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Abstract

This paper introduces a new Lorenz dominance criterion that allows ranking income distributions according to centrist measures à la Seidl and Pfingsten (1997). In doing so, it defines α-Lorenz curves by adapting the generalized Lorenz curves to this case. In addition, it provides an empirical illustration of these tools using Australian income data for the period 2001–2008. The results suggest that despite the reduction of relative inequality, inequality increased for most centrist value judgments.

JEL classification: D63

Keywords: Income distribution, Lorenz dominance, intermediate inequality indices, Ray-invariance, Australia

1. Introduction

In the literature on income distribution, there is a wide consensus on the properties an inequality measure has to satisfy when using it to compare income distributions having the same mean: Symmetry (S) and the Pigou-Dalton principle of transfers (PD).[1] The former axiom guarantees anonymity, and the latter requires a transfer of income from a richer to a poorer person to decrease inequality. When comparing two income distributions that differ in their means, another value judgment has to be invoked—the one regarding the type of mean-invariance the index satisfies—and no agreement has been reached with respect to this matter. Most scholars choose relative indexes, which implies that inequality remains unaltered when all incomes increase/decrease by the same proportion (scale invariance). Others opt, instead, for absolute measures so that inequality does not change if all incomes are augmented/diminished by the same amount (translation invariance). However, following Dalton (1920), several reports on questionnaires indicate that many people believe that an equiproportional increase in all incomes raises income inequality whereas an equal increment decreases it.[2] Kolm (1976) labeled these measures "centrist" (intermediate) and considered "rightist" (relative) and "leftist" (absolute) measures extreme cases of this more general view.[3]

Relative inequality indexes that verify the population principle (PP), together with PD and S, are consistent with the Lorenz dominance criterion (Foster, 1985). In other words, if the Lorenz curve of an income distribution lies at no point below that of another and at some point above, the former distribution will have lower inequality than the latter according to any relative inequality index satisfying the above axioms, which makes Lorenz dominance an attractive tool. Absolute indexes verifying PD, S, and PP are also consistent with a Lorenz-type dominance criterion, in this case given by "absolute" Lorenz curves (Moyes, 1987). However, as opposed to what happens with

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[1] Properties such as normalization, continuity, differentiability, and replication invariance are also commonly invoked, but they are of a more technical nature.

[2] See Amiel and Cowell (1992), Harrison and Seidl (1994), and Seidl and Theilen (1994), among others. In particular, Ballano and Ruiz-Castillo (1993) find that 27% of individuals support this perception of inequality.

[3] The literature offers several intermediate notions. Some of them lead to iso-inequality contours that are linear (Bossert and Pfingsten, 1990; Seidl and Pfingsten, 1997; Del Río and Ruiz-Castillo, 2000; Chakravarty and Tyagarupananda, 2008; Del Río and Alonso-Villar, 2010), whereas others are non-linear (Krtscha, 1994; Yoshida, 2005; Zheng, 2007). For a discussion on these notions, see Del Río and Alonso-Villar (2008).

relative and absolute measures, in the centrist context, there has been almost no discussion regarding the Lorenz dominance that could be defined. An exception is Yoshida (2005), who not only offers a centrist notion that generalizes the “fair compromise” concept proposed by Krtscha (1994) but also introduces a concept of Lorenz dominance, which allows ranking income distributions according to non-linear centrist notions. The centrist attitude behind the “fair compromise” notion is rather challenging because it approaches the absolute view soon when income increases, which makes it difficult for inequality to decrease when analyzing an economy over time (Del Río and Alonso-Villar, 2008). When using instead intermediate measures that consider that iso-inequality contours are straight lines, the centrist attitude remains constant when income increases. In other words, these measures do not allow any change in individuals’ value judgments regarding inequality when varying aggregate income, which seems plausible for analysis in the short and medium run, bringing a complementary perspective to the former. As far as we know, no dominance criterion has been proposed in the literature for ray-invariant centrist measures.

To close this gap, this paper aims to introduce a Lorenz dominance criterion that allows comparisons among income distributions according to the ray invariance proposed by Seidl and Pfingsten (1997), which is a general centrist notion that has given rise to measures with a clear empirical implementation (Del Río and Ruiz-Castillo, 2000; Del Río and Alonso-Villar, 2010). In doing so, this paper adapts the generalized Lorenz curve (Shorrocks, 1983) to our case. The advantage of using a dominance criterion is that it allows the discovery of cases in which one distribution has a higher inequality than another, not only according to a particular index but according to all those indexes consistent with the dominance criterion, which adds robustness to empirical findings.

The α-inequality concept proposed by Seidl and Pfingsten (1997) is rather simple: To keep inequality unaltered, any extra income should be distributed in fixed proportions given by vector α. Based on this idea, Del Río and Ruiz-Castillo (2000) introduced a similar notion but with the advantage of having economic meaning and satisfying the horizontal equity axiom. Their intermediate approach requires considering only those α-rays resulting from linear combinations between the relative and the absolute ray. Thus, when total income increases, inequality remains unchanged if π 100% of the income surplus is allocated preserving income shares in the distribution of reference and

$(1-\pi)100\%$ is distributed in equal absolute amounts. This notion approaches the rightist view when π is close to 1 and the leftist view when π is close to 0. Because the centrist notion proposed by Del Río and Ruiz-Castillo (2000) can be seen as a particular case of the α-inequality proposed by the Seidl and Pfingsten (1997), the new dominance criterion is also valid for it.[4]

It is important to note that intermediate measures are not only a theoretical refinement of relative and absolute measures but a helpful tool for applied research because they allow delving deeper into situations in which income growth is accompanied by a decrease in inequality according to the relative Lorenz criterion together with an increase according to the absolute Lorenz criterion.[5] In those cases, several centrist measures can be used to ascertain to what extent relative inequality has decreased and how far the economy is from experiencing a reduction in absolute inequality. In doing so, this paper draws upon the centrist approach proposed by Del Río and Ruiz-Castillo (2000) and uses different πs to ascertain the range of values under which one distribution has lower inequality than another. Thus, we can determine the set of value judgments that would make society prefer one distribution over the other.[6]

As opposed to centrist and leftist inequality indices, rightist indices are cardinally unaffected by the unit of measurement, which has made them very popular. However, as recently pointed out by Zheng (2007), to measure inequality it is not necessary to ask for this property but to require that inequality rankings between income distributions remain unchanged when all incomes are multiplied by a (positive) scalar—the unit-consistency axiom. This guarantees that rankings are unaffected by the currency unit. In this new scenario, not only relative or rightist measures but also intermediate or centrist measures that satisfy the unit-consistency axiom, as it is the case of the Del Río and

[4] The new criterion is also valid for the intermediate measures proposed by Del Río and Alonso-Villar (2010), which can be seen as extensions of those by Del Río and Ruiz-Castillo (2000).
[5] This circumstance is not unusual because when the mean of an income distribution rises, absolute measures are more demanding than relative. This is so because giving an equal amount of income to every individual leads to a more egalitarian distribution than giving to each of them an amount that keeps the original income shares.
[6] This approach was used by Del Río and Ruiz-Castillo (2001) to compare income distributions in Spain between 1980 and 1990. They concluded that for those people whose opinions are closer to the relative inequality notion (that is, if $\pi \in [0.87,\ 1]$), inequality would have decreased in Spain during that decade. However, for people more skewed towards the left side of the political spectrum (that is, if $\pi \in [0,\ 0.71]$), it would be the opposite.

Ruiz-Castillo's (2000) notion used in this paper, appear as plausible options for empirical research.[7]

These tools are illustrated by using Australian income data for the period 2001–2008. Along this period, households witnessed a general increase in their incomes and a reduction in relative inequality together with an increase in absolute inequality. It seems, therefore, convenient to explore inequality in more detail to ascertain whether inequality has also decreased according to most centrist views and how far Australia is from reaching a reduction according to a leftist view of inequality. For this purpose, the Lorenz dominance defined in this paper is applied to the ray-invariant notion proposed by Del Río and Ruiz-Castillo (2000).

The paper is structured as follows. Section 2 presents the intermediate inequality approach followed in this paper and defines a Lorenz-type curve which gives rise to a dominance criterion consistent with this centrist view. Section 3 offers an empirical illustration of these tools using Australian data for the period 2001–2008. Finally, Section 4 presents main conclusions.

2. Ray invariance and the Lorenz criterion

In this paper, an inequality index, *I*, is a real function defined on the set income distributions *x*, and satisfying the following basic properties:[8]

a) *Symmetry*: $I(x) = I(x\Pi)$, where Π is a permutation matrix.

b) *Replication invariance*: $I(x) = I(\underbrace{x,...,x}_{k})$, where $(\underbrace{x,...,x}_{k})$ is a k-fold replication of x.

c) *Schur-convexity*: $I(Bx) \leq I(x)$ for all bistochastic matrices B that are not permutation matrices.

[7] As opposed to what happens when using the linear inequality notion proposed by Bossert and Pfingsten (1990), the notion proposed by Del Río and Ruiz-Castillo (2000) is unaffected by the currency unit, as shown in Del Río and Alonso-Villar (2008).

[8] Note that Schur-convexity implies symmetry and the Pigou-Dalton principle of transfers (Berge, 1963).

This index is labeled intermediate or centrist if $I(ax) > I(x)$ and $I(x+b1^n) < I(x)$, for any $a > 1$ and $b \in \Re_{++}$ where $1^n \equiv (\underbrace{1,...,1}_{n})$.

Because we focus on symmetric indexes, we can restrict our analysis to the set of all possible ordered income distributions $x_1 \leq x_2 \leq ... \leq x_n$ ($2 \leq n < \infty$), denoted by $D \subset \Re^n$.

2.1 The ray invariance of Seidl and Pfingsten (1997)

A centrist inequality attitude can be modeled in various ways, depending on the shape of the set of inequality equivalent income distributions. In what follows, we present the α-inequality concept proposed by Seidl and Pfingsten (1997) (hereinafter S-P). Accordingly, any extra income should be distributed in fixed proportions (α) in order to keep inequality unaltered.

Let $\alpha \in D$ be a vector of the simplex (i.e., $\sum_{i=1}^{n} \alpha_i = 1$) that is Lorenz-dominated by vector $\frac{1^n}{n}$. The set of distributions for which α represents an intermediate notion is denoted by $\Gamma(\alpha) = \{x \in D : \alpha \succcurlyeq_L v_x\}$, where $\succcurlyeq_L$ denotes (weak) Lorenz dominance and $v_x \equiv \left(\frac{x_1}{\sum_{i=1}^{n} x_i}, ..., \frac{x_n}{\sum_{i=1}^{n} x_i} \right)$ is the vector of income shares associated with distribution *x*. Therefore, this inequality notion requires a certain relationship between the direction of the invariance line, given by α, and distribution *x* if one wants it to represent an intermediate attitude. In other words, α cannot be used for all distributions but only for those that it Lorenz-dominates. Conversely, given a distribution $x \in D$, not all α vectors are suitable if they are to represent intermediate notions for *x*. Only those included in the set $\Omega(x) = \{ \alpha : \alpha \in D, \sum_{i=1}^{n} \alpha_i = 1, \alpha \succcurlyeq_L x \}$ are admissible.

Given a distribution $x \in \Gamma(\alpha)$, the corresponding iso-inequality line is defined by $E_\alpha(x) = \{y \in D : y = x + \tau\alpha, \tau \in \Re \}$ (see Figure 1). Note that, on the one hand, any distribution in $E_\alpha(x)$ with $\tau > 0$ is less egalitarian in the Lorenz sense than $x + \tau 1^n$ and more egalitarian than λx when $\lambda > 1$. On the other hand, those distributions in $E_\alpha(x)$ having $\tau < 0$ are more equally distributed than $x + \tau 1^n$ and less equally distributed than λx when $0 < \lambda < 1$.

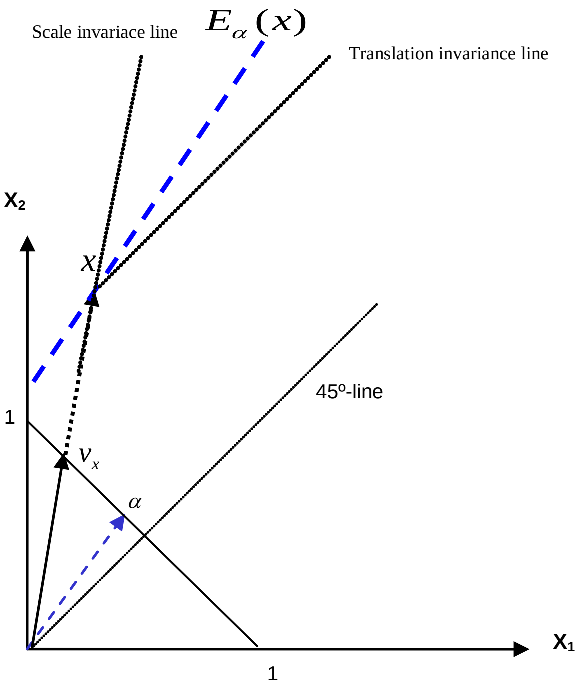

Figure 1. Ray-invariance in Seidl and Pfingsten and scale and translation invariances (n=2).

An intermediate inequality measure, I_α, is labeled ray invariant if $I_\alpha(x) = I_\alpha(x + \tau\alpha)$, where $x \in \Gamma(\alpha)$ and $\tau \in \Re$. A binary relation can be then defined as follows:

$$x \succcurlyeq_\alpha y :\Leftrightarrow I_\alpha(x) \leq I_\alpha(y),$$

where $x, y \in \Gamma(\alpha)$.

2.2 A new Lorenz dominance criterion

Let $\tilde{x}$ denote a distribution obtained through distribution $x \in \Gamma(\alpha)$ allocating its total income among individuals according to income shares given by α (i.e., $\tilde{x} \equiv \alpha \sum_{i=1}^{n} x_i$, see Figure 2). Using $\tilde{x}$, we construct distribution x_α as the ordered vector resulting from the losses and gains experienced by individuals when moving from distribution x to $\tilde{x}$ (i.e., x_α is obtained from $x - \tilde{x}$).

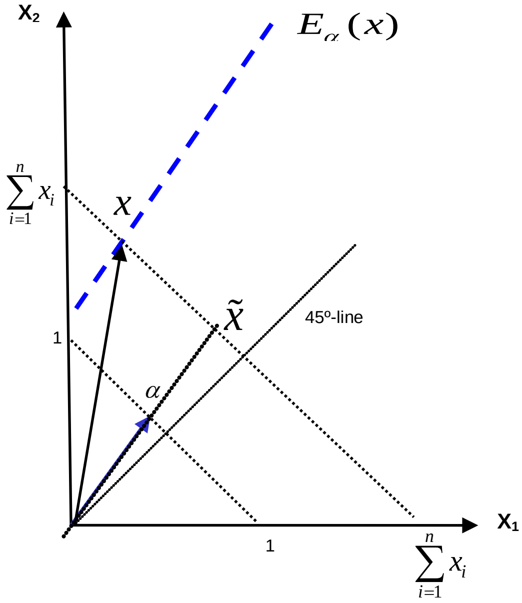

Figure 2. Constructing distribution $\tilde{x}$.

Definition. We define the α-Lorenz curve for distribution $x \in \Gamma(\alpha)$ as the generalized Lorenz curve (Shorrocks, 1983) of distribution x_α:

$$L_\alpha(p,x) = GL(p,x_\alpha),$$

for any $p \in (0,1]$ and adopting the convention $L_\alpha(0,x)=0$ (where the generalized Lorenz curve of a distribution *y* is $GL(\frac{k}{n},y)=\frac{1}{n}\sum_{i=1}^{k} y_i$) This function is convex with respect to *p*, takes no positive values within the interval $(0,1)$, is equal to zero when *p* is equal to 0 and 1, and reaches a minimum at a point $p^* = \frac{k^*}{n}$, where k^* represents the individual with the smallest loss (i.e., $(x_\alpha)_{k^*} < 0$ and $(x_\alpha)_{k^*+1} \geq 0$, see Figure 3).

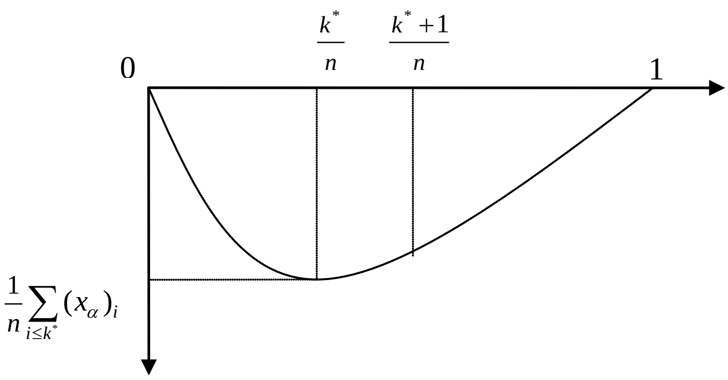

Figure 3. The α-Lorenz curve

Definition. For any $x, y \in \Gamma(\alpha)$ we define an α-Lorenz-dominance criterion as follows:

$$x \succcurlyeq_{L\alpha} y :\Leftrightarrow L_{\alpha}(p,x) \geq L_{\alpha}(p,y).$$

The binary relation given by $\succcurlyeq_{L\alpha}$ allows a partial ordering of income distributions following the α-ray invariance notion proposed by S-P.

Proposition. *The ranking given by an α-Lorenz curve is consistent with that of any index satisfying symmetry, replication invariance, Schur-convexity, and invariance along α-rays. Namely, if $x, y \in \Gamma(\alpha)$, $x \succcurlyeq_{\alpha} y \Leftrightarrow x \succcurlyeq_{L\alpha} y$.*

Proof

Firstly, we prove that for any $x, y \in \Gamma(\alpha)$, $x \succcurlyeq_{\alpha} y \Rightarrow x \succcurlyeq_{L\alpha} y$.

From $x \succcurlyeq_{\alpha} y$, it follows that $I_{\alpha}(x) \leq I_{\alpha}(y)$ for any α-invariant intermediate inequality index. Because I_{α} satisfies symmetry and is α-invariant, we have that $I_{\alpha}(x) = I_{\alpha}(x_{\alpha}) \leq I_{\alpha}(y_{\alpha}) = I_{\alpha}(y)$. Using Theorem 1 in Dasgupta et al. (1973), the Schur-convexity of I_{α} implies that $(x_{\alpha})_1 + ... + (x_{\alpha})_k \geq (y_{\alpha})_1 + ... + (y_{\alpha})_k \;\; \forall k \leq n$ and $(x_{\alpha})_1 + ... + (x_{\alpha})_n = (y_{\alpha})_1 + ... + (y_{\alpha})_n$. Consequently, $GL(p, x_{\alpha}) \geq GL(p, y_{\alpha})$. In other words, $L_{\alpha}(p,x) \geq L_{\alpha}(p,y)$.

Secondly, we prove that for any $x, y \in \Gamma(\alpha)$, $x \succcurlyeq_{L\alpha} y \Rightarrow x \succcurlyeq_{\alpha} y$.

If $L_{\alpha}(p,x) \geq L_{\alpha}(p,y)$, then $\frac{1}{n}\left[(x_{\alpha})_1 + ... + (x_{\alpha})_k\right] \geq \frac{1}{n}\left[(y_{\alpha})_1 + ... + (y_{\alpha})_k\right] \;\; \forall k \leq n$ (with strict inequality in the case k=n). From the aforementioned theorem, it follows that the value of any Schur-convex function evaluated at x_{α} is lower than at y_{α}. Consequently, $I_{\alpha}(x_{\alpha}) < I_{\alpha}(y_{\alpha})$. Because I_{α} is α-invariant $I_{\alpha}(x) = I_{\alpha}(x_{\alpha})$ and $I_{\alpha}(y_{\alpha}) = I_{\alpha}(y)$. Therefore, $x \succcurlyeq_{\alpha} y$. □

2.3 Interpreting α according to Del Río and Ruiz-Castillo (2000)

The ray invariance concept proposed by S-P has no clear economic interpretation—which makes it difficult its use in empirical analyses—and violates the horizontal equity axiom because individuals who have the same income level initially may be treated differently (Zoli, 2003). To solve these problems, Del Río and Ruiz-Castillo (2000) offered a ray-invariant notion that can be considered as a special case of the former. According to their proposal, inequality depends on two parameters, instead of one: the income shares in the distribution of reference that gives rise to the rightist and leftist views, denoted by simplex vector v, and $\pi \in [0, 1]$, which is used to define a convex combination between them.[9] Once v and π are fixed, it is possible to calculate the n-dimensional simplex vector $\alpha = \pi v + (1-\pi)\frac{1^n}{n}$ that defines the direction of the ray. The economic meaning of this invariance notion is simple: When total income increases, inequality remains unchanged if π 100% of the income surplus is allocated preserving income shares in the distribution of reference and $(1-\pi)$ 100% is distributed in equal absolute amounts. In other words, α can be interpreted as a convex combination of the value judgments behind the distribution of reference, v, and those behind the equalitarian distribution, $\frac{1^n}{n}$ (see Figure 4). If we chose a value of π close to 1, the notion represents rightist value judgments while if π is close to 0, the inequality attitude is rather leftist, as compared to the distribution of reference.

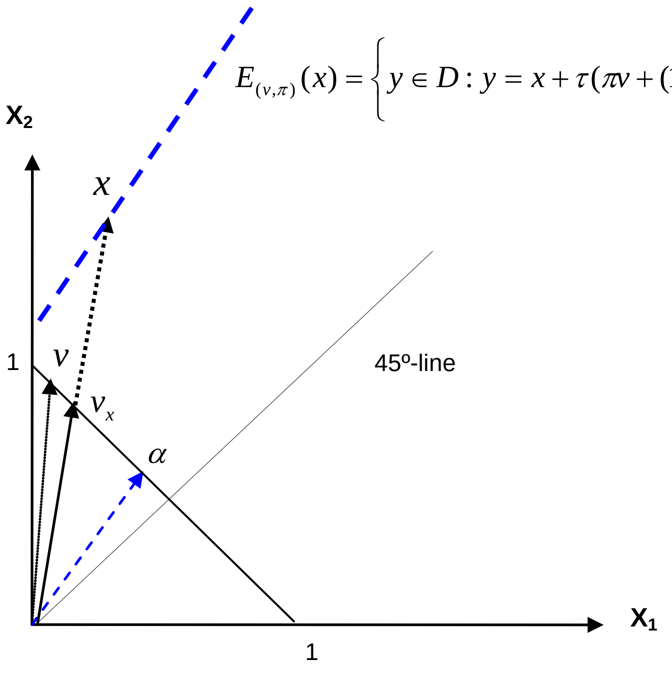

Figure 4. Invariance in Del Río and Ruiz-Castillo (2000) ($n = 2,\ \pi = 0.25$).

[9] Vector v is assumed to represent an ordered distribution.

The distribution of reference, v, plays an important role in this approach. Note that vector v does not necessarily have to coincide with vector v_x (as shown in Figure 4). However, in comparing distribution x and distribution y (which can be assumed to have a higher mean without loss of generality), vector v could be chosen as the income shares of x, i.e., $v = v_x$. By using this benchmark, together with the parameter π reflecting the inequality-invariance value judgments of society, it would be possible to determine whether y has a lower inequality than the distribution reached if π 100% of the income gap had been distributed according to income shares in x and $(1-\pi)$ 100% in equal amounts among individuals. Note that, in doing so, the same vector of reference ($v = v_x$) has to be used for both distributions x and y (which makes this centrist notion path independent, as explained in Del Río and Alonso-Villar, 2010). It would not be possible to use $v = v_x$ for measuring the inequality level corresponding to x while using $v = v_y$ in the case of distribution y because that would imply that different inequality attitudes would be used for each distribution. In other words, once v and π are chosen, they cannot be changed: The same intermediate notion must be used when comparing any two income distributions. Therefore, when studying the evolution of an economy over time, this approach allows the possibility of taking into account the starting point.

3. An Illustration: Recent Evolution of Income Inequality in Australia

In this section, we provide an empirical illustration of the new intermediate dominance criterion using Australian income data for the period 2001–2008.[10] During this period, Australia experienced high and sustainable economic and employment growth, the real economy growing on average by more than 3.5 percent every year (ABS 2011). As will be shown, the evolution of the income distribution in Australia during this period provides a suitable case for an intermediate inequality analysis.

The data used in our analysis come from the first and eighth waves of the Household, Income and Labour Dynamics in Australia (HILDA) Survey conducted by the

[10] For studies focusing on earlier periods using other methodologies, see, for example, Saunders et al. (1991), Blacklow and Ray (2000), Athanasopoulos and Vahid (2003), and Atkinson and Leigh (2007).

Melbourne Institute of Applied Economics and Social Research to analyze the change in income inequality that took place in Australia during that period. We look at changes in the distribution of annual private income before taxes and transfers from the public sector. This income variable is defined as the sum of market income including labour income in the form of wages and salaries, capital income from businesses, investments, and private pensions plus the value of all non-market private transfers received by any household member. Differences in non-income needs across households of different size and composition are considered in the analysis, so household income is converted into household equivalent income using an equivalence scale. Thus, we use the parametric family of equivalence scales introduced by Buhmann *et al.* (1988):

$$e(s,\Theta) = s^{\Theta},$$

where s is the size of the household and Θ is the elasticity of the scale rate. Adjusted income values are computed by dividing household income by scale factor s^{Θ}, with different values of Θ being used to check the robustness of the results. All income values correspond to real values expressed in constant 2008 Australian dollars derived using Consumer Price Index figures provided by the Australian Bureau of Statistics. Finally, all the estimates are computed using the population weights reported in HILDA to obtain population rather than sample estimates.

Table 1 summarizes the main changes in the distribution of household income that took place between 2001 and 2008. In this period, households in Australia witnessed a general increase in their incomes, with the mean household income growing more than $1,163 every year, equivalent to an annual rate of growth of 2.7 percent. This growth, however, was not uniform across the whole distribution. Thus, the first column in Table 1 suggests that the *absolute* income gains among the top three quintiles were larger than those experienced by the bottom positions, whose mean income grew less than the overall mean. On the contrary, the largest *relative* income growth was for the poorest three quintiles, which grew above the population average, with the growth rate declining as we move up in the distribution. This growth pattern is consistent with the increase in the income share of the three bottom quintiles and the corresponding decline of the share accumulated by the two top quintiles (columns 3 and 4).[11]

[11] Analyzing the top incomes in Australia, Atkinson and Leigh (2007) found that between the 1920s and 1980s top income shares fell for most of the period, while they rose rapidly between the 1980s and 1990s.

Table 1
Annual Changes in Mean Income and Income Shares by Income Quintiles in Australia between 2001 and 2008

Quintile	Absolute change (AUD $)	Relative change (%)	Income Shares (%) 2001	Income Shares (%) 2008
1	178.99	19.33	0.26	0.74
2	929.13	5.71	6.92	8.48
3	1,039.46	2.87	16.75	16.96
4	1,237.54	2.22	26.39	25.53
5	2,465.89	2.34	49.68	48.29
Total	1,163.84	2.71	100	100

Note: Equivalent incomes computed assuming a value for Θ equal to 0.5.

These changes in the income distribution have important implications in terms of inequality. The fact that the absolute difference between top and bottom positions widened suggests an increase in absolute inequality. On the contrary, the convergence in the income shares of the groups points toward a reduction in relative inequality. These findings are corroborated in Figure 5, which shows the absolute and relative Lorenz curves for the 2001 and 2008 income distributions. In the relative case (Figure 5.a), the Lorenz curve for 2008 dominates the initial one, which implies that these two distributions will be unambiguously ranked by the class of scale invariant inequality indexes (Foster, 1985). In contrast, the absolute Lorenz curve for 2001 lies above that of 2008 at every point (Figure 5.b), so that any inequality measure verifying the translation invariance property would indicate an increase in the level of absolute inequality during this period.[12]

[12] The dominance results presented in Figure 5 are not sensitive to the choice of the equivalence scale parameter. Thus, for any value of $\Theta \in [0,1]$, we find that absolute (relative) inequality in Australia increased (decreased) between 2001and 2008.

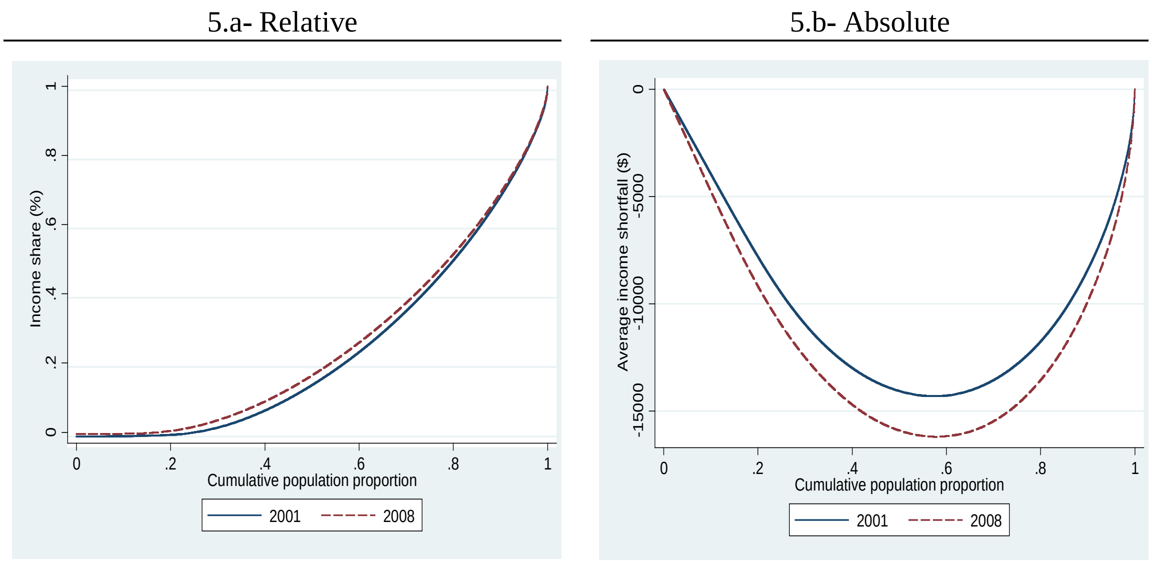

Figure 5. Absolute and relative Lorenz curves for Australia, years 2001 and 2008 (Θ = 0.5).

It is important to note that the relative and absolute dominance criteria do not inform us about the changes in inequality for intermediate notions of inequality lying between the "rightist" and "leftist" extreme cases. To deal with this issue, we now use our α-Lorenz curves and the corresponding dominance criterion to compare the 2001 and 2008 distributions adopting centrist notions of inequality. This methodology allows us to search for unambiguous rankings among the classes of intermediate inequality measures. For that purpose, we use the class of inequality indexes consistent with the ray-invariant notion proposed by Del Río and Ruiz-Castillo (2000).

Let x and y denote the income distributions in Australia in 2001 and 2008. Also, let v_x, v_y, and $\frac{1^n}{n}$ be the vectors of the simplex associated with the x, y, and egalitarian distributions, respectively. Taking the initial distribution as the distribution of reference, we consider inequality-invariance value judgments, α, of the form

$$\alpha = \pi v_x + (1-\pi)\frac{1^n}{n},$$

where $\pi \in [0, 1]$. If $\pi = 0$, then α becomes the absolute ray-invariant notion, whereas $\pi = 1$ leads to the relative ray. Further, for a particular α to represent a centrist attitude,

α must Lorenz-dominate both v_x and v_y. This condition holds for $\pi = 0$ but it is not satisfied when π is set equal to 1 because v_x is Lorenz-dominated by v_y.[13]

We denote by π^M the maximum value of π for which a valid intermediate notion can be derived. For a given vector α, the ordered distributions x_α and y_α defined in Section 2.2 have to be constructed to check for the existence of α-Lorenz dominance. When x and y come from populations of different sizes, this requires the use of replications of the initial distributions to ensure that the vectors are conformable. Notice that this does not impose any limitation on the validity of the analysis because inequality rankings are unaffected by population replication. In fact, for the general class of inequality indexes satisfying the replication invariance principle considered in this paper, the original and replica distributions exhibit exactly the same level of inequality independently of the mean-invariance notion. Furthermore, for any value of π, the invariance value judgment given by α is equivalent to the one obtained for the replicated population because both α vectors have the same Lorenz curve. This means that to keep inequality unaltered, both notions require an analogous allocation of the income growth among individuals.

As an example, Figure 6 shows the α-Lorenz curves for 2001 and 2008 for an equivalence scale parameter Θ of 0.5 and values of π equal to 0.25, 0.5, 0.75, and 0.9. As the figure shows, it is possible to derive unambiguous rankings of the initial and the final distributions for some centrist notions of inequality. Thus, when π is set equal to 0.25 or 0.5, the curve for 2001 α-Lorenz dominates that of 2008 and, therefore, it follows from our proposition that any inequality index consistent with these centrist attitudes would conclude that income inequality increased during this period. Interestingly, this result does not hold anymore when centrist views that are closer to the relative notion of inequality are considered. For both π equal to 0.75 and 0.9, the α-Lorenz curves for 2001 and 2008 cross multiple times, which implies that no unambiguous ranking of these two distributions can be derived for these attitudes towards inequality.

[13] This comes from the fact that x is Lorenz-dominated by y.

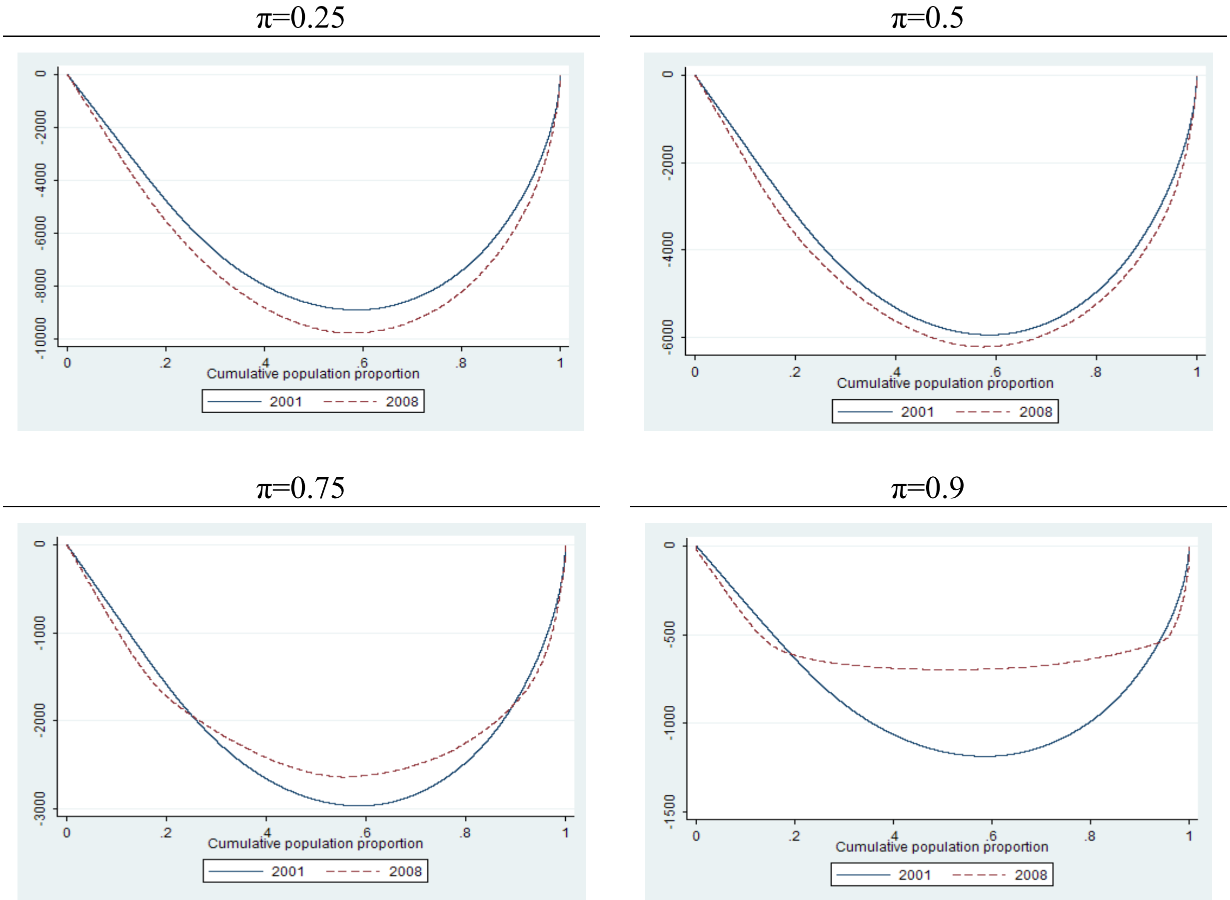

Figure 6. α -Lorenz curves for Australia, years 2001 and 2008 (Θ = 0.5).

Table 2 summarizes the main results of the intermediate inequality analysis for different values of the equivalence scale parameter Θ. For each case, the value of π^M and three sets of centrist notions expressed in terms of π are provided: The values according to which income inequality increased during the period as the distribution in 2001 dominates that in 2008; those π s for which these two distributions cannot be unambiguously ranked; and the set of notions for which one could claim income inequality declined from 2001–2008.

We find that the initial and the final distributions are comparable for most of the centrist notions that can be constructed as a convex combination of the vectors v_x and $\frac{1^n}{n}$. Thus, π^M is around 0.9 for all the equivalence scales considered.

Table 2
Intermediate Inequality in Australia: 2001–2008

Θ	π^M	Values of π according to which inequality increased, remained unaltered, or diminished		
		Increased ($x \succcurlyeq_\alpha y$)	Unaltered	Diminished ($y \succcurlyeq_\alpha x$)
0	0.88	[0,0.69]	(0.69, 0.88]	-
0.25	0.91	[0,0.64]	(0.64, 0.91]	-
0.5	0.94	[0,0.65]	(0.65, 0.94]	-
0.75	0.92	[0,0.62]	(0.62, 0.92]	-
1	0.87	[0,0.60]	(0.60, 0.87]	-

Remarkably, our results suggest that income inequality in Australia increased for most of the intermediate value judgments. In fact, inequality increased for all those centrist attitudes that require the equal distribution of at least 31–40 percent, depending on the value of parameter Θ, of the income gains in order to keep inequality unchanged. Thus, for example, when $\Theta = 0.5$, inequality increases if using centrist views where at most 65 percent of the income growth is distributed across individuals according to income shares in 2001 (Table 2, column 2) and, consequently, at least 35 percent of the growth is allocated in equal amounts.

On the other hand, for $\Theta = 0.5$, the initial and the final distributions cannot be unambiguously ranked when using invariant notions according to which inequality is maintained if the proportion of the income growth that is equally distributed among individuals ranges between 6 and 35 percent (Table 2, column 3). For the remaining equivalence scale values, the lower and upper limits of the interval are 9–13 and 31–40 percent, respectively.

Finally, for none of the values of π for which a valid centrist notion can be defined, we can claim income inequality in Australia actually declined between 2001 and 2008.

4. Conclusions

This paper has introduced a dominance criterion that allows ranking income distributions according to the centrist inequality notion proposed by Seidl and Pfingsten (1997). In doing so, α-Lorenz curves, which are related to the generalized Lorenz curves (Shorrocks, 1983), are defined. Our proposal allows finding cases in which one distribution has higher inequality than another not only according to a particular α-index but according to all those α-indexes consistent with our dominance criterion (as also happens with the relative and absolute Lorenz dominance criteria and the indexes verifying the scale invariance and the translation invariance axioms, respectively). We have used his dominance criterion to rank income distributions in Australia between 2001 and 2008 according to the family of centrist notions proposed by Del Río and Ruiz-Castillo (2000), which are particular cases of those proposed by Seidl and Pfingsten (1997). The results show that even though relative inequality decreased during the period, according to most ray-invariant centrist views of inequality, inequality increased.

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