h}_j^y - \overline{h}_j^y | \left(\gamma_{0j} = \gamma_0^*\right) = \gamma_{0j} - \gamma_0^* \tag{18}$$

$$\frac{\overline{h}_j^y}{h_*^y} - \frac{\overline{h}_j^y}{h_*^y} | \left(\gamma_{0j} = \gamma_0^*\right) = \frac{1}{h_*^y}\left(\gamma_{0j} - \gamma_0^*\right) \tag{19}$$

Finally, it can be shown that the equivalence biases when both $\eta_j = \eta^*$ and $\gamma_{0j} = \gamma_0^*$ are imposed are

$$\overline{h}_j^y - \overline{h}_j^y | \left(\eta_j = \eta^*, \gamma_{0j} = \gamma_0^*\right) = \left(\gamma_{0j} - \frac{\eta_j}{\eta^*}\gamma_0^*\right) + \left(1 - \frac{\eta_j}{\eta^*}\right)\widetilde{c}_* + \left(\frac{\eta_j}{\eta^*} - 1\right)h_*^y \tag{20}$$

$$\frac{\overline{h}_j^y}{h_*^y} - \frac{\overline{h}_j^y}{h_*^y} | \left(\eta_j = \eta^*, \gamma_{0j} = \gamma_0^*\right) = \left(\frac{\eta_j}{\eta^*} - 1\right) + \frac{1}{h_*^y}\left[\gamma_{0j} - \frac{\eta_j}{\eta^*}\gamma_0^* + \left(1 - \frac{\eta_j}{\eta^*}\right)\widetilde{c}_*\right]. \tag{21}$$

Equations (20) and (21) imply that the assumption of homogeneity in both welfare sensitivity and consumption stickiness may over- or under- state the equivalance scale. Ceteris paribus, however, greater fixed discretionary consumption, $\gamma_{0j} > \gamma_0^*$, will result in understated equivalent income when homogeneity is imposed. Conversely, greater sensitivity to an income or expenditure shock, such that $\eta_j < \eta^*$, will result in over-compensation when homogeneity is imposed. The reason for this result is that η_j requires a smaller income shift to equivalise $V\left(h_j^y, X_j\right)$ with $V\left(h_*^y, X_*\right)$ than does η^*.[6]

[6] In the case $\eta_j < \eta^*$, the slope of household type j's welfare curve is flatter when η^* is imposed instead of η_j. As such, achieving the absolute welfare change $\left|\Delta V\left(h_j^y, X_j\right)\right|$ requires a greater change in household income h_j^y than if η_j were adopted.

4 Model estimation

Data

Data are obtained from the Household, Income and Labour Dynamics in Australia (HILDA) survey which commenced in 2001 and is currently in its 12th wave. The survey is undertaken annually and collects information about economic and subjective well-being, labour market dynamics and family dynamics in Australian households. The first wave of the survey consisted of 7,682 households, with an additional 2,153 households added in 2011 (viz. wave 11). Although respondents are allocated a unique household identifier for each wave, this household identifier is limited to the particular wave for which it is allocated. Consequently, individuals need to be tracked over waves to construct a set of inter-temporal households that can be used to estimate the model specified in Section 2. Given the model's dependence on household income levels, we identify households by reference to the main income earner. If individual i is identified as the main income earner in waves k, $k+1$ and $k+2$ then we identify the set of information (personal and household) associated with individual i over that period, $\{r_{ik}, r_{ik+1}, r_{ik+2}, X_{ik}, X_{ik+1}, X_{ik+2}\}$ as belonging to a single household. For notational convenience, we assume $X_{it} = \{x_{it}, z_{it}\}$ where x_{it} and z_{it} are associated with minimal variable allocation v_{1t}^* and unanticipated fixed costs $\varepsilon_{0t}(q)$ respectively (the variables $\{x_{it}, z_{it}\}$ are listed in Appendix A).

The 'residual income' r_{it} is constructed as the difference between gross household income and direct housing accommodation costs defined as the household's annual mortgage repayments or housing rent. A very small number of households made both rental and mortgage repayments. In this case, we took the sum of both values as their housing-related expenditure. Without loss of generality, we estimate the model after dividing r_{it} by $fs_{it} \times 5000$, where fs is household size, such that we estimate the per-person threshold $k_{it}/(fs_{it} \times 5000)$ in bundles of \$5000. We also convert r_{it} to real terms such that k_{it} represents the real stress threshold and any time effects in k_{it} are not the result of inflation.[7] To avoid excessive notation, in the remaining sections of the paper any reference to k_{it} or r_{it} is on a real per-person basis except where stated otherwise.

We also require the indicator m_{it} which determines whether household i is financially stressed at time t. This indicator is constructed based on household responses to a set of financial stress variables in HILDA. In particular, we assume that a household is financially stressed (viz. $m_{it} = 1$) if it could not pay its utility bills, mortgage or rent on time, if it requested financial help from friends or family, pawned or sold something to make ends meet, was unable to heat its home, went without meals or requested help from a charity or similar organisation.

Households

We assume that households are dynamic and do not restrict our dataset to households with constant characteristics. Consequently, households are allowed to add or remove new members, change location, or may be renters in period k and home owners in period $k+1$. The only restriction we impose is an identification restriction that the primary wage earner remain the same. As such, if the primary wage earner changes in period $k+3$ then a new household is identified and this household is allowed to have its own consumption stickiness and welfare sensitivity. We iterate this process commencing at wave 2 and ending at wave 12 of the survey, identifying households h_i and their corresponding dataset $\{m_{ik}, m_{ik+1}, ..., r_{ik}, r_{ik+1}, ..., X_{ik}, X_{ik+1}, ...\}$.[8]

To examine sensitivity to identification we also estimated the model based on identifying households using the first survey respondent and by limiting estimation to continuous households (viz. avoiding households where the household is identified in period t and $t+2$, but treated

[7] Real values are constructed by reference to the ABS' Consumer Price Index using 2012 as the base year.

[8] The iteration is commenced at wave 2 since the variables used to estimate z_{it} are not available in wave 1.

as a distinct household in period $t+1$). Identification using the first survey respondent yields longer-lived households since a change in the main income earner does not produce a second household. For example, a household whose main income earner is jailed is treated as a separate household, with distinct requirements and a potentially different level of minimal necessary expenditure and welfare sensitivity, for the period of imprisonment when using the primary wage earner for household identification. This is not the case where identification is based on the first respondent except in the rare case where the jailed party is also the first respondent. The results were, however, similar using either identification method or when restricting estimation to continuous households. The equivalency scales obtained in Section 6, in particular, were essentially unchanged.

To enable estimation, we restrict our sample to households for which at least one switch in the indicator variable takes place, and also remove observations for which we are unable to deduce household income, financial stress or for which we do not observe x_{it} or z_{it}. We also adopt a similar inclusion restriction to Krueger and Perri (2006) and only include households that have been interviewed at least 6 times.[9] Accordingly, the model is estimated on a panel containing 26,397 observations of the indicator variable m_{it} (with 9,608 instances where $m_{it}=1$) covering $N=3,103$ households. The estimation produces values for the γ and β parameters associated with x_{it} and z_{it} respectively, in addition to values for each household's time-invariant variable outlay requirement γ_{0i} and each household's uncertainty η_i. In particular, the stress thresholds $\widehat{k}_{it}$ for the ith household, $i \in \{1,2,...,N\}$, are estimated for a minimum of $T_i=6$ periods and a maximum of $T_i=10$ periods, with 10 periods being the most common (1091 households) and 6 periods being the least common (403 households).

Parameter estimates

The parameter estimates for the coefficients γ and β and the parameters chosen for the prior distributions are presented in Appendix B. These estimates may be contrasted with those from a reduced-form Probit model which are also presented in Appendix B. The Appendix contains a set of estimation statistics for both the estimation dataset (comprising the 26,397 observations used to estimate the basic model) and the full dataset. The full dataset contains 52,421 observations covering $6,154$ households, whereas the estimation dataset pertains to the 50 per cent of households that exhibit switching between $m=0$ and $m=1$. The other fifty per cent of households are either always $m=0$ ($2,921$ households and $24,957$ observations) or $m=1$ (130 households and $1,067$ observations). The results for the full dataset are presented to provide insight into the performance of the model across all households, rather than only those that inform the likelihood function.[10]

The interpretation of the structural parameter estimates differs from those for the Probit model and is based on the impact of the chosen covariates on the value of the stress threshold k_{it}. Although the objective of the covariates is to yield estimates of v^*_{1it} and ε_{0it}, it is still informative to consider the individual parameters estimated by the model. As expected, the presence of a shock such as death or serious injury significantly inflates ε_{0it} and therefore k_{it}. The minimum required level of variable consumption v^*_{1it} increases when a household's main income earner shifts from being self-employed (or an employer) to being an employee. Similarly, a change in residence, job or number of jobs also results in a higher required level of consumption v^*_{1it}. Ostensibly, household i moves to a larger residence or household i's main income earner changes jobs or holds a greater number of jobs to satisfy his or her family's greater variable consumption

[9] The model was also estimated subject to the restriction that households be interviewed at least 8 or 10 times with little change to the parameters β, γ or the distribution of the stress thresholds.

[10] The γ_{0i} and η_i parameters are determined by the prior distribution for the subset of households that do not exhibit switching between $m=0$ and $m=1$.

requirements v_{1it}^*. It is important to note, however, that minimum variable consumption v_{1it}^* also depends on $\widehat{\gamma}_{0i}$ with $x_{it}'\widehat{\gamma} + z_{it}'\widehat{\beta}$ explaining *changes* in household i's variable consumption. The coefficients for the time indicators in x_{it} do not vary markedly over the period 2004 to 2008 suggesting that the marginal impact of time on v_{1it}^* changes little for this period. The situation appears to change in 2011 and 2012 when minimum variable expenditure requirements rise significantly.

In contrast to the Probit model, the model clearly distinguishes between financially stressed and non-stressed thresholds with the average predicted outcome during periods of stress, $E(\widehat{m}_{it}|m_{it} = 1)$, being 58 per cent and the average predicted outcome during periods without stress, $E(\widehat{m}_{it}|m_{it} = 0)$, being just under 28 per cent. The Probit model correctly predicts about 35 per cent of financially stressed households, compared to approx. 71 per cent for the structural model. The structural model also provides an accurate depiction of the dispersion of financial stress, with predicted levels of stress being close to actual levels. In particular, the model predicts financial stress 37.38 per cent of the time, a little above the actual value of 36.39 per cent. The results for the full dataset improve on those observed for the estimation dataset, with $E(\widehat{m}_{it}|m_{it} = 1) - E(\widehat{m}_{it}|m_{it} = 0)$ close to 44 per cent, and 67 per cent of stress incidence being correctly predicted. Overall, 8 in 10 observations are correctly predicted when using the full dataset.

5 Characteristics of the stress threshold

5.1 Distribution of the stress threshold

Figure 2 presents the estimated distribution of stress thresholds $\widehat{k}_{it}$ across all households and time periods, in addition to the constituent elements of the threshold $\widehat{\gamma}_0$, $\left(x_{it}'\widehat{\gamma} + z_{it}'\widehat{\beta}\right)$, and welfare sensitivity $\widehat{\eta}$. Corresponding descriptive statistics are presented in Table 1. For ease of exposition, the stress thresholds in this figure and hereafter are multiplied by \$5000 thereby reversing the scaling of residual incomes undertaken for estimation purposes. On this basis, the presented stress threshold is $\widehat{k}_{it} = 5000 \times \left(\widehat{\gamma}_0 + x_{it}'\widehat{\gamma} + z_{it}'\widehat{\beta}\right)$. The stress threshold's distribution (Figure 1a) is unimodal with a right tail stretching to just under \$70,000. Around 80 per cent of the distribution is located between \$10,000 and \$35,000, and the median household requires residual income of approximately \$20,500 per person to accommodate its variable allocations v_{1t}^* and state-contingent fixed requirements ε_{0t} to avoid stress. The inter-quartile range in the stress threshold is around \$13,000, with the central fifty per cent of households requiring between \$14,500 and \$27,500 for variable allocations and state-contingent spending. The distribution of the threshold is distinct from that of residual income (and household income generally), exhibiting substantially smaller levels of standard deviation, skewness and kurtosis (Table 1). Although the stress threshold clearly differs across households, it exhibits substantially less dispersion than overall household income.

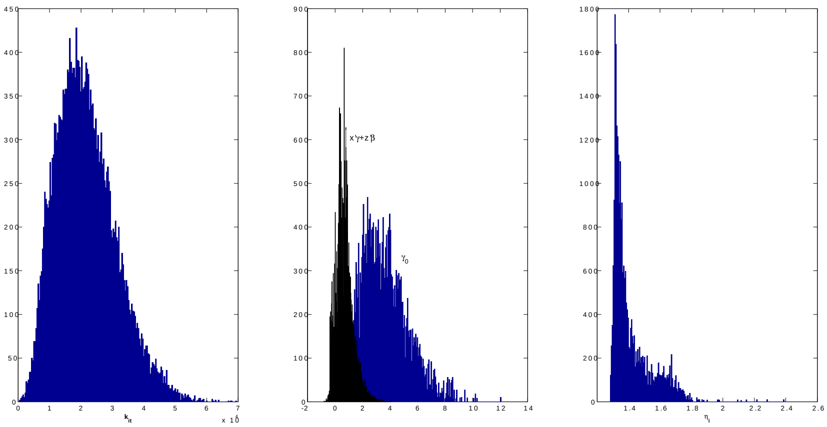

Figure 2. Histogram of: (a) Stress Thresholds $\widehat{k}_{it}$, (b) Permanent $\widehat{\gamma}_0$ and time-varying $(x_{it}'\widehat{\gamma} + z_{it}'\widehat{\beta})$ components of Stress Thresholds, and (c) Welfare Sensitivity $\widehat{\eta}_i$

The household's permanent and time-varying expenditure requirements, $\widehat{\gamma}_0$ and $\left(x_{it}'\widehat{\gamma} + z_{it}'\widehat{\beta}\right)$ respectively, are presented in Figure1b. The permanent component of variable expenditure, $\widehat{\gamma}_0$, is larger in magnitude than $x_{it}'\widehat{\gamma} + z_{it}'\widehat{\beta}$, with the latter typically accounting for about 15 per cent of the stress threshold. The time-varying component, however, exhibits substantially larger kurtosis which is reflective of the large impact of unanticipated fixed shocks z_{it} for a relatively small number of households. The two distributions differ significantly in terms of their dispersion, indicating that the variation in stress thresholds depends predominantly on permanent (or at least, enduring) differences between households (such as household-specific habits and consumption preferences), and less so on time-varying factors such as unanticipated, necessary expenditures which are fairly similar for the majority of households.

In contrast to the typical assumption of common welfare sensitivity, it is clear from Figure 1c that welfare sensitivity differs substantially across households. The mean level of welfare sensitivity is 1.415, although the distribution is associated with a high level of kurtosis such that large numbers of households exhibit less sensitivity to income or expenditure shocks (and, therefore, have a flatter welfare curve) than the mean household (viz. have higher values of $\widehat{\eta}$). Approximately 34 per cent of households have welfare sensitivity greater than 1.415, with a small number of households exhibiting dramatically lower levels of welfare sensitivity than the mean or median household (the maximal value of $\widehat{\eta}$ is 2.4). Assuming a typical non-stressed household where $k_{it} - r_{it} = -1$, the impact of a two unit (i.e. \$10,000) decline in per-capita income increases the probability of financial stress by 45 per cent for a household with η based on the 95th percentile of the distribution of welfare sensitivities, relative to 56 per cent when η is based on the 5th percentile of this distribution.

Table 1: Descriptive statistics for $r_{it}, \widehat{k}_{it}$ and $\widehat{\eta}_i$.

	r_{it}	$\widehat{k}_{it}$	$\widehat{\gamma}_{0i}$	$x'_{it}\widehat{\gamma} + z'_{it}\widehat{\beta}$	$\widehat{\eta}_i$
Mean	28,836	21,628	3.629	0.696	1.415
Median	22,058	20,534	3.417	0.645	1.362
Std deviation	24,138	9,572	1.693	0.620	0.126
Skewness	3.375	0.627	0.756	0.816	1.736
Kurtosis	29.515	3.298	3.538	4.155	7.122
25th percentile	13,943	14,545	2.367	0.288	1.326
75th percentile	37,207	27,544	4.644	1.020	1,467

Note: $\widehat{k}_{it} = 5000 \times \left(\widehat{\gamma}_{0i} + x'_{it}\widehat{\gamma} + z'_{it}\widehat{\beta}\right)$.

5.1.1 Welfare Sensitivity and Household Income

To establish the relationship between welfare sensitivity and household income, we derive household income (and household income per family member) quintiles, and evaluate the mean value of $\widehat{\eta}_i$ for households belonging to each quintile. We find that household sensitivity declines with income (Table 2), with households in higher income quintiles typically having higher values of $\widehat{\eta}$ and flatter welfare curves. The welfare sensitivity parameter for quintiles based on household income per family member moves from 1.372 for the lowest income quintile to 1.515 for the highest income quintile; this implies a fall in sensitivity to an income or expenditure shock for households in larger income quintiles. These results are consistent with Blundell et al (2008) who, using US data, find a positive relationship between the level of insurance for transitory shocks (being $z'_{it}\widehat{\beta}$ in our model) and household income. In turn, the results are consistent with Krueger and Perri (2006) who find a positive relationship between consumption stability and the use of state-contingent credit.

We also find that $\widehat{\eta}_i$ falls, hence welfare sensitivity increases, with larger family size which suggests that the capacity to insure against permanent or transitory shocks declines with family size. Accordingly, larger-sized families typically have greater sensitivity to income or expenditure shocks than their smaller-sized counterparts.

Table 2: Mean welfare sensitivity $\widehat{\eta}_i$ when grouped by income quintiles

Quintile	Household Income	Per-capita Household Income
1	1.372	1.372
2	1.395	1.372
3	1.423	1.390
4	1.427	1.430
5	1.461	1.515

5.2 Stress thresholds, family size and age

Table 3 presents the distribution of stress thresholds by family size and age of the primary income earner. The Table suggests the presence of returns to scale in the household's setting of its stress threshold with the threshold declining with family size. In particular, the stress threshold declines from a median of \$22,390 for a single person household to \$16,900 per person for households containing five or more persons. The distributions also become more peaked with family size, with larger households exhibiting a smaller inter-quartile range and a lower likelihood of extreme stress thresholds.

Based on the typical scale fs^{ϱ}, with ϱ acting as the scale parameter, the values in Table 3 imply an economies of scale parameter of 0.86 for the median household, 0.40 for households at the 5th percentile, 0.96 at the 25th percentile and about 0.81 for households at the 95th percentile.[11] These scales suggest that, with the exception of a small number of households at the left tail of the stress threshold distribution, households exhibit relatively small economies of scale. The results also indicate economies of scale significantly smaller than the $\varrho = 0.5$ typically assumed (see, for example, Burkhauser et. al., 1996). In terms of the latter, the use of $\varrho = 0.5$ implies requirements that are substantially smaller than what would appear to be necessary for a household to avoid financial stress; a household of size 5 or more, for example, has per-capita requirements of $16,897 per person relative to $10,013 if $\varrho = 0.5$ is assumed.[12]

Table 3: Distribution of Stress Threshold $\widehat{k}_{it}$ according to family size fs_{it} and age of primary income earner

	Frequency	5%	25%	50%	75%	95%	IQR
Family size fs							
1	7,854	8,166	14,672	22,390	30,377	42,401	15,705
2	7,956	7,956	14,493	21,170	28,559	40,429	14,067
3	4,116	4,116	15,732	21,133	27,247	36,920	11,515
4	3,946	3,946	14,871	19,276	24,570	33,631	9,699
5+	2,525	2,525	12,661	16,897	21,490	29,667	8,829
Age of primary income earner							
18 - 29	4,834	10,261	16,920	22,600	29,065	39,431	12,145
30 - 39	6,047	10,003	16,646	22,160	29,094	41,301	12,448
40 - 49	6,090	9,044	15,885	21,659	28,672	40,492	12,787
50 - 59	4,300	8,633	15,830	21,554	28,056	39,740	12,226
60 - 69	2,489	6,342	10,860	15,862	22,939	33,199	12,079
70+	2,492	6,154	8,983	12,214	16,776	25,848	7,793

Note: Summation across age and family size differs by 145 observations. This difference is based on primary income earners aged less than 18.

Stress thresholds also decline with the age of the primary income earner. The median stress threshold for persons aged 18-29 ($22,600) is significantly greater than that estimated for persons aged over 60 ($13,650). The parameter estimates indicate a substantial decline in both the mean (or median) and dispersion of minimal expenditure requirements for persons aged above 60, predominantly due to a decline in minimal discretionary expenditure v^*_{1t}. This result is consistent with the general notion of declining consumption over the life cycle (see, for example, Gourinchas and Parker, 2002). However, v^*_{1t} represents the amount of discretionary expenditure required to *avoid* financial stress. As such, older households appear to require substantially smaller levels of expenditure to satisfy their consumption requirements and avoid financial stress. At the same time, welfare sensitivity levels are fairly similar for households across each age bracket, with older households being slightly less resilient to shocks than younger households.[13] Consequently, our evidence suggests an increasing capacity for older households to engage in efficient expenditure. This is consistent with the notion that, due to factors such as a typically lower opportunity cost of time, older households are better able to engage in substitution that minimizes expenditure whilst maintaining consumption levels (see, also, Aguiar and Hurst, 2009).

[11] The addition of each household's housing expenditure to their stress threshold produced an economies of scale parameter (0.83) only marginally smaller than that based solely on the stress threshold.

[12] The equivalent stress thresholds for the median household if we assume $\rho = 0.5$ are: $22,390; $15,832; $12,926; $11,195 and $10,013 for family sizes 1 to 5+ respectively.

[13] Mean welfare sensitivities per age bracket are: 1.421 (18-29 years); 1.429 (30-39 years); 1.419 (40-49 years); 1.428 (50-59 years); 1.397 (60-69 years); 1.361 (70+).

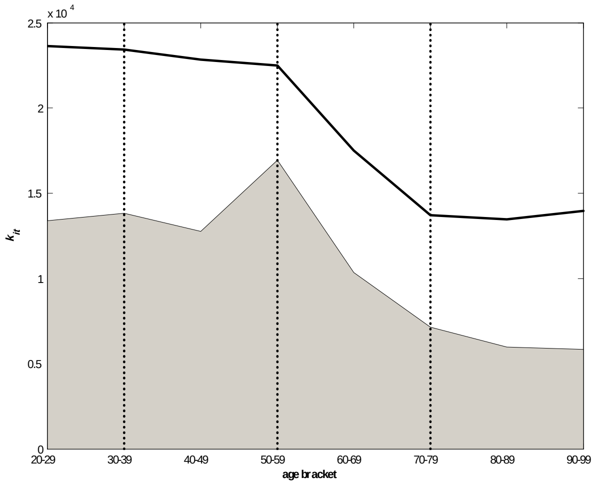

Figure 3. Stress thresholds and age (shaded area is the gap between mean household income and mean stress threshold per age bracket).

Figure 3 provides a striking analogue to the household consumption lifecycle, showing how stress thresholds change with age. The Figure suggests two core periods of change that take place when moving from the 30s to the 40s and from the 50s to the 60s. Household stress thresholds are essentially flat over the period where the primary income earner is between 20 and 40. Over this period, household requirements exhibit little change. Requirements begin to decline over the age period 40 to 60, albeit still at a relatively modest pace. Over the period covering 60 to 80 years of age, however, requirements fall dramatically and households are able to avoid financial stress notwithstanding sharp declines in expenditure. This period typically coincides with (full or partial) retirement and the concomitant decline in the opportunity cost of time.

It is important, however, to distinguish between the magnitude of the stress threshold and the typical welfare level associated with each age bracket. In this respect, we find it sensible to measure welfare as the typical difference between the household's income and stress threshold. Maximising this difference is analogous to minimizing the probability of financial stress for a given level of welfare sensitivity. Pursuant to this measure, welfare is maximised at the median age of 55, with welfare levels rising fastest over the period 45 - 55 years and declining at their sharpest rate over the period 55 - 65 years.

5.3 Stress threshold to household income ratio

It is informative to evaluate the household's total minimum expenditure requirements as a proportion of household income. The total minimum expenditure requirement is determined as the sum of the stress threshold and housing accommodation expenditure, and the proportion is estimated as $\widehat{p}_{it} = \left(\widehat{k}_{it} \times fs_{it} + \overline{v_{0t}}\right)/h^{y}_{it}$, across all households and conditional on m_{it}. This is depicted in Figure 4, and indicates that the median household's stress threshold and accommodation expenditure constitutes approximately 85 per cent of household income. For this household $\left(\widehat{k}_{it} \times fs_{it} + \overline{v_{0t}}\right) < h^{y}_{it}$ such that the median household is not in a state of financial stress. Accordingly, the median household can handle a negative income shock (or an expenditure shock) of approximately 15 per cent before it switches from $m_{it} = 0$ to $m_{it} = 1$ thereby being in a position of financial stress.

There is, however, substantial variation around the median, with 50 per cent of households having minimum expenditure requirements between 64 and 120 per cent of household income. As expected, $\widehat{p}_{it}$ declines with each income quintile, although households are susceptible to financial stress irrespective of income quintile. This is also observed in Lusardi et al's (2007) study of financial fragility in the US, where a sizeable proportion of middle income households reported an inability to come up with \$2000 of emergency funds within 30 days.

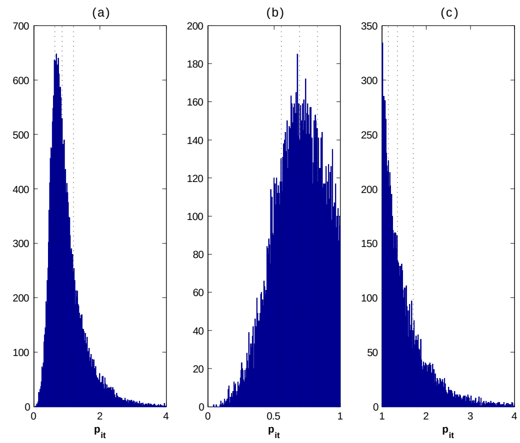

Figure 4. Stress Threshold and housing accommodation expenditure as a proportion of household income for (a) all households $\widehat{p}_{it}$, (b) non-stressed households $\widehat{p}_{it}|\widehat{m}_{it} = 0$, and (c) stressed households $\widehat{p}_{it}|\widehat{m}_{it} = 1$.

Table 4: Distribution of stress threshold and housing accommodation expenditure as a proportion of household income $\widehat{p}_{it}$ per income quintile

Quintile	5%	25%	50%	75%	95%
1	0.529	0.788	1.163	1.704	3.096
2	0.480	0.724	0.978	1.364	2.311
3	0.456	0.677	0.887	1.204	1.867
4	0.407	0.636	0.816	1.043	1.490
5	0.295	0.498	0.647	0.816	1.124
All	0.408	0.640	0.856	1.200	2.098

The median household in the bottom income quintile is associated with $\widehat{p}_{it} > 1$ indicating a position of financial stress (Table 4). Outside of this quintile, however, the median household does not exhibit financial stress. Financially stressed households are clearly present in the inter-quartile range pertaining to the first three income quintiles. Conversely, households in the highest income quintile exhibit relatively low stress levels with the median household in this quintile possessing household income that is approximately 50 per cent greater than the sum of its stress threshold and housing accommodation expenditure.

The conditional distribution of $\widehat{p}_{it}$ provides a clear depiction of the distributional differences in the stress threshold for families that are stressed (Figures 4b,c). It is clear that the distribution of $\widehat{p}_{it}|\widehat{m}_{it} = 1$ differs substantially from its counterpart for non-stressed households. The median value of p_{it} for non-stressed households is just over 68 per cent, indicating that the median household not currently in financial stress is capable of bearing a negative income shock of about 30 per cent before becoming financially stressed, with 50 per cent of households exhibiting the capacity to bear negative income shocks of between 17 and 45 per cent before exhibiting financial stress (Table 5).

Table 5: Descriptive statistics for the stress threshold and housing accommodation expenditure as a proportion of household income $\widehat{p}_{it}$, conditional on financial stress

	$\widehat{p}_{it}\vert\widehat{m}_{it}=0$	$\widehat{p}_{it}\vert\widehat{m}_{it}=1$	All
Mean	0.682	1.790	1.097
Median	0.692	1.355	0.856
Std deviation	0.183	7.827	4.818
Skewness	-0.035	58.575	94.321
Kurtosis	2.545	4,184	10,950
25th percentile	0.554	1.142	0.640
75th percentile	0.826	1.730	1.200

The process whereby a household moves from being financially stressed to non-stressed appears significantly more convoluted than the converse situation. The median financially stressed household requires a positive income shock of about 35 per cent to move from $m_{it} = 1$ to $m_{it} = 0$. The situation is, however, complicated by the level of volatility in $\widehat{p}_{it}$ for financially stressed households and the presence of a large number of households in the right tail of the conditional distribution of p_{it}. Given these properties, the median financially stressed household is only weakly reflective of the set of financially stressed households. Accordingly, 50 per cent of financially stressed households require subsidisation of between 14 and 73 per cent of their household income to shift to $m_{it} = 0$.

6 Household uncertainty and equivalence rates

6.1 The impact of welfare sensitivity and permanent consumption on welfare levels

This section examines the distortions derived in Section 3. First, we evaluate whether there are meaningful group-wise differences in permanent consumption requirements and welfare sensitivity. Using various diagnostics, it is reasonably clear that the distribution of household specific consumption requirements and welfare is not uniform across the various household types. The permanent component of discretionary consumption is greatest for lone households, group households and households without children, and falls significantly in the presence of children. The results are broadly consistent with the presence of consumption lifecycle patterns where demand for some subset of goods is age specific (eg. goods consumed by children) (Deaton et al. 1989; Fernandez-Villaverde and Kruger, 2007; Gourinchas and Parker, 2002).

This is portrayed clearly when $\widehat{\gamma}_{0j}$ is disaggregated by family size, with the permanent component of the stress threshold falling consistently from 86 per of minimum requirements to 79 per cent for a 6 person household (Table 6). Welfare sensitivity also appears to differ by household type and size. In particular, we find that $\widehat{\eta}_i$ falls, hence welfare sensitivity increases, with larger family size. This suggests that the capacity to insure against permanent or transitory shocks declines with family size, thereby inducing greater sensitivity to income or expenditure shocks in larger–sized families.

The Impact column in Table 6 shows the impact of a \$1000 per-person fall in household income for each group in terms of the change in the group's probability of financial stress. A fall in household income for a household with children or a household with more family members triggers a greater fall in utility than for a single person household or a couple with no children. In particular, a \$1000 fall in household income for a single person household has a relatively small impact on the probability of stress, which rises by about 1.5 per cent. This increases to a

5.2 per cent impact for a typical 4-person household. and a 5.7 per cent impact for a 5 person household.

Table 6: Welfare sensitivity $\widehat{\eta}_j$ and permanent consumption $\widehat{\gamma}_{0j}$ by household type and family size

	Freq.	$\widehat{\gamma}_{0j}$	$\widehat{\eta}_j$	$\frac{\widehat{\gamma}_{0j}}{\widehat{k}_j}$	Impact
Household type					
Couple family wo children or others	4237	3.717**	1.425**	0.857	0.6%
Couple family with children < 15 wo others	4244	3.274	1.390	0.821	5.4%
Couple family with depst wo others	513	3.449**	1.397	0.820	1.6%
Couple family with ndepchild wo others	641	3.716**	1.418**	0.845	0.9%
Lone parent with children < 15 wo others	1432	3.090**	1.379**	0.802	4.1%
Lone parent with depst wo others	284	3.539**	1.385	0.814	5.6%
Lone parent with ndepchild wo others	551	3.611**	1.408**	0.847	3.9%
Lone person	5527	3.880**	1.428**	0.856	1.5%
Group household	353	3.829**	1.422**	0.814	2.3%
Multi family household	213	2.986**	1.376	0.815	4.3%
Family size					
1	5527	3.979**	1.428**	0.856	1.5%
2	5855	3.700**	1.419**	0.848	1.4%
3	2904	3.546**	1.404**	0.824	4.3%
4	2785	3.289	1.390	0.822	5.2%
5	1127	2.868**	1.371**	0.805	5.7%
6	370	2.808**	1.369**	0.793	5.5%

#Values marked with ** indicate that the sample for the particular group is statistically different (at the .05 level) to the sample associated with the benchmarket household (being a Couple family with children < 15 wo others or a family of size 4) using a Kolmogorov-Smirnov test. The colum entitled 'Impact' is the percentage impact of a $1000 per person fall in household income on the probability of financial stress.

6.1.1 Policy Implications

There are clear policy implications stemming from the results in Table 6. Households have stickiness in their stress threshold that differs by household type and household size. In particular, households with more family members exhibit less stickiness, and greater fluctuation, in their discretionary expenditure requirements. Households also exhibit differences in welfare sensitivity, with welfare outcomes for households with more members being somewhat more sensitive to income shocks. Both of these outcomes are inconsistent with the general estimation of minimal expenditure requirements based on households sharing a common level of stickiness in minimal discretionary consumption and a common welfare sensitivity.

Consider a policy-maker compensating household *groups* by adjusting pre-tax household income with the objective of equivalizing V_j with $\overline{V}$. The policy-maker is required to make a choice regarding the extent to which household specific welfare sensitivity and stickiness requirements (in minimal discretionary consumption) are accounted for in determining the level of any compensation. The welfare distortions associated with the assumptions depend on the biases (16) - (21). Although the biases are non-zero unless $\eta_j = \eta^*$ and $\gamma_{0j} = \gamma_0^*$, the pertinent consideration is whether assumptions regarding η_j and γ_{0j} produce economically substantive differences between

$\overline{h}_j^y$, $\overline{h}_j^y| (\eta_j = \eta^*)$, $\overline{h}_j^y| (\gamma_{0j} = \gamma_0^*)$ and $\overline{h}_j^y| (\eta_j = \eta^*, \gamma_{0j} = \gamma_0^*)$ (or in the associated equivalence scales).

Table 7: Equivalence scales $\overline{h}_j^{y,total}/h_*^{y,total}$

	Restriction			
	None	$\widehat{\eta}_j = \eta^*$	$\widehat{\gamma}_{0j} = \gamma_0^*$	both
Household type				
Couple family wo children or others	0.519	0.517	0.472	0.470
Couple family with children < 15 wo others	1.000	1.000	1.000	1.000
Couple family with depst wo others	0.927	0.927	0.893	0.892
Couple family with ndepchild wo others	0.856	0.854	0.779	0.777
Lone parent with children < 15 wo others	0.688	0.689	0.717	0.717
Lone parent with depst wo others	0.646	0.647	0.611	0.612
Lone parent with ndepchild wo others	0.549	0.548	0.511	0.510
Lone person	0.270	0.269	0.238	0.237
Group household	0.618	0.617	0.553	0.552
Multi family household	1.165	1.166	1.244	1.246
Family size				
1	0.279	0.278	0.248	0.247
2	0.540	0.538	0.496	0.495
3	0.799	0.798	0.758	0.757
4	1.000	1.000	1.000	1.000
5	1.131	1.134	1.243	1.245
6	1.350	1.354	1.503	1.507

Table 7 presents equivalence scales based on household type and family size conditional on alternative assumptions regarding consumption stickiness and welfare sensitivity. The scales in the column for which equivalence scales are computed with no restrictions (with the heading 'None') involve a policy maker that compensates for differences (between group j and the reference household) in both welfare sensitivity and the stickiness of discretionary consumption. In contrast, the equivalence scales under the columns $\widehat{\eta}_j = \eta^*, \widehat{\gamma}_{0j} = \gamma_0^*$ involve compensating only for differences in welfare sensitivity and consumption stickiness respectively. The scale under the column 'both' pertain to a policy-maker who does not compensate for either γ_{0j} or η_j, but rather assumes that stickiness and welfare sensitivity is the same for all households. This final approach is reflective of the approach adopted in the literature to date.

It is clear that the equivalence scales $\overline{h}_j^y/h_*^y$ involving no restrictions are substantially different to those where it is assumed that $\eta_j = \eta^*$ and $\gamma_{0j} = \gamma_0^*$. When distinguishing by household type, the assumptions $\eta_j = \eta^*$ and $\gamma_{0j} = \gamma_0^*$ typically produce smaller equivalence scales thereby underestimating the level of compensation needed to equivalise welfare levels. On the other hand, when distinguishing by family size, a policy maker setting equivalence scales based on $\eta_j = \eta^*$ and $\gamma_{0j} = \gamma_0^*$ will substantially over-compensate larger households, and under-compensate smaller (up to 3 person) households. Taking into account differences in the stickiness of necessary discretionary consumption and welfare sensitivity, a household with 5 members requires an additional 13 per cent in pre-tax income (relative to the benchmark household's income), compared to an additional 24.5 per cent if it is assumed that $\eta_j = \eta^*$ and $\gamma_{0j} = \gamma_0^*$.

Almost all of the difference in the equivalence scales is attributed to the assumption $\widehat{\gamma}_{0j} = \gamma_0^*$. Consequently, decisions regarding the extent to which welfare equivalisation accounts for differences in the stickiness of required discretionary consumption have a meaningful impact on equivalence scales. The nature of required discretionary consumption changes with household size (and household type) such that households require less to achieve the same stress level; essentially learning to substitute in better ways. *If* the policy-maker holds larger households to this improved level of substitution, the assumption $\gamma_{0j} = \gamma_0^*$ yields equivalence scales that tend to over-compensate as household size increases.

6.2 The importance of intra-group differences for determining equivalence scales

The calculations above indicate that γ_{0j} is responsible for a greater level of the variation in the equivalence scales than welfare sensitivity η. This is supported when looking at the unconditional distributions of the differences between the equivalence scales with and without restrictions on γ_{0j} and η (using the average 4 person household as the benchmark or reference household). Figure 5 shows that the difference between the restricted and unrestricted equivalence scales is significantly greater following an assumption of homogeneous discretionary expenditure stickiness ($\gamma_{0j} = \gamma_0^*$) rather than homogeneous welfare sensitivity ($\eta_j = \eta^*$). In the case of the former, equivalence scales may be either positively or negatively distorted, whereas a restriction on welfare sensitivity typically results in under-compensation.

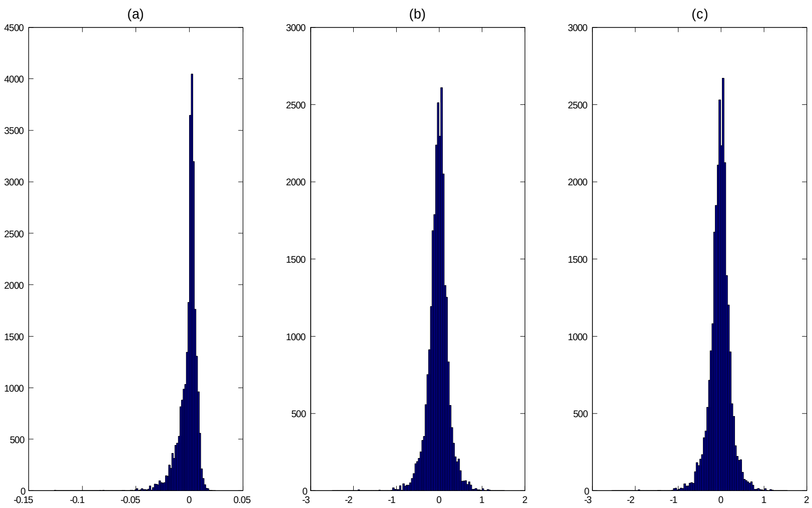

Figure 5: Distribution of: (a) $\frac{h_j^y}{h_*^y}|\left(\eta_j = \eta^*\right) - \frac{h_j^y}{h_*^y}$, (b) $\frac{h_j^y}{h_*^y}|\left(\gamma_{0j} = \gamma_0^*\right) - \frac{h_j^y}{h_*^y}$, and (c) $\frac{h_j^y}{h_*^y}|\left(\eta_j = \eta^*, \gamma_{0j} = \gamma_0^*\right) - \frac{h_j^y}{h_*^y}$

Although the impact of welfare sensitivity on equivalence scales is smaller than that of consumption preferences, welfare sensitivity nevertheless changes significantly across households, ranging from 1.28 to 2.39. At these two levels, the welfare impact for two households with otherwise equal stress thresholds and household incomes is substantial.[14] Table 7, on the other hand, gives the impression that the effect of household differences in welfare sensitivity on equivalence

[14] Assume a single person household with income of \$75,000 and a stress threshold of \$65,000. A \$5000 negative shock in income increases the probability of stress from 6 to 22 per cent (when η is 1.3), and from 20 per cent to 33 per cent when η is 2.4.

levels is minor. This is conditional, however, on the acceptance that the location estimate η_j is a reliable representation of the welfare sensitivity of group j. It is instructive, therefore, to consider the range of equivalence scales observed for commonly adopted household groupings when heterogeneity in welfare sensitivity is allowed.

Table 8 shows the distribution of the household welfare sensitivities across families of size 1 to 6. The 90 per cent range for welfare sensitivities tends to lie between between 1.3 and about 1.65 suggesting a reasonable level of dispersion in welfare sensitivity within groups. The dispersion is largest for single person households, and declines with family size; the implication is that family response to an income shock tends to become more homogeneous as family size increases. Accordingly, the assumption of a common group level welfare is questionable, particularly for smaller sized households.

Table 8: Distribution of $\widehat{\eta}_j$ across family size

Family size	5%	25%	50%	75%	95%	IQR	Mean
1	1.305	1.330	1.378	1.503	1.699	0.172	1.434
2	1.306	1.325	1.366	1.491	1.687	0.166	1.424
3	1.310	1.329	1.361	1.448	1.663	0.119	1.409
4	1.304	1.322	1.351	1.423	1.649	0.101	1.394
5	1.304	1.325	1.347	1.395	1.570	0.070	1.377
6	1.310	1.327	1.348	1.389	1.510	0.061	1.373

6.2.1 The tails of welfare sensitivities - under/over compensation

To examine the impact of within-group heterogeneity in welfare sensitivity, we derive equivalence scales at the left and right tails of the welfare sensitivities for each group. In particular, we derive the equivalent income for each group by setting η_j such that $P\left(\eta < \eta_{j\_low}\right) = 0.01$ (viz. a 1 per cent probability of observing η less than η_{j_low}) and $P\left(\eta < \eta_{j\_high}\right) = 0.99$. By using η_{j_high}, for example, we assume that the policy-maker estimates equivalence scales on the basis that compensation for group j will equivalise 99 per cent of households as opposed to the average household in the group. As such, since $\eta_{j\_high} \geq \eta_j$, the equivalence income based on η_{j_high} will be greater than or equal to the equivalence income obtained by adopting η_j. In the case where welfare sensitivity is homogeneous for group j, the equivalence scales will be identical to $\overline{h}_j^y / h_*^y$ observed in Table 7.

Table 9 indicates that the equivalence scales exhibit a relatively small change when η_{j_low} is adopted in place of η_j for households of all sizes. This is expected since the bias stemming from choosing a lower value of welfare sensitivity is bounded on the left In the converse situation, however, the equivalence scale increases, with the absolute increase varying across the family sizes. In the case $fs = 2$ or 6 the equivalence scale when adopting η_{j_high} increases by 5.9 and and 5.6 percentage points respectively, while the adoption of η_{j_high} for $fs = 4$ or 5 results in a greater than 4 percentage point increase in the equivalence scale.

Table 9: Household equivalence scales $\overline{h}_j^{y,total}/h_*^{y,total}$ using $\widehat{\eta}_j, \widehat{\eta}_{j\_low}, \widehat{\eta}_{j\_high}$

Family size	$\widehat{\eta}_j$	$\widehat{\eta}_{j\_low}$	$\widehat{\eta}_{j\_high}$
1	0.279	0.275	0.289
2	0.540	0.533	0.599
3	0.799	0.790	0.829
4	1.000	0.989	1.041
5	1.131	1.120	1.174
6	1.350	1.338	1.406

Since a change in welfare sensitivity for a particular family size can produce meaningful changes in the equivalency scale, it is instructive to consider the general appropriateness of grouping households by reference to typical demographic features such as family size. A substantial level of dispersion in the equivalence scales following a grouping by family size or type implies that these groupings may fail to provide an adequate characterisation of the welfare differences between households.

Figure 6 shows the dispersion in equivalence scales for all households, and for households characterised by family size. For each family size, the level of within-group dispersion, due to both consumption preferences and welfare sensitivities, is substantial. The results are consistent with evidence that, even after accounting for household size, substantial lifecycle effects are observed in (durable and non-durable) consumption expenditure (Fernandez-Villaverde and Kruger, 2007; Gourinchas and Parker, 2002). Similarly high levels of within-group dispersion are observed when grouping by household type, and are not mitigated by sub-grouping by reference to income quintiles or geographical characteristics.

As such, we measure the 'tastes' and preferences highlighted by Fisher (1987) as being relevant to any welfare comparison and find them to be of major importance (see, also, Jorgenson and Slesnick, 1984). In particular, the choice of a location measure such as the 'average' equivalence scale is likely to substantially under-compensate a large proportion of households belonging to the particular group. Interestingly, the distribution of equivalence scales also differs for each family size, especially in terms of kurtosis. As family size increases, the proportion of values observed in the far right tail increases (Table 10). Accordingly, the welfare loss associated with the policy-maker's choice of an 'average' equivalence scale appears to increase with family size.

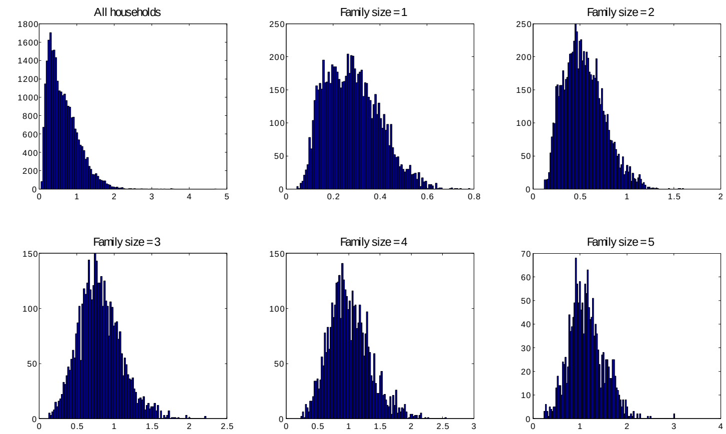

Figure 6: Distribution of equivalence scales $\overline{h}_j^{y,total}/h_*^{y,total}$ when grouped by family size

Table 10: Dispersion of equivalence scales $\overline{h}_j^{y,total}/h_*^{y,total}$

Family size	Mean	Std	IQR	Skewness	Kurtosis	Sig#
1	0.283	0.115	0.168	0.489	2.813	<0.01
2	0.546	0.215	0.297	0.565	3.113	<0.01
3	0.808	0.284	0.370	0.529	3.580	<0.01
4	1.012	0.328	0.427	0.550	3.399	<0.01
5	1.142	0.368	0.472	0.500	3.579	-
6	1.361	0.517	0.613	0.935	4.949	<0.01

#Based on a Kolmogorov-Smirnov test that the sample of equivalence scales for family size j is from the same underlying distribution as that for a family with 4 members.

7 Conclusion

A model of household stress is developed that is used to estimate household-specific financial stress thresholds using data from the Household, Income and Labour Dynamics in Australia survey. The framework defines the stress threshold as the point at which household income is insufficient to meet household expenditure requirements. The model is based on the understanding that household expenditure can be disaggregated into fixed and variable costs and we distinguish between these two types of costs based on the household's capacity to reduce or avoid the cost in the short term. Fixed costs - which the household cannot avoid in the short term - are further divided into those costs that depend only on time, and costs that depend both on time and some unknown state contingent measure. Variable costs, on the other hand, may be adjusted with households being able to shrink and substitute expenditure for the goods and services that fall under this cost type in order to satisfy their budget constraint.

The stress thresholds are estimated as the unobserved level of fixed and variable expenditure that the household treats as being necessary expenditure. Necessary expenditure is identified indirectly using information from household financial stress indicators. We provide the distribution of stress thresholds across households and show that the distributions vary across household size and the age of primary income earners. We also show that the distribution of stress thresholds differs markedly depending on whether households are currently stressed. Household stress thresholds for households in financial stress exhibit significantly higher levels of skewness and kurtosis than for non-stressed households. Accordingly, we provide evidence that the range of income or (non-discretionary) expenditure shocks necessary to place a non-stressed household in financial stress is substantially less dispersed than the income shift required to transition a household from the stressed to the non-stressed state.

We find heterogeneity in both discretionary consumption preferences and welfare sensitivity across households, with the financial stress thresholds being particularly influenced by the former. Smaller sized households typically exhibit greater stickiness in their stress threshold, and this appears to be related to the ability of smaller sized households to generate smoother expenditure. In contrast, larger households exhibit greater levels of state-contingent expenditure. Interestingly, however, the range of stress thresholds declines with family size, with larger households exhibiting greater homogeneity in their minimum expenditure requirements and greater sensitivity to income or expenditure shocks.

Our framework leads naturally to the computation of equivalence scales and we show that equivalence scales depend significantly on assumptions regarding heterogeneity. We derive the distortions associated with assuming homogeneous welfare sensitivity and discretionary consumption, with the resulting estimates indicating that the assumption of homogeneity leads to both under- and over- estimation of group-wise equivalence scales. The impact of the erroneous assumption of homogeneous discretionary consumption stickiness is particularly large, resulting in equivalence scales that typically over-compensate larger households and under-compensate smaller households.

The importance of accounting for heterogeneity is further magnified when we consider variations within a household grouping such as by household type or size. There is substantial variation in discretionary consumption stickiness and welfare sensitivity within each group, resulting in a wide range of within-group equivalence scales. In particular, we show that the choice of a location measure (such as the average equivalence scale for a group) results in under-compensation for a significant proportion of households within each group, with the level of potential under-compensation increasing with family size.

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Appendix A: Covariates

The estimation of v_{1it}^{*} and ε_{0it} requires covariates x_{it} and z_{it}. The variables $\{x_{it}, z_{it}\}$ are listed in Table A1. The inclusion of renter and mortgagee indicator variables is motivated by research suggesting that renter's non-housing consumption is different to that of owners (Yang, 2009). In the estimation dataset, approximately 59 per cent of observations pertain to households that partially or fully own a home, whereas approximately. 41 per cent are renters. About 24 per cent of home owners own their home outright with the remaining 35 per cent making mortgage repayments. These numbers are fairly consistent with the Australian Bureau of Statistics' Survey of Income and Housing showing that, in 2011-12, renters constituted a little over 30 per cent of all households, about 37 per cent of households were paying off a mortgage, and approximately

31 of households owned their home outright.[15] We include variables pertaining to employment status and number of jobs as indicators of the stability of income earned over the financial year, and use variables relating to health or major events such as pregnancy or retirement as potential indicators of a consumption shift.

Table A1: Covariates used to estimate stress threshold k_{it}

x_{it}	x_{it} (cont.)	z_{it}
employee	promoted	death of close friend
self-employed	back together with spouse	weather disaster
employer	retired	death of close relative
unpaid worker	year = 2003	death of spouse
# jobs in last FY	year = 2004	fired from job
long-term health condition	⋮	serious injury to family member
renter	year = 2012	serious personal injury
mortgagee		family member jailed
changed jobs		jailed
married		victim of property crime
changed residence		separated
pregnancy		victim of physical crime

Since z_{it} constitutes a set of instruments for unanticipated, unavoidable expenditure ε_{0it}, the variables chosen represent a set of major, largely unpredictable events that impact on the household's fixed cost v_{0it}. We note that indicator variables for the years 2001 and 2010 are not included since the variables listed in z_{it} (and some of the variables in x_{it}) are not available for 2001 and the questions used to construct the dependent variable m_{it} were omitted from the survey in 2010.

Appendix B: Estimated results for the structural model

Table B1 provides parameter estimates for both the estimation dataset (comprising the 26,397 observations used to estimate the basic model) and the full dataset contains 52,421 observations covering 6,154 households. Estimation of the structural model requires us to choose a set of priors (μ_0, σ_0^2) for the expected threshold $\widehat{k}_{it}$ and a set of priors (n_0, s_0) for the uncertainty parameter η_i. We choose the following relatively uninformative priors for k_{it}: $\mu_0 = 23,000/5000, \sigma_0 = 4$. Essentially we assume that each person requires expenditure of approximately $23,000 to avoid stress. This figure is based on the average residual income for households when $m_{it} = 1$ and assumes that a level of expenditure in this ballpark is reflective of the expenditure that a person requires for consumption v_{1it}^* and unanticipated, fixed expenditure ε_{0it}. We adopt a large standard deviation to reflect our considerable uncertainty regarding the expected value of k_{it}.

We set (n_0, s_0) such that we assume that η_i has a prior mean of unity and a standard deviation of 0.5. These values are set such that, given a typical level of residual income per person of around $25,000 (which is approximately the mode of the distribution of residual incomes), the unconditional probability of a household being financially stressed is about 1 in 3 (with a 90 per cent chance of being between 15 and 41 per cent). This is consistent with both the HILDA dataset and ABS data on household poverty (ABS, 2004; 2011). As a point of comparison, Lusardi et al. (2011), found that around 25 per cent of US respondents to a 2009 survey were certain that

[15] Our proportions cannot be compared directly to those of the ABS. Our proportions are based on all observations whereas the ABS proportions refer to a single point in time.

they could not come up with \$2000 within 30 days (and can, therefore, reasonably be considered to be financially fragile), with nearly 50 per cent responding that they would probably be unable to come up with \$2000 within 30 days.

Table B1: Parameter estimates for x_{it}, z_{it} for the structural model

x_{it}	Coefficient	z_{it} *(cont.)*	Coefficient
employee	0.266**	death of spouse or child	-0.068
self-employed	-0.099	fired from job or redundant	0.241**
employer	-0.056	serious injury to family member	0.227**
unpaid worker	-0.184	serious personal injury	0.207**
# jobs in last FY	0.341**	family member jailed	-0.064
long-term health condition	-0.009	jailed	-0.262
renter	0.095	victim of property crime	0.161**
mortgagee	0.047	separated	0.709**
changed jobs	0.227**	victim of physical crime	-0.015
married	0.027		
changed residence	0.448**	*Estimation dataset*	
pregnancy	0.104	*Obs*	26,397
promoted	0.102	$E(\widehat{m}_{it} \mid m_{it}=0)$	0.277
back together with spouse	-0.024	$E(\widehat{m}_{it} \mid m_{it}=1)$	0.582
retired	-0.152	$E(\widehat{m}_{it} > 0.5 \mid m_{it}=1)$	0.616
year = 2003	-0.134**	$E(\widehat{m}_{it} \leq 0.5 \mid m_{it}=0)$	0.764
year = 2004	-0.223**	$E(\widehat{m}_{it} > 0.5)$	0.373
year = 2005	-0.323**	$E(m_{it})$	0.364
year = 2006	-0.307**		
year = 2007	-0.256**	*Full dataset*	
year = 2008	-0.371**	*Obs*	52,421
year = 2009	0.012	$E(\widehat{m}_{it} \mid m_{it}=0)$	0.201
year = 2011	0.214**	$E(\widehat{m}_{it} \mid m_{it}=1)$	0.637
year = 2012	0.047**	$E(\widehat{m}_{it} > 0.5 \mid m_{it}=1)$	0.670
		$E(\widehat{m}_{it} \leq 0.5 \mid m_{it}=0)$	0.826
z_{it}	Coefficient	$E(\widehat{m}_{it} > 0.5)$	0.274
death of close friend	0.229**	$E(m_{it})$	0.203
weather disaster	-0.173		
death of close relative	0.108**		

Coefficients denoted ** are significantly different to zero at the .05 level..

Appendix C: Estimation of reduced form models

To investigate the properties of the dataset, we estimated reduced form linear, Probit and logit models. The reduced form models use $\{r_{it}, x_{it}, z_{it}\}$ as explanatory variables for m_{it}. The results were similar across the reduced-form models, therefore we restrict attention to the Probit model.

The Probit model's coefficients in Table C1 imply that the probability of financial stress declines with residual income r_{it} and paid employment. The ownership of a house also reduces the probability of financial stress, whereas renting or repaying a mortgage increases the probability of financial stress. There are clear time effects that appear to be associated with general macroeconomic conditions that show declining probabilities of financial stress until 2008, after which the probability of financial stress rises until 2012. Not surprisingly, major events such as death, injury or separation increase the probability of financial stress.

Overall, the Probit model has difficulty distinguishing between financially stressed and non-stressed households with the difference between the expected conditonal probabilities $E(\widehat{m}_{it}|m_{it} = 1) - E(\widehat{m}|m_{it} = 0)$, being just under 13 per cent. The model also tends to under-predict, thereby failing to adequately capture the prevalence of financial stress across households. If we assume that households with a greater than 50 per cent probability of $m_{it} = 1$ are financially stressed, only 20.13 per cent of observations are associated with financial stress compared to the 36.39 per cent observed in the dataset. Likewise, only 35 per cent of financially stressed households are correctly predicted.

Table C1: Parameter estimates from a reduced-form Probit model

x_{it}	Coefficient	z_{it} (*cont.*)	Coefficient
employee	-0.134**	death of spouse or child	0.014
self-employed	-0.228**	fired from job or redundant	0.178**
employer	-0.123**	serious injury to family member	0.182**
unpaid worker	0.017	serious personal injury	0.183**
# jobs in last FY	0.001	family member jailed	0.316**
long-term health condition	0.017	jailed	-0.201
renter	0.339**	victim of property crime	0.256**
mortgagee	0.220**	separated	0.433**
changed jobs	0.302**	victim of physical crime	0.345**
married	-0.026		
changed residence	0.197**	r_{it}	-0.000**
pregnancy	0.122**		
promoted	0.089**	*Estimation dataset*	
back together with spouse	0.090	*Obs*	26,397
retired	-0.129**	$E(\widehat{m}_{it}\|m_{it} = 0)$	0.323
year = 2003	-0.236**	$E(\widehat{m}_{it}\|m_{it} = 1)$	0.453
year = 2004	-0.336**	$E(\widehat{m}_{it} > 0.5\|m_{it} = 1)$	0.354
year = 2005	-0.475**	$E(\widehat{m}_{it} \leq 0.5\|m_{it} = 0)$	0.886
year = 2006	-0.545**	$E(\widehat{m}_{it} > 0.5)$	0.201
year = 2007	-0.551**	$E(m_{it})$	0.364
year = 2008	-0.673**		
year = 2009	-0.596**	*Full dataset*	
year = 2011	-0.471**	*Obs*	52,421
year = 2012	-0.569**	$E(\widehat{m}_{it}\|m_{it} = 0)$	0.183
		$E(\widehat{m}_{it}\|m_{it} = 1)$	0.372
z_{it}		$E(\widehat{m}_{it} > 0.5\|m_{it} = 1)$	0.257
death of close friend	0.138**	$E(\widehat{m}_{it} \leq 0.5\|m_{it} = 0)$	0.959
weather disaster	0.051	$E(\widehat{m}_{it} > 0.5)$	0.086
death of close relative	0.086**	$E(m_{it})$	0.203

Coefficients are based on estimation dataset. Coefficients denoted using ** are significantly different to zero at the .05 level.