Income Distribution in Discrete Hours Behavioural Microsimulation Models: An Illustration of the Labour Supply and Distributional Effects of Social Transfers

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Abstract

The distributional implications of an extreme hypothetical reform of the Australian tax-transfer system are examined using the Melbourne Institute Tax and Transfer Simulator. This simulation model predicts labour supply using a probabilistic discrete hours approach. The analysis of the distribution of income is difficult because, even for a small sample with a modest range of labour supply points, the range of possible combinations over the sample is extremely large. This paper demonstrates the use of a pseudo income distribution, where the probability of a particular labour supply value occurring (standardised by the population size) is used to refer to a particular position in the pseudo income distribution. The superior performance of the pseudo approach compared to using the expected income or taking 10 draws from the possible set of distributions is clearly illustrated by the simulation example for sole parents, in which the whole social security system is abolished. The example shows further that even if sole parents could move freely into the labour force, they do not manage to compensate fully for their loss of income when no benefit payments are available.

1 Introduction

Behavioural microsimulation models, designed to simulate tax policy changes, are frequently based on discrete hours labour supply models. The strategy adopted in the discrete hours approach to labour supply modelling is to replace the full budget set with a finite number of net incomes corresponding to discrete hours points.[1] This type of modelling is essentially probabilistic. That is, it does not identify a particular level of hours worked for each individual, but generates for each individual a probability distribution over the discrete hours levels used. Hence the standard approach to poverty and inequality measurement cannot be applied.

This paper has two main aims. First, it examines alternative methods of obtaining summary measures of inequality and poverty using an example in the context of behavioural microsimulation modelling of tax policy changes based on a discrete choice labour supply model. Section 2 introduces the outcomes available after simulating policy reforms using a discrete hours labour supply model, and discusses three alternative approaches to poverty and inequality measurement. Section 3 describes the alternative approaches in more detail. Special attention is given to the case of the variance, because the formulae for other measures are far less tractable.

A second aim of the paper is to examine an extreme policy change involving the elimination of the 'safety net' for sole parents, provided by the Australian tax and transfer system. This also has the advantage of allowing empirical comparisons of the results of applying the alternative distributional approaches, and contrasts behavioural with non-behavioural simulations in the context of a large policy reform where the assumption of fixed labour supplies would be quite unrealistic. The simulation also gives an indication of the potential magnitude of the overall effect of social transfers on labour supply, government expenditure and inequality. The simulations use the Mel-

[1] The discrete approach allows both for random preference heterogeneity and state-specific errors in perception, and can incorporate either directly estimated or indirectly imputed fixed costs in estimation. See, for example, Van Soest (1995) and Keane and Moffitt (1998). For a survey of discrete and continuous approaches, see Creedy and Duncan (2002).

bourne Institute Tax and Transfer Simulator (MITTS), described very briefly in section 4. The results of the hypothetical reform are reported in section 5. Section 6 concludes.

2 The Distribution of Labour Supply and Income

This section shows the implication for the expected income distribution arising from the use of discrete labour supply models in policy simulations. The discrete choice model of labour supply and the associated probability distribution over hours of work are described in subsection 2.1. Subsection 2.2 considers several possible ways of dealing with such probability distributions in constructing summary measures of the income distribution across a sample of households.

2.1 The Discrete Choice Model

Suppose there are k discrete hours levels $h_1, ..., h_k$. The utility associated with each hours level is denoted U_i^* and is a function of 'measured' utility $U\left(h_i|\, X\right)$ plus an 'error term', v_i, arising from factors such as measurement errors concerning the variables in X, optimisation errors of the individual or the existence of unobserved preference characteristics. Hence:[2]

$$\begin{aligned} U_i^* &= U\left(h_i|\, X\right) + v_i \\ &= U_i + v_i \end{aligned} \tag{1}$$

Any observation on h is associated with a set of possible 'draws' of the k random variables v_i from their respective distributions. For hours level, i, utility maximisation implies that this hours level is chosen if:

$$U_i^* \geq U_j^* \qquad \text{for all } j \tag{2}$$

[2]Although utility is considered to be a function of hours worked and net income, the latter is determined directly from the associated hours level and the wage and other characteristics of the individual.

Substituting for U_i^*, using (1), and rearranging, this condition is equivalent to the requirement that:

$$v_j \leq v_i + U_i - U_j \qquad \text{for all } j \tag{3}$$

Hence, for any given value of v_i, the probability of U_i^* exceeding all other values is equal to the joint probability that $v_i + U_i - U_1 \geq v_1$ and $v_i + U_i - U_2 \geq v_2$ and so on for all j. If the various distributions are independent, this joint probability is the product of the separate probabilities, $P(v_j \leq v_i + U_i - U_j)$. Therefore, for any given value of v_i, the probability that hours level i produces maximum utility is equal to:

$$\prod_{j \neq i} P(v_j \leq v_i + U_i - U_j) \tag{4}$$

This is the conditional probability, for a given value of v_i. The overall probability is found by aggregating terms like (4) over all possible values of v_i. Suppose v is a continuous random variable, where $f(v)$ and $F(v)$ are the density and distribution functions respectively of v. Then:

$$p_i = \int_{-\infty}^{+\infty} \left\{ \prod_{j \neq i} F(v_i + U_i - U_j) \right\} f(v_i)\, dv_i \tag{5}$$

Suppose the distribution of v is described by the Extreme (Maximum) Value Type I distribution with $f(v) = e^{-v} e^{-e^{-v}}$. It can be shown that:

$$p_i = \frac{e^{U_i}}{\sum_{j=1}^{k} e^{U_j}}$$

Given information about individuals' preference functions, and their net incomes at each hours point, equation (5) can be used to compute the probability distribution of hours worked for each individual.

2.2 Distributions across the Sample

With n individuals and k hours levels, there would be k^n possible combinations of labour supply, and thus income distributions, each resulting in a different value of poverty and inequality. Under the reasonable assumption that individuals' distributions of hours are independent, the probability of each income distribution, P_q, is given by the product of the relevant probabilities. Hence, if $p_{i,j}$ is the probability that individual j is at hours level i, the joint probability P_q is equal to $p_{i,1}p_{j,2}...p_{r,n}$, where q runs from 1 to k^n; and i, j, r can attain values between 1 and k (indicating the labour supply points chosen in combination q for each individual). In principle, an inequality or poverty measure can be calculated as a weighted average over all possible outcomes, with weights equal to the probabilities P_q. However, for any realistic sample size, even for few discrete labour supply points, the large number of possible combinations makes it computationally impractical to calculate all k^n distributions and associated probabilities P_q.[3]

Instead of examining all combinations of income levels, a sampling approach could be adopted. A large number of possible income distributions could be obtained by taking random draws from each individual's hours distribution. With a sufficiently large number of randomly selected samples, the proportion of each hours combination would replicate the precise probabilities discussed above. This approach still requires a large computational effort, depending on the number of draws needed to obtain a good approximation. However, it provides a valuable way of examining the performance of alternative less computer-intensive approaches against this benchmark.

In considering alternatives which offer more practical solutions, the most obvious is perhaps a simple approach where the expected income is calculated for each individual and is used as if it were a single 'representative' level of income for that individual.[4] An alternative is to treat all possible outcomes for every individual as if they were separate observations, weighted by the individual probabilities of labour supply to produce a pseudo distribution.

To illustrate the alternative approaches imagine a two-person population with two hours levels available, and wage rates as shown in Table 1. There are therefore four possible alternative combinations, with associated probabilities, on the assumption that the probability distributions for different

[3]The appropriate weighted average could be obtained cumulatively using an algorithm for systematically working through all the k^n combinations. However, the computing time needed would be extremely long.

[4]See for example, Gerfin and Leu (2003).

individuals are independent, as shown in Table 2. The poorest person would have proportions of total income of 0.40, 0.43, 0.25 and 0.40 respectively. On average (using the above probabilities) the poorest person would have 0.3178 of total income.

Table 1: Income Distribution: A Two-Person Example

	Probabilities		Incomes	
Hours	person 1	person 2	person 1	person 2
10	0.8	0.3	20	30
20	0.2	0.7	40	60
Wage	2.0	3.0		

Table 2: Alternative Possible Income Distributions

	Incomes		
Combination	Person 1	Person 2	Probability
1	20	30	0.24
2	40	30	0.06
3	20	60	0.56
4	40	60	0.14

The expected number of hours of persons 1 and two respectively are 12 and 17 hours, giving incomes of 24 and 51 respectively. The poorest person would have 0.32 of total income. For the pseudo income distribution, there are income outcomes 20, 30, 40 and 60 with probabilities 0.4, 0.15, 0.1 and 0.35. Thus the poorest 40 per cent of the pseudo population have an income proportion of 0.13, the poorest 55 per cent have 0.33 of total income, and 65 per cent of the poorest 'people' have 0.6 of total income.

3 Analytical Comparison of Alternative Approaches

This section explores analytically the differences between the alternative approaches to constructing inequality measures, following Creedy *et al.* (2003). It is found that measures such as the Gini coefficient become intractable, so most attention is concentrated on the variance. Numerical comparisons are provided in the next section. First, the notation is introduced.

Let $y_{j,i}$ and $p_{j,i}$ denote respectively the i^{th} person's income at hours level j, and the probability of hours level j. Assuming independence, the joint probabilities of the k^n possible alternative combinations are as shown in Table 3. An arbitrary combination m from the set of all possible combinations consists of the set of hours points $h_{1m}, h_{2m}, ..., h_{nm}$ for persons 1 to n respectively.

Table 3: Alternative Possible Income Distributions

	Incomes of persons:				
Combination	1	2	...	n	Probability
1	$y_{1,1}$	$y_{1,2}$	...	$y_{1,n}$	$p_{1,1}p_{1,2}\cdots p_{1,n}$
m	$y_{h_{1m},1}$	$y_{h_{2m},2}$	...	$y_{h_{nm},n}$	$p_{h_{1m},1}p_{h_{2m},2}\cdots p_{h_{nm},n}$
k^n	$y_{k,1}$	$y_{k,2}$	...	$y_{k,n}$	$p_{k,1}p_{k,2}\cdots p_{k,n}$

3.1 The Gini Coefficient

If we allow ρ_i to refer to the ranking of each observation in the population, ordered from richest to poorest, the standard Gini measure can be written as:[5]

$$G = \frac{n+1}{n-1} - \frac{2\sum_{i=1}^{n} \rho_i y_i}{n(n-1)\frac{1}{n}\sum_{i=1}^{n} y_i} \tag{6}$$

with $\rho_i = \rho_{i-1} + 1$ and $\rho_1 = 1$.

The Gini Coefficient for all combinations is expressed as:

$$\frac{n+1}{n-1} - 2\sum_{m=1}^{k^n} p_{h_{1m},1}p_{h_{2m},2}\cdots p_{h_{nm},n} \frac{\sum_{i=1}^{n} \rho_{l_{im}} y_{h_{l_{im}m}l_{im}}}{n(n-1)\frac{1}{n}\sum_{i=1}^{n} y_{h_{im},i}} \tag{7}$$

[5] Sample weights are straightforward to include in all formulae of this section. For example for the Gini: $G = \frac{N+1}{N-1} - 2\sum_{i=1}^{n} \rho_i w_i y_i / \left\{ N(N-1)\frac{1}{N}\sum_{i=1}^{n} w_i y_i \right\}$, with $N = \sum_{i=1}^{n} w_i$ and $\rho_i = \rho_{i-1} + w_{i-1}$. However, to keep the notation easier to read, weights are omitted.

with l_{im} indicating the index for the household with the ith ranked income in the mth combination and $\rho_{l_{im}} = \rho_{l_{i-1,m}} + 1$. Using expected incomes it is:

$$\frac{n+1}{n-1} - 2\frac{\sum_{i=1}^{n} \rho_{l_i^*} \sum_{h=1}^{k} p_{hl_i^*} y_{hl_i^*}}{n(n-1)\frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{k} p_{hi} y_{hi}} \tag{8}$$

with l_i^* indicating the index for the household with the ith ranked expected income and $\rho_{l_i^*} = \rho_{l_{i-1}^*} + 1$. Using the pseudo method the Gini measure is:

$$\frac{n+1}{n-1} - 2\frac{\sum_{i=1}^{n}\sum_{h=1}^{k} \rho_{v_{h(i-1)+h},\widehat{l}_{h(i-1)+h}} p_{v_{h(i-1)+h},\widehat{l}_{h(i-1)+h}} y_{v_{h(i-1)+h},\widehat{l}_{h(i-1)+h}}}{n(n-1)\frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{k} p_{hi} y_{hi}} \tag{9}$$

with $\widehat{l}_{h(i-1)+h}$ indicating the index for the household with the $(h(i-1)+h)^{th}$ ranked income within the nh possible incomes, $v_{h(i-1)+h}$ indicating the index for the discrete hours point with the $(h(i-1)+h)^{th}$ ranked income and $\rho_{v_{h(i-1)+h},\widehat{l}_{h(i-1)+h}} = \rho_{v_{h(i-1)+h-1},\widehat{l}_{h(i-1)+h-1}} + p_{v_{h(i-1)+h},\widehat{l}_{h(i-1)+h}}$.

It is clear that the explicit analysis of the properties of the different methods is intractable for the Gini Coefficient and other such measures. Monte Carlo experiments in Creedy *et al.* (2003) have revealed the properties of the alternative methods for the Gini Coefficient and for other poverty and inequality measures.

3.2 The Mean and Variance

However, for the mean and variance analytical expressions can be found for the properties of the different methods. First, the arithmetic means of each possible combination, denoted $\overline{Y}_i$, can be used to produce the arithmetic mean of all these means, $\overline{Y}$:

$$\overline{Y} = p_{1,1}p_{1,2}...p_{1,n}\overline{Y}_1 + ... + p_{h_{1m},1}p_{h_{2m},2}...p_{h_{nm},n}\overline{Y}_m + ... + p_{k,1}p_{k,2}...p_{k,n}\overline{Y}_{k^n} \tag{10}$$

Substituting for $\overline{Y}_m = \frac{1}{n}\sum_{i=1}^{n} y_{h_{im},i}$, gives:

$$\overline{Y} = \frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{k} p_{h,i} y_{h,i} \tag{11}$$

The arithmetic mean, $\overline{S}^2$, of the variances for each possible combination, S_i^2, is:

$$\overline{S}^2 = p_{1,1}p_{1,2}...p_{1,n}S_1^2+...+p_{h_{1m},1}p_{h_{2m},2}...p_{h_{nm},n}S_m^2+...+p_{k,1}p_{k,2}...p_{k,n}S_{k^n}^2 \quad (12)$$

Where $S_m^2 = \frac{1}{n}\sum_{i=1}^n y_{h_{im},i}^2 - \overline{Y}_m^2$.

Hence:[6]

$$\begin{aligned}
\overline{S}^2 &= \sum_{m=1}^{k^n} p_{h_{1m},1}p_{h_{2m},2}...p_{h_{nm},n}\frac{1}{n}\sum_{i=1}^{n} y_{h_{im},i}^2 \\
&\quad -\sum_{m=1}^{k^n} p_{h_{1m},1}p_{h_{2m},2}...p_{h_{nm},n}\left(\frac{1}{n}\sum_{i=1}^{n} y_{h_{im},i}\right)^2 \\
&= \frac{1}{n}\sum_{h=1}^{k}\sum_{i=1}^{n} p_{h,i}y_{h,i}^2 - \frac{1}{n^2}\sum_{h=1}^{k}\sum_{i=1}^{n} p_{h,i}y_{h,i}^2 \\
&\quad -\frac{1}{n^2}\sum_{m=1}^{k^n} p_{h_{1m},1}p_{h_{2m},2}...p_{h_{nm},n}\left(2\sum_{i=1}^{n}\sum_{j=1}^{i-1} y_{h_{im},i}y_{h_{jm},j}\right)
\end{aligned} \quad (13)$$

3.2.1 The Expected Income Method

Consider the use of the arithmetic mean income for each individual as a representative income.[7] Using expected incomes, $\overline{y}_1 = \sum_{h=1}^k p_{h,1}y_{h,1}$ to $\overline{y}_n = \sum_{h=1}^k p_{h,n}y_{h,n}$, the overall mean of the n individual means, $\overline{Y}_m$, is identical to (11). The variance of the individual mean incomes, S_m^2 is:

$$\begin{aligned}
\overline{S}_m^2 &= \frac{1}{n}\sum_{i=1}^{n}\overline{y}_i^2 - \overline{Y}_m^2 \\
&= \frac{1}{n}\sum_{i=1}^{n}\left(\sum_{h=1}^{k} p_{h,i}y_{h,i}\right)^2 - \left(\frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{k} p_{h,i}y_{h,i}\right)^2
\end{aligned} \quad (14)$$

The terms in (14) contain powers of the various probabilities. Hence $S_m^2 \neq \overline{S}^2$. The arithmetic means, as linear functions, are identical, but the

[6] More elaborate derivations of the formulae in this section can be found in Creedy *et al.* (2003).

[7] Other possible candidates are the median and the mode of the hours distribution for each individual. These are rejected here on the grounds that they ignore potentially important information, as they are based on just one value in the distribution, and the arithmetic means of resulting income distributions do not correspond to $\overline{Y}$.

variances, involving nonlinear functions of the various terms, are different. A similar feature is expected for any inequality measure that is expressed as a nonlinear function of incomes. The difference between the method using all combinations and the method using the expected income is:

$$\begin{aligned} \overline{S}_m^2 - \overline{S}^2 &= \left(\frac{1}{n} - \frac{1}{n^2}\right)\left[\sum_{i=1}^{n}\left(\sum_{h=1}^{k} p_{h,i}y_{h,i}\right)^2 - \sum_{i=1}^{n}\sum_{h=1}^{k} p_{h,i}y_{h,i}^2\right] \\ &= \left(\frac{n-1}{n^2}\right)\sum_{i=1}^{n}\left(\left(\overline{y_i}\right)^2 - \overline{y_i^2}\right) \leq 0 \end{aligned} \tag{15}$$

This confirms the trivial case where the two approaches give identical results for the variance if either the hours distributions are concentrated on a single hours level for each individual or the incomes are the same irrespective of the hours worked. The method using the expected income always underestimates the true expected variance. This is not surprising, given that the use of the expected income understates the variation in incomes in the population.

3.2.2 The Pseudo Income Distribution Method

Consider the pseudo income distribution with nk income levels, each associated with a corresponding probability. The incomes are $y_{h,i}$, where h ranges from 1 to k and i ranges from 1 to n, and associated probabilities are $p_{h,i}/n$. The division by n ensures that the sum of the probabilities adds to 1. The $y_{h,i}$ values are placed in a single vector, $z = \{y_{h,i}\}$ with nk elements, with the associated probabilities given by $p' = \{p_{h,i}/n\}$. Hence:

$$\sum_{j=1}^{nk} p'_j = \sum_{i=1}^{n}\sum_{h=1}^{k} p_{h,i}/n = 1 \tag{16}$$

The arithmetic mean of this pseudo distribution, $\overline{Y}_p$, is equal to $\overline{Y}$ in (11) above. The variance of this pseudo distribution, S_p^2, is given by:

$$\overline{S}_p^2 = \frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{k} p_{h,i}y_{h,i}^2 - \left[\frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{k} p_{h,i}y_{h,i}\right]^2 \tag{17}$$

Again, this expression depends on the powers of the various probabilities, so it cannot be expected to equal the arithmetic mean of the individual sample variances given in (13). The difference between the method using all combinations and that using the pseudo method is:

$$\begin{aligned}
\overline{S}_p^2 - \overline{S}^2 &= -\frac{1}{n^2}\left(\sum_{h=1}^{k}\sum_{i=1}^{n}\left(p_{h,i}^2 y_{h,i}^2 - p_{h,i} y_{h,i}^2\right) + 2\sum_{i=1}^{n}\sum_{h=1}^{k}\sum_{l=1}^{h-1} p_{h,i} y_{h,i} p_{l,i} y_{l,i}\right) \\
&= -\frac{1}{n^2}\sum_{i=1}^{n}\left(\left(\overline{y}_i\right)^2 - \overline{y_i^2}\right) \geq 0 \qquad (18)
\end{aligned}$$

The pseudo method always overestimates the true expected variance, because it exaggerates the true variety of incomes by treating all individual hours points as separate observations with weights relative to the probability of occurring. Comparing this difference to that obtained for the method using expected income demonstrates that, for samples of more than two persons, estimates of the variance using the pseudo method are closer to the variance calculated in the method using all combinations, compared with estimates using the expected income method. Furthermore, the true outcome lies between the pseudo method and the expected income method. The difference is likely to become smaller for the pseudo method as the sample size increases, indicating that the exaggeration of income variability becomes less important when the number of individuals in the sample increases.

4 The MITTS Simulation Model

The numerical comparisons of the alternative approaches reported in section 5 are obtained using the Melbourne Institute Tax and Transfer Simulator (MITTS). Here a brief description of this model is presented.[8] MITTS provides both behavioural and non-behavioural simulations of reforms to the Australian direct tax and transfer system, and is based on the Housing and Income Cost Survey of 1997/98, the latest year for which data are available.

The behavioural responses in MITTS are based on the use of quadratic preference functions whereby the parameters are allowed to vary with an individual's characteristics. These parameters have been estimated for five demographic groups, which include married or partnered men and women,

[8] For a full description, see Creedy *et al.* (2002).

single men and women, and sole parents. Labour supply is divided into 11 discrete hours points.[9] Using the expression given in Section 2.1, a probability distribution is generated for each individual over the discrete hours levels used.

The behavioural simulations begin by taking the discrete hours level for each individual that is closest to the observed hours level. Then, given the parameter estimates of the quadratic preference function (which vary according to a range of individual and household characteristics), a random draw is taken from the distributions of the 'error' terms. This draw is rejected if it results in an optimal hours level that differs from the discretised value observed. The accepted drawings are then used in the determination of the optimal hours level after the policy change. A user-specified total number of 'successful draws' (that is, drawings which generate the observed hours as the optimal value under the base system for the individual) are produced. In computing the transition matrices, which show probabilities of movement between hours levels, the labour supply of each individual before the policy change is fixed at the observed discretised value and a number of transitions are produced for each individual, which is equal to the number of successful draws specified.

In some cases, the required number of successful random draws producing observed hours as the optimal hours cannot be generated from the model within a reasonable number of total drawings. The number of random draws tried, like the number of successful draws required, is specified by the user. If after the total draws from the error term distribution, the model fails to predict the observed labour supply, the individual is left at the observed hours in policy simulations. In the simulation example used here, the number of successful random draws required is set to 100 with a maximum of 1000

[9] For those individuals in the data set who are not working, and who therefore do not report a wage rate, an imputed wage is obtained. This imputed wage is based on estimated wage functions, which allow for possible selectivity bias, by first estimating probit equations for labour market participation. However, some individuals are excluded from the database if their imputed wage or their observed wage (obtained by dividing total earnings by the number of hours worked) is unrealistic. The wage functions are reported in Kalb and Scutella (2002) and the preference functions are in Kalb (2002): these are updated versions of results reported in Creedy *et al.* (2002).

tries per draw.

To keep the present exercise manageable, the following comparisons are based on a particular demographic group, rather than the complete database. The results relate to sole parents, of which there are 560 in the survey. Of these, 9 individuals could not be calibrated at their observed (discretised) hours point. A further 61 sole parents had their labour supply fixed in the behavioural simulation. These included 14 disabled, 27 students, 27 self employed, 1 over 65 years old, and 1 with an imputed wage that was considered too low to be reliable.

5 Eliminating the Safety Net

Using Monte Carlo experiments Creedy *et al.* (2003) show that, when examining summary measures of the income distribution in discrete hours labour supply modelling, the method of constructing a pseudo income distribution is superior to using either an expected income approach, or a random sampling approach for a limited number of draws. In this section the performances of the three methods are illustrated by examining the results of a hypothetical policy simulation.

In considering a hypothetical policy reform, it was decided to examine the distributional and labour supply consequences for sole parents of completely eliminating the safety net that the social security system provides. All basic benefits and family related payments are thus removed without changing the income tax system, including rebates. Such a drastic policy change is clearly expected to have substantial effects on labour supply, so that a comparison of behavioural and non-behavioural results is likely to reveal large differences both in terms of government tax and expenditure, and the distribution of income.[10] The revenue and labour supply effects are summarised in subsection 5.1. Subsection 5.2 compares the outcomes for a range of distributional measures, using the alternative methods discussed above. Subsection 5.3 dis-

[10] It is not expected that the effects of such a drastic change could be accurately predicted by a model based on more subtle differences in incomes over the range of labour supply. However, it gives some indication of the effect. The results obtained here can be compared with results obtained from a similar exercise using US data.

cusses the distributional implications of the policy change and compares the results to some U.S. results.

5.1 Revenue and Labour Supply Changes

When examining average hours, the labour supply after the change for each individual is based on the average value over the successful draws, for which the error term leads to the correct predicted hours before the change. This is equivalent to calculating the expected hours of labour supply after the change, conditional on starting from the observed hours before the change. In computing the tax and revenue levels, an expected value is also obtained after the policy change. That is, the tax and revenue for each of the accepted draws are computed for each individual and an average over these is taken.

A summary of the tax and revenue implications is provided in Table 4. Here the column heading 'LS' indicates that labour supply variations have been modelled, while the heading 'Fixed' applies to a non-behavioural simulation in which all individuals have their discretised labour supplies held constant. In the latter case, income taxes fall because some of the eliminated benefits are taxable. With an increase in labour supply, income tax revenue increases. Expenditure on benefits does not change after taking labour supply responses into account as no benefit payments are available, regardless of other income received by the household.

A summary of the behavioural responses is given in Table 5, based on expected hours for each person. This shows a doubling in the preference of sole parents for participation in the labour force. This shift from the no-work corner solution dominates the responses, with only about 12 per cent working longer hours. The very small proportion moving from work to non-work is caused by one person in the data set. This person (a 50 year old woman with one child aged between 5 and 12 years) reported working 11 hours a week, at a low gross wage of just over \$5 per hour.[11] After the change she has a probability of 0.36 of not working, 0.44 of working 10 hours and 0.11 of working 45 hours, while the other discrete hours levels attract

[11] Only five sole parents are found to have such a low wage rate.

Table 4: Tax and Expenditure Changes

	Pre-Reform	Changes			
		LS[a]		Fixed[a]	
	Abs. Value	Abs.	%	Abs.	%
	($m)	($m)		($m)	
Government Revenue					
Income Tax Revenue[b]	1796.9	754.7	42	-313.7	-17.5
Government Expenditure					
Tax Rebates	278.3	-257.5	-92.5	-267.4	-96.1
Basic Benefits	4008.5	-4008.5	-100.0	-4008.5	-100.0
Family Related Benefits	3099.6	-3093.3	-99.8[c]	-3093.3	-99.8
Total Expenditure	7386.4	-7359.3	-99.6	-7369.3	-99.8
Net Expenditure	5589.5	-8114	-145.2	-7055.5	-126.2

Notes: a) LS indicates the columns with behavioural changes and Fixed indicates the columns without behavioural changes.
b) Includes Medicare levy
c) Maternity Allowance and Child Disability Allowance have not been eliminated, impacting on the total change in family related benefits.

lower probabilities. At this low wage level and without any social transfers, it is difficult to earn more than the estimated fixed cost of working, which is around 200 dollars for sole mothers. In total, 423 sole parents would have zero income at zero hours if no benefits were available. Of these sole parents, 172 have a non-zero probability of being at zero hours. When weighting by the probability of being at zero hours, these 172 sole parents, a total of 56 (or 10 per cent) are predicted not to participate at zero hours out of all sole parents.

Further details of the transitions are in Table 6, which shows the transition probabilities for movements, from rows to columns, among the discrete hours levels used. It can be seen that the proportions of workers before and after the reform match those in Table 5, and the majority of movements is to full time work of 35 hours or more. The probability of moving into full time from part-time work is also high for those working 10 and 15 hours.

Table 5: Summary of Behavioural Responses

Behavioural Response	Percentage
Workers (base)	42.44
Workers (reform)	85.14
Non-work to work	42.83
Work to non-work	0.14
Workers working more	11.98
Workers working less	0.00
Average hours change	21.23

Table 6: Transition Proportions: Movements Between Hours Levels

Hours before reform	Hours after reform											
	0	5	10	15	20	25	30	35	40	45	50	Total
0	25.6	0.0	0.0	0.2	0.9	2.0	5.1	10.4	14.1	18.4	23.3	57.6
5	-	36.5	0.1	0.1	2.0	6.4	2.0	5.8	13.7	18.9	14.6	3.6
10	4.3	-	17.6	-	0.5	1.4	5.2	7.3	12.9	23.0	27.9	3.2
15	-	-	-	25.4	1.6	8.9	2.7	6.8	11.1	15.5	28.0	2.8
20	-	-	-	-	54.9	0.1	0.6	2.5	10.0	11.3	20.5	3.9
25	-	-	-	-	-	73.5	0.8	2.3	2.7	7.8	12.9	3.7
30	-	-	-	-	-	-	74.4	2.1	3.4	8.8	11.3	3.3
35	-	-	-	-	-	-	-	89.4	1.6	2.2	6.8	3.2
40	-	-	-	-	-	-	-	-	90.6	1.7	7.7	12.2
45	-	-	-	-	-	-	-	-	-	98.3	1.7	1.0
50	-	-	-	-	-	-	-	-	-	-	100	5.5
Total	14.9	1.3	0.6	0.8	2.7	4.4	5.8	9.7	21.1	14.7	23.9	100

5.2 Comparison of the Alternative Approaches

To examine the effects of this policy change on the distribution of income before and after labour supply changes, several summary measures are presented. First, two commonly used measures of inequality, the Gini coefficient and the Atkinson measure of inequality using an inequality aversion parameter of 0.5 are examined.[12] For the measurement of poverty, three special cases of the Foster *et al.* (1984) family of measures are used, with a fixed poverty line set at $176 per week.[13]

As shown above, evaluation of the distributional implications of reforms is not feasible using a complete enumeration of all possible income distributions. Instead, comparisons are based on the use of a benchmark case involving 100,000 samples.[14] Table 7 shows alternative values. The first row of the table shows the pre-reform values, obtained using discretised hours levels, while the second row is the benchmark case. The last three rows of the table show the differences between the benchmark values and the measures obtained using the alternative strategies, of respectively using expected incomes, the pseudo distribution and just 10 random draws from all conceivable distributions. The results show that the performance of the pseudo distribution method is superior to the other two methods for all measures.

5.3 The Implications of the Reform

The non-behavioural results are summarised in Table 8. These results are based on the actual observed hours, rather than the discretised hours levels used in the behavioural model. For this reason, the 'before reform' values differ slightly from those shown above, which are based on discretised hours.

[12] Some sole parents have zero net incomes in the post-reform situation; these are set equal to 1 for the purpose of computing the Atkinson measure.

[13] This value is half the median income over the entire population, with income defined as income unit income per equivalent adult. The equivalence scales used are those proposed by Whiteford (1985) and use a scaling of 1 for the first adult, 0.52 for second and subsequent adults and 0.32 for each child.

[14] Convergence of the mean of the distribution is shown to be obtained at roughly 50,000 draws in Creedy *et al.* (2003), so at 100,000 draws the calculated measures should be stable.

Table 7: Alternative Summary Measures Resulting From Behavioural Simulation

	G	$A\,(0.5)$	P_0	P_1	P_2	$\overline{X}$	variance
pre-reform	0.2185	0.0411	0.0432	0.0103	0.0029	311.71	22216.35
post reform using different methods							
10^5draws	0.3508	0.1644	0.2400	0.1620	0.1439	264.37	32762.20
Expected	0.3201	0.1253	0.2445	0.1156	0.0948	264.37	29107.13
Pseudo	0.3508	0.1644	0.2400	0.1620	0.1439	264.37	32768.89
10 draws	0.3512	0.1639	0.2470	0.1624	0.1431	264.10	32639.13
Difference from the method using 10^5 (100,000) draws							
Δexpected	-0.0307	-0.0392	0.0045	-0.0464	-0.0491	0.0001	-3655.06
Δpseudo	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	6.70
Δ10 draws	0.0004	-0.0006	0.0070	0.0004	-0.0008	-0.2752	-123.06

As anticipated, all the inequality and poverty measures show substantial increases, particularly the poverty measures.

Table 8: Summary of Non-Behavioural Results

	Before reform	After reform	Change
Inequality			
G	0.2157	0.7138	0.4981
$A(0.5)$	0.0398	0.5280	0.4883
Poverty (poverty line \$176 per week)			
P_0	0.0273	0.6731	0.6458
P_1	0.0043	0.6060	0.6018
P_2	0.0009	0.5783	0.5774

The increase in inequality and poverty when allowing for labour supply changes is substantially lower than is indicated by this non-behavioural simulation exercise. However, it is clear from Table 7 that, despite the large positive impact on labour supply behaviour, there are still large increases in poverty and inequality. Thus, even if sole parents were able to move freely within the labour force, as a group sole parents would still be worse off if the social security system were not to provide a safety net, due to their demographic structure and poor labor market opportunities. In addition to low wage rates, many sole parents may have low skills (as reflected by their educational level) making it difficult for them to find employment even if they were willing to work. This would mean more sole parents may find themselves in poverty as a result of the change than are predicted by the model. Furthermore the measures used here are based on incomes only and do not account for the lower social welfare of sole parents resulting from the expected increase in labour supply, which is substantial at an average of more than 21 hours per week. The distributional changes based on the behavioural outcomes in Table 7 are therefore probably the least negative possible. The actual outcome is likely to lie somewhere in between the static and the behavioural simulation result.

Hoynes (1996) carried out a comparable exercise for the U.S. and found similar results for couples participating in welfare, although the predicted increase in hours worked is lower than here. She found that 'if the AFDC-UP program was eliminated, the increase in labor force participation and hours of work is not sufficient to make up for the loss of welfare benefits' (1996, p. 325).[15] A similar observation was made in the review paper by Moffitt (1992) for sole parents. He reported an average hours of work increase of 1 to 9.8 hours per week, and concluded, 'the labor supply effects, while statistically significant are not large enough to explain the high rates of poverty among female heads; most AFDC women would, apparently, be poor even in the absence of the AFDC program' (1992, p. 56). In other words, the predicted increase in labour supply after abolishing AFDC is not enough to compensate for the loss in welfare income. A possible explanation for the larger simulated effect in Australia compared to the U.S. is Australia's more generous welfare system.

6 Conclusions

This paper has compared alternative approaches to the measurement of inequality and poverty indices in the context of behavioural microsimulation

[15] AFDC-UP stands for Aid to Families with Dependent Children - Unemployed Parent Program, which is a welfare programme in the U.S.

with discrete hours labour supply models. Special consideration is needed because microsimulation modelling using a discrete hours approach does not identify a particular level of hours worked for each individual after a policy change, but generates a probability distribution over the discrete hours levels used. This makes analysis of the distribution of income difficult because, even for a small sample with a modest range of hours points, the range of possible labour supply combinations becomes too large to handle.

The approaches examined include the use of an expected income level for each individual. Alternatively, a simulated approach could be used in which labour supply values are drawn from each individual's hours distribution and summary statistics of the distribution of income are calculated by taking the average over each set of draws. A benchmark value for the measures was created by taking 100,000 draws from the hours distribution. It was found that for so many draws the outcomes of the measures had converged. Finally, the construction of a pseudo income distribution was proposed. This uses the probability of a particular labour supply value occurring (standardised by the population size) to refer to a particular position in the pseudo income distribution.

By examining an extreme reform of the Australian social security system it was shown that the pseudo method clearly performs better than the expected income method or the sampling approach with just 10 draws. The latter has a similar computational burden to the pseudo approach. The approach using 100,000 draws takes much more time to run than the pseudo method, whereas the measures calculated with these two methods are very close.

Another major finding of the paper is that even if sole parents were able to move freely into the labour force to help compensate for a loss of social security benefits, they would remain worse off financially than under the current system where a safety net is in place. U.S. researchers have found similar results for sole parents and couples on welfare (Hoynes, 1996; Moffitt, 1992). These calculations do not allow for a potential lack of demand for the sole parent's skills or for the decrease in social welfare through the increased labour supply necessary to provide an income. Therefore these calculations are likely to provide a picture of the situation for sole parents which is overly optimistic.

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