Public Capital, Congestion and Private Production in Australia*

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Abstract

This paper is an empirical investigation into the impact of public capital on the private sector's economic activity in Australia. In particular, it is assumed that the contribution of public capital to private factor productivity is subject to congestion. New data sets of capital stocks and private output are constructed for the Australian economy. By estimating flexible functional forms of private sector production functions with congestion in public capital services, the paper shows that public capital is productive in private production but is subject to congestion. A one per cent decrease in the public capital output ratio would result in about 0.5 per cent decrease in private output. Empirical evidence also suggests that the restriction of constant returns to scale over private inputs, labour and private capital, and public inputs is valid.

Key words: Public investment; Infrastructure; Congestion; Productivity

JEL classification: E23; E62; O47

1 Introduction

This paper is an empirical investigation into the impact of public capital on the private sector's economic activity. Public infrastructure of an economy, such as railways, water systems, police and fire services, courts, and educational and health care facilities, are all vital components in the engine of production. Spawned by early research by Aschauer (1989b; 1989a), which showed that public capital exerts a strong positive effect on private production and ascribed the United States productivity decline of the 1970s to under-investment in infrastructure, much effort has been devoted to assess the role of public capital in production. The Gramlich (1994) survey of this literature indicates that the issue has not been settled.

Aschauer's work also prompted a number of theoretical papers which view public investment as a potential source of endogenous growth (see Barro, 1990; Jones and Manuell, 1997). Congestion models of productive government services, such as those in Barro and Sala-I-Martin (1992) and Glomm and Ravikumar (1994), are among them. Specifically, public infrastructure is introduced as an external input to firms' production function. It is assumed that the contribution of infrastructure to private factor productivity is subject to congestion. In congestion models, an individual firm's decision to expand own output (with an increase in its own capital or employment) congests the facilities available to other producers. Infrastructure's contribution to private firms is decreasing in the private capital stock, employment or total output. Due to the lack of use fees, this distortion leads to the usual excessive use of public services.

A feature of the empirical literature on public capital is that it is largely focused on the US economy. Recently, however, a growing literature on other countries has began to emerge using a variety of methodologies (for example, Berndt and Hansson, 1992; Lynde and Richmond, 1993; Kavanagh, 1997; Demetriades and Mamuneas, 2000; Everaert and Heylen, 2001). Otto and Voss (1994; 1996; 1998) conduct a series of empirical research on quantifying the role of public services in private production for the Australian economy. Otto and Voss rely mainly on a Cobb-Douglas production

function of the private economy with three factors: labour and the stocks of private and public capital. Otto and Voss (1998) also consider a more flexible representation of production, which is a Cobb-Douglas function of labour and a CES aggregate of private and public capital, permitting a high degree of substitutability between the two capital sectors. Their results provide support of the importance of public capital for private production and constant returns to scale (CRS) across three inputs in the Australian economy.

This paper extends previous studies in three ways. First, the effect of congested public services on private production is assessed empirically. Second, this paper applies more flexible functional forms of the production function, the Translog and CES, in the empirical study. Finally, new data sets are constructed for the Australian economy to examine the role of public capital in private production. In particular, two sets of capital stock data, complied by the ABS and the Treasury respectively, are used in the paper.

The paper adopts a general to specific approach. A more flexible production function, the Translog, is first examined empirically. After testing for certain restrictions, a less flexible production function, a CES production function of three inputs, is set up with a specific role for public capital. The derived labour and investment demand equations are estimated and the parameters of the production function can then be obtained.

The rest of the paper is structured as follows. Section 2 describes public and private capital in Australia. Section 3 sets up a three-factor-Translog production function to model input and output decisions in a cost-minimisation framework. After testing the restrictions of homogeneity in prices and constant returns to scale in the Translog function, Section 4 goes on to specify a CES production function, in which public infrastructure is subject to congestion. The labour and investment demand equations are then derived from the production function. Section 5 presents the estimation results of the CES production function. The last section concludes the paper.

2 Public and private capital in Australia

Taking into account the privatisation of public enterprises in the 1990s, this paper defines Australian private sector as including both private business and public enterprises to internalise such capital movements. Public enterprises are institutions which provide marketed services and are not considered in this paper as in the public sector. The public sector is then defined as the general government sector, which comprises national, state and local governments (see Australian Bureau of Statistics, 2000).

The Australian Bureau of Statistics (ABS) publishes two sets of capital stocks, one in *Australian System of National Accounts* (annual estimates, called ABS data set in this paper) and the other one in *Modellers' Database* (quarterly estimates, called TRYM data thereafter because the data are used in the Treasury Macroeconomic Model).[1] While the two data sets are highly correlated, the ABS measures of capital stocks are much greater than the TRYM measures.

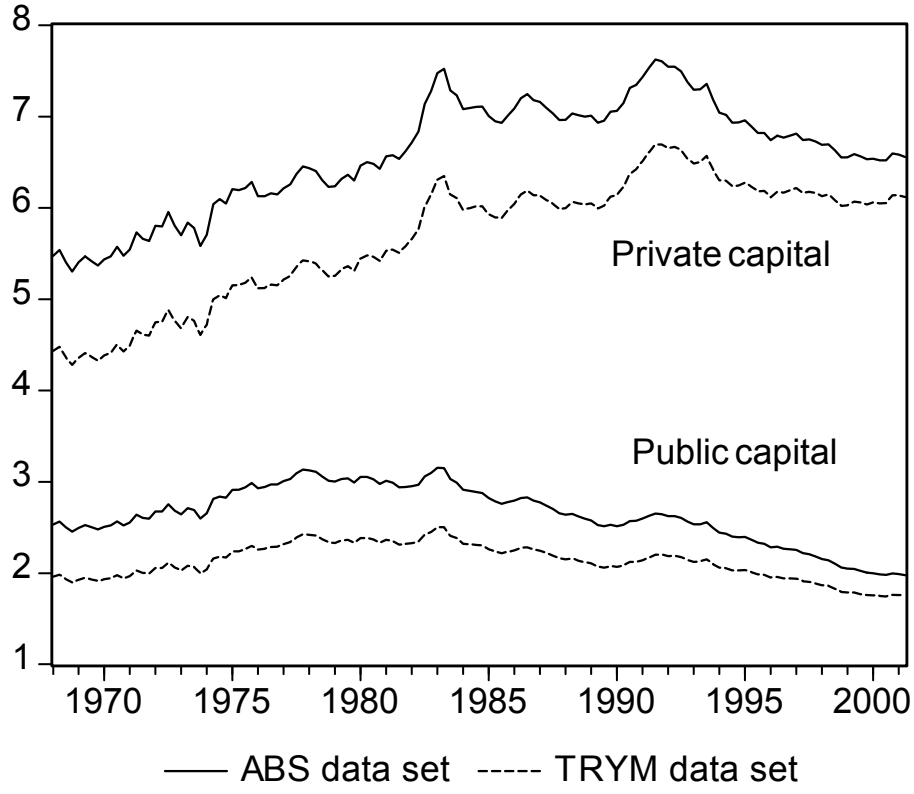
The capital output ratio in the private sector had increased until the early 1990s. In the past ten years, this ratio has decreased from its peak in 1992, though it was stabilised in the past few years. The ratio of public capital with respect to private output, however, has been falling steadily since the early 1980s and the decrease seems to accelerate in the past ten years. Figure 1 plots these ratios calculated from both ABS and TRYM data sets. The output measure used in the graph is total private non-dwelling output, derived from the ABS income-based GDP. It can be seen that the two data sets follow similar patterns though are different in scale. Full details on data sources, the methods of construction and descriptive statistics are provided in an appendix.

One caveat of using the ABS data set is that for each time period prior to the referencing year (1999/2000), the chain volume measures of the capital stock and respective investment and consumption of fixed capital in each sector are "not additive and the capital stock accumulation identity no longer holds".[2] The capital stock measures in the

[1] The author understands that the capital stock data in TRYM are complied by the Treasury.

[2] ABS officials explain that "to keep all year to year growth rates of capital stock volume estimates unchanged when the reference year is changed, each variable is re-referenced separately, causing discrepancies to arise between individual elements and their totals" (private communication). The non-additivity

Figure 1: Capital-output ratios: 1968Q1-2001Q2



TRYM data set are complied so as to avoid such a problem. When implementing chain volume measures, ABS favoured the resultant better measures of economic growth at the cost of loss of additivity. Chaining capital stock was also to comply with the international convention which states that the discrepancies (in additivity) should remain in the published data without adjustment as any adjustment would again distort growth rates. The capital stock estimates in the TRYM data set are not consistent with other variables that are chain volume measures. As far as consistency concerned, the ABS data set is preferred.

3 Translog cost function

3.1 Theoretical model

It is assumed that labour (L) and private capital (K) are privately purchased inputs and public capital (G) are freely provided by the government to firms.[3] The production function is assumed as

$$Y = F(L, K, G, t) \tag{1}$$

where Y is total private output, t represents the level of technology, and F is homogeneous of degree one in L, K and G. Let p_L and p_K be the wage rate and the implicit price of capital services; p is the price of output. Define private costs as $C = p_L L + p_K K$. With competitive markets and cost-minimisation behaviour, the cost function takes the form

$$C = C(p_L, p_K, Y, G, t) \tag{2}$$

By Shepard's Lemma, the conditional factor demand functions for labour and private capital are

$$L^* = \frac{\partial C}{\partial p_L}, \text{ and } K^* = \frac{\partial C}{\partial p_K}$$

problem explains why the capital stock identity no longer holds for chain volume measures.

[3]The method employed in this section is similar to those in Lynde and Richmond (1992) and Vijverberg, Vijverberg, and Gamble (1997). Lynde and Richmond (1992) assume Hicks neutral technology in the production function (equation 1 in their paper). This section shows that the same set of share equations can be derived without the assumption of Hicks neutral technology.

Define the cost share of labour and private capital as

$$s_L = \frac{p_L L^*}{C} = \frac{\partial \ln C}{\partial \ln p_L}, \tag{3a}$$

$$\text{and } s_K = \frac{p_K K^*}{C} = \frac{\partial \ln C}{\partial \ln p_L} \tag{3b}$$

Since competitive output markets imply $p = \partial C/\partial Y$, the "cost share of output" is $s_Y = pY/C = \partial \ln C/\partial \ln p$. It is easily seen that $s_L + s_K = 1$. By the envelope theorem, $\partial C/\partial G = -pF_G$; and by Euler's theorem and the assumption of homogeneous of degree one in L, K, and G, then

$$pY - pF_G G = p_L L + p_K K$$

implying

$$s_Y + s_G = 1 \tag{4}$$

$$\text{where } s_G = \frac{-pF_G G}{C} = \frac{\partial \ln C}{\partial \ln G}$$

The "cost share" of public capital, s_G, can be interpreted as the reduction in total cost by using public capital services. If the cost function C is assumed as a translog function of its arguments, p_L, p_K, Y, G and t, then

$$\begin{aligned}
\ln C = {} & \ln \alpha_0 + \alpha_Y \ln Y + \alpha_G \ln G + \alpha_t t + \sum_{i=L,K} \alpha_i \ln p_i \\
& + \frac{1}{2}\left\{ \sum_{i=L,K} \sum_{j=L,K} \beta_{ij} \ln p_i \ln p_j + \beta_{YY} (\ln Y)^2 + \beta_{GG} (\ln G)^2 + \beta_{tt} t^2 \right\} \\
& + \sum_{i=L,K} \beta_{iY} \ln p_i \ln Y + \sum_{i=L,K} \beta_{iG} \ln p_i \ln G + \sum_{i=L,K} \beta_{it} t \ln p_i \\
& + \beta_{YG} \ln Y \ln G + \beta_{tG} t \ln G + \beta_{tY} t \ln Y
\end{aligned} \tag{5}$$

Using (3) and (4), the share equations of labour, private capital, output and public capital can be derived from this translog function as follows

$$s_i = \alpha_i + \beta_{it}t + \beta_{iG}\ln G + \beta_{iY}\ln Y + \sum_{j=L,K}\beta_{ij}\ln p_j, \qquad i = L, K \tag{6a}$$

$$s_Y = \alpha_Y + \beta_{tY}t + \beta_{YG}\ln G + \beta_{YY}\ln Y + \sum_{j=L,K}\beta_{jY}p_j \tag{6b}$$

$$s_G = \alpha_G + \beta_{tG}t + \beta_{GG}\ln G + \beta_{YG}\ln Y + \sum_{j=L,K}\beta_{jG}p_j \tag{6c}$$

3.2 Empirical model

The data employed here are annual time series over the period 1968-2001 (financial years) from both ABS and TRYM data sets. The shares of labour, private and public capital, and output can be calculated from the data sets. The empirical model therefore comprises four equations analogous to those in (6), with the left-hand variables replaced by the measured shares and stochastic errors added on the right-hand side. Given additive disturbance terms and the restrictions on shares in (3) and (4), these equations fall into two pairs with only one independent equation for each pair. The equations for s_L and s_G are chosen to estimate the parameters of the model

$$s_L = \alpha_L + \beta_{Lt}t + \beta_{LG}\ln G + \beta_{LY}\ln Y + \beta_{LL}\ln p_L + \beta_{LK}\ln p_K + u_L \tag{7a}$$

$$s_G = \alpha_G + \beta_{Gt}t + \beta_{GG}\ln G + \beta_{YG}\ln Y + \beta_{GL}\ln p_L + \beta_{GK}\ln p_K + u_G \tag{7b}$$

where $u = (u_L, u_G)$ is assumed normally distributed with zero mean and covariance matrix Ω. Applicable restrictions are:

(1) homogeneity in prices: $\beta_{LL} + \beta_{LK} = 0$ and $\beta_{GL} + \beta_{GK} = 0$;

(2) homogeneity in L, K and G, giving the linear restriction (4) on s_G and s_Y: $\beta_{LG} + \beta_{LY} = 0$ and $\beta_{GG} + \beta_{YG} = 0$; and

(3) symmetry: $\beta_{LG} = \beta_{GL}$.

Table 1: Unrestricted estimates of the cost function model

Data set	**ABS**		**TRYM**	
equation	s_L	s_G	s_L	s_G
Intercept	-2.61 (0.46)	32.2 (0.99)	-3.65 (2.58)**	26.3 (1.30)
Trend	-0.00 (0.02)	0.02 (0.85)	-0.00 (1.12)	0.04 (1.01)
$\ln p_L$	0.06 (0.58)	0.25 (0.82)	0.08 (2.67)**	-0.07 (0.30)
$\ln p_K$	-0.17 (15.1)**	0.31 (2.56)**	-0.20 (32.6)**	0.43 (15.0)**
$\ln Y$	0.05 (0.37)	-0.78 (1.34)	0.07 (0.67)	-0.68 (1.64)*
$\ln G$	0.19 (0.39)	-2.18 (0.76)	0.27 (2.26)**	-1.74 (1.05)
ρ	0.67 (2.70)**	0.77 (3.62)**	0.29 (1.45)	0.74 (4.71)**
Sample period	1968-2001		1969-2001	
R^2	0.99	0.93	0.99	0.96
D.W.	1.86	1.52	2.14	1.54
Log-likelihood	195.4		204.0	
Wald test on restrictions				
(1)+(2)	4.29		22.4**	
(1)+(2)+(3)	5.27		43**	

Note: Asymptotic t-scores are in parentheses.

ρ: coefficient of first-order autocorrelation in the residuals.

* and **: significant at the 5 per cent and 10 per cent levels, respectively.

$\chi^2_{(4)}(0.05) = 9.49$ and $\chi^2_{(5)}(0.05) = 11.07$.

Since the error terms u may be serial correlated and an endogenous variable (Y) is on the right-hand side of the model, Vijverberg, Vijverberg, and Gamble (1997) show that full information maximum likelihood (FIML) is the preferred estimation method. The validity of the restrictions can be tested by *Wald* statistics, which have a *chi*-squared distribution. It should be noted that there is some evidence that the *chi*-squared test is biased towards rejection of the null hypothesis in small samples (see, for example, the discussion in Judge, Hill, Griffiths, Lutkepohl, and Lee, 1985, p.475).

Table 1 reports the cost function estimates and the restriction test statistics for the two data sets. The results, however, show a high degree of multicollinearity, and for the ABS data set in particular. *Wald* tests on the restrictions are not significant for the ABS data set, but the test statistics strongly reject the constraints for the TRYM data set.

Table 2: Restricted estimates of the cost function model

Data set	**ABS**		**TRYM**	
equation	s_L	s_G	s_L	s_G
Intercept	1.05 (4.98)**	-2.13 (2.89)**	1.57 (8.58)**	-0.29 (0.32)
Trend	-0.01 (4.78)**	0.01 (5.20)**	-0.01 (2.39)**	0.01 (4.00)**
$\ln p_L$	0.17 (17.8)**	-0.25 (4.95)**	0.20 (34.3)**	-0.41 (12.9)**
$\ln p_K$	-0.17 (17.8)**	0.25 (4.95)**	-0.20 (34.3)**	0.41 (12.9)**
$\ln Y$	0.25 (4.95)**	-0.65 (4.08)**	0.41 (12.9)**	-0.43 (2.06)**
$\ln G$	-0.25 (4.95)**	0.65 (4.08)**	-0.41 (12.9)**	0.43 (2.06)**
ρ	0.79 (5.00)**	0.45 (1.87)*	0.88 (7.74)**	0.49 (2.22)**
Sample period	1968-2001		1969-2001	
R^2	0.98	0.89	0.98	0.93
D.W.	1.92	1.56	1.70	1.56
Log-likelihood	188.7		190.0	

Note: Asymptotic t-scores are in parentheses.

ρ: coefficient of first-order autocorrelation in the residuals.

* and **: significant at the 5 per cent and 10 per cent levels, respectively.

Table 2 reports the results of estimating the equations, imposing price homogeneity, constant returns to scale and symmetry. All the coefficients except one become highly significant and the equations appear to fit the data well. The problem of multicollinearity disappears and the estimates for the two data sets become similar. The coefficients on the prices of labour and private capital services have the expected signs and the signs of the other estimated coefficients are sensible. The coefficient on public capital in the public capital share equation (the s_G column in the Table) is positive, implying a diminishing marginal product of public capital. It should be noted that the marginal product of public capital is positive, due to the negative cost shares of public capital in the sample period (see equation 4).

From the definitions of the cost shares of labour and private capital in (3), the elasticities of the demands for labour and private capital with respect to public capital

can be calculated from the estimated coefficients

$$\epsilon_{LG} = \frac{\partial \ln L}{\partial \ln G} = s_G + \frac{\beta_{GL}}{s_L} \tag{8a}$$

$$\epsilon_{KG} = \frac{\partial \ln K}{\partial \ln G} = s_G + \frac{\beta_{GK}}{s_K} \tag{8b}$$

Using the estimates in Table 2 and the mean values of cost shares in the sample period, it is found that ϵ_{LG} and ϵ_{KG} are -0.78 and 0.36 for the ABS data set, and -0.98 and 0.84 for the TRYM data set, respectively.

For the ABS data set, the likelihood ratio test statistic of imposing three restrictions is slightly greater than the critical value of the 5 per cent significant level, but cannot lead to the rejection at the 1 per cent level of significance. Taking account of the bias of the χ^2 test towards rejection of the null in small samples, both the *Wald* and likelihood tests suggest the acceptance of all of these constraints for the ABS data set. The TRYM data set, however, rejects the constraints.

This section estimates two cost share equations of labour and public capital, derived from a translog cost function, for two data sets of the Australian economy. It is found that public capital is productive in private production and its marginal product is diminishing. There is evidence that public capital may be complementary to private capital. The statistical tests indicate that the restriction of constant returns to scale is valid for the cost function, in which public capital is included with private capital and labour. One setback of the cost function approach is that it treats output as an exogenous variable. Subsequently, the next section specifies a more restrictive production function, which models explicitly input demands and output as endogenous variables. Moreover, public capital can be modelled to play a more specific role in private production.

4 CES production function with congestion

4.1 Congestion in public services

As noted by Stiglitz (1988), a substantial part of public capital such as roads and highways, airports, harbours, etc., are clearly not pure public goods and are subject to congestion. Individuals can be excluded at some cost from using a public facility. To allow for this possibility, this section assumes that the public capital stock is non-exclusive and is a complementary input in private production. As in the previous section, there are three factors of production in the economy: private capital, labour and public capital. Each firm produces output y_t at time t according to the technology

$$y_t = f(k_{t-1}, l_t, \hat{G}_{t-1}) \tag{9}$$

where k is the amount of capital rented by the firm, l is the amount of labour hired, and $\hat{G}$ is aggregate stock of public capital (subject to congestion) available to all firms at time t. Public capital is a common external input to each firms's production process and its quantity is determined by the government. As in Barro and Sala-I-Martin (1992) and Glomm and Ravikumar (1994), congestion is modelled in terms of output as follows:

$$\hat{G}_{t-1} = \frac{G_{t-1}}{Y_t} \tag{10}$$

where Y is aggregate output. Because of congestion, an increase in Y for given G lowers public capital available for each firm and therefore reduces total output. The formulation assumes that G has to rise in relation to total output in order to expand the public capital stock available to each user. The production function f is assumed constant returns to scale between private labour and capital but allow increasing returns to scale across all three inputs for each firm. In the aggregate level, however, the technology exhibits constant returns to all three factors, which rules out the possibility of public capital as a pure public good. Glomm and Ravikumar (1994) prove that under such a specification (with decreasing returns to private and public capital factors), the economy

will converge to a steady state.

4.2 CES technology

The CES production function with congestion is defined as following for individual firms:

$$y_t = \left(\frac{G_{t-1}}{Y_t}\right)^{\theta} (\alpha k_{t-1}^{\rho} + \beta l_t^{\rho})^{1/\rho} \tag{11}$$

where the elasticity of substitution between labour and private capital is $\sigma = 1/(1-\rho)$ and θ is positive if public capital services are productive in private production. It is clear that an increase in G relative to total output boosts the productivities of labour and private capital at the firm level. Given the congestion factor, the technology exhibits constant returns to scale between private labour and capital. Aggregating identical firms gives the aggregate production function

$$Y_t = \left(\frac{G_{t-1}}{Y_t}\right)^{\theta} (\alpha K_{t-1}^{\rho} + \beta L_t^{\rho})^{1/\rho} \tag{12a}$$

$$\text{or} \qquad Y_t = G_{t-1}^{\theta/(1+\theta)} (\alpha K_{t-1}^{\rho} + \beta L_t^{\rho})^{\frac{1}{\rho(1+\theta)}} \tag{12b}$$

which is constant returns to scale across three factors, private and public. It is also assumed that the technology is Harold neutral and $L_t = H_t e^{\lambda t}$ where H is the total hours worked and λ the rate of technological progress.

Under the case of constant returns to scale across three factors, if private factors are paid according to their marginal products, private output will not be fully distributed. It is necessary to make some assumption about the manner in which the rents from public services are appropriated by the private factors of production. It is simply asserted in this paper that private factor shares are proportionally related to their respective true marginal productivities. Therefore, given the congestion factor $\hat{G}$, the wage rate and the rental rate of private capital when firms maximise profits are following (all time

subscripts are dropped for simplicity):

$$\frac{p_L}{p} = \alpha \left(\frac{G}{Y}\right)^{\theta\rho} \left(\frac{L}{Y}\right)^{\rho-1} \tag{13a}$$

$$\frac{p_K}{p} = RR = \beta \left(\frac{G}{Y}\right)^{\theta\rho} \left(\frac{K}{Y}\right)^{\rho-1} \tag{13b}$$

where p_L, p_K and p are the prices of labour services, capital services and output, respectively. RR is the real rate of return of private capital services. Subsequently, labour demand in equilibrium can be derived as

$$L = Y \left[\alpha \left(\frac{G}{Y}\right)^{\theta\rho} \left(\frac{p_L}{p}\right)^{-1}\right]^{1/(1-\rho)} \tag{14a}$$

$$\ln L = \ln Y + \frac{1}{1-\rho}\left[\alpha + \theta\rho \ln\left(\frac{G}{Y}\right) - \ln\left(\frac{p_L}{p}\right)\right] \tag{14b}$$

Labour demand in the short run is specified in an error correction form that incorporates both short run dynamics and long run responses. Population growth, GDP growth and real wage growth are expected to affect labour demand in the short run. In addition, a term of changes in public investment I_g relative to the last-period public capital stock is added into the specification

$$\begin{aligned}\Delta \ln H_t = {} & n_t + \frac{b_1}{1-\rho}\left[\Delta \ln\left(\frac{p_{L,t}}{p_t}\right) - \lambda\right] + b_2\left[\Delta \ln Y_t - \lambda\right] + b_3 \Delta \ln\left(\frac{I_{g,t}}{G_{t-1}}\right) \\ & + b_0 \left\{ \begin{array}{c} \ln H_{t-1} + \lambda(t-1) - \ln Y_{t-1} \\ -\frac{1}{1-\rho}\left(\alpha + \theta\rho \ln\left(\frac{G_{t-2}}{Y_{t-1}}\right) - \ln\left(\frac{p_{L,t-1}}{p_{t-1}}\right) - \lambda(t-1)\right) \end{array} \right\} + \upsilon_{1,t}\end{aligned} \tag{15}$$

The implicit price of private capital services, or the user cost of capital, comprises real interest costs, depreciation and an adjustment of risk, and can be defined as (in the absence of risk)[4]

$$p_K = p_I(r + \delta - \pi) \tag{16}$$

[4]The following estimations assume the risk parameter of 0.5 per cent per year. It is found that different values of the risk parameter only affect the results marginally. The average yield spreads of AA corporate bonds over Commonwealth government bonds with equivalent maturity are about 0.5 per cent per year (see F03hist.xls in www.rba.gov.au).

where p_I is the price of capital goods, r the nominal interest rate, δ the depreciation rate and π the inflation rate (see for example Jorgenson, 1963). Therefore a marginal Tobin's Q can be defined as

$$Q = \frac{p}{p_K} RR \tag{17}$$

and in equilibrium Q should be equal to unity when the optimal condition (13b) is met. In the short run, however, it can vary, affecting investment demand by firms.

If assuming the economy will ultimately converge to a steady state, then at the steady state the ratio of investment to capital is constant and equals $(n + \lambda + \delta)$, where n is population growth.[5] In the short run, however, the demand for investment is affected by capacity utilisation (actual output relative to production capability), the difference between the rate of return and user cost of private capital. The higher capital utilisation and the rate of return of private capital are, the higher the ratio of investment to the last-period capital stock (I_t/K_{t-1}). Allowing for partial adjustment towards equilibrium, the following equation specifies the demand for investment in the short run

$$\begin{aligned} \frac{I_t}{K_{t-1}} &= \left(1 - \sum_{i=1}^{3} a_i\right)(n_t + \lambda + \delta_t) + \sum_{i=1}^{3} a_i \frac{I_{t-i}}{K_{t-i-1}} + a_4 \Delta \ln\left(\frac{I_{g,t}}{G_{t-1}}\right) \\ &\quad + a_5 \frac{\ln Y_t}{\ln Y_t^*} + a_0(Q_t - 1) + \upsilon_{2,t} \end{aligned} \tag{18}$$

where Y^* is the production capability determined by the production function, given the current levels of private capital, employment and public capital.

As in the last section, each row of the residual $\upsilon = \{\upsilon_1, \upsilon_2\}$ is assumed to be multivariate normally distributed, with a zero mean and a covariance matrix Ω. Since labour demand (15) and investment demand (18) are simultaneously determined by firms, the FIML estimators are efficient. The quarterly data series from the ABS and TRYM data sets are used in estimation.[6] The construction of the quarterly data series

[5] If certain conditions are satisfied, an economy with a neoclassical production function, such as the one presented here, will exhibit a unique steady state, in which per capita quantities will be growing at the rate of technological progress. See Barro and Sala-I-Martin (1995, Ch.1). The capital accumulation identity immediately gives $i_t/k_{t-1} = n + \lambda + \delta$.

[6] The specifications of the investment and labour demand equations are similar to those in TRYM. Unlike TRYM, however, the output price equation is not jointly estimated with the investment and

of capital stocks is described in the appendix.

5 Empirical results of the congestion function

The data used to estimate the CES production function are quarterly time series over the period 1976Q1-2001Q2.[7] The FIML estimates of the labour and investment demand equations for the ABS and TRYM data sets, respectively, are reported in Table 3.

The parameter estimates of the production function for the two data sets are sensible and are similar to each other. The annual labour-augmenting rate of technological progress is about 2 per cent for the TRYM data set and 2.7 per cent for the ABS data set. The elasticity of substitution between labour and private capital is about 0.6 for both data sets. At the 10 per cent level, the estimate of the parameter on the congestion factor (θ) from the ABS data set is significant, while the one from the TRYM data set is close to the significance level. The estimates of θ indicate that a 1 per cent decrease in the public capital output ratio would lead to a decrease in private output by about 0.4 to 0.6 per cent, *ceteris paribus*. The implied elasticities of output with respect to public capital ($\frac{\theta}{1+\theta}$) are positive and are significant at the 1 and 5 per cent levels for both data sets, respectively. The size of this elasticity is a little different between the data sets: about 0.27 for the TRYM data and 0.38 for the ABS data.

With different data sets and production function specifications, the estimates of the output elasticity of public capital presented here appear a little higher than the findings in the literature. In their first study, Otto and Voss (1994) report an estimate of 0.4 for the output elasticity of public capital, though they prefer a specification of increasing returns to scale across all inputs with constant returns to scale over private

labour demand equations. Under the assumption of perfect competition, firms choose two private inputs to maximise profits. The system may be over-identified if the output price is also determined by firms. The joint estimation of the labour and investment equations with the output price equation, however, does not affect the final results much.

[7] Estimations were also conducted for the sample period 1966Q1-2001Q2. It was found that the estimates from the TRYM data set are similar to those reported here though θ is smaller (0.15) while the estimates from the ABS data set are not sensible as λ is negative and σ is greater than one. The reason for the differences is that the price of capital goods p_I and subsequently the implicit price of capital services p_K is very volatile in the period 1973-1975 and adversely affects the results from the ABS data set. Therefore the sample period 1976Q1-2001Q2 is chosen in the paper.

Table 3: Estimates of CES with congestion

Data set	**ABS**		**TRYM**	
equation	Investment	Labour	Investment	Labour
a_0	0.12×10^{-3} (1.04)		0.43×10^{-3} (0.26)	
a_1	0.84 (8.08)**		0.89 (8.30)**	
a_2	0.13 (1.06)		0.26 (2.06)**	
a_3	-0.12 (1.11)		-0.31 (2.69)**	
a_4	0.68×10^{-3} (0.49)		-0.21×10^{-3} (0.17)	
a_5	0.01 (2.54)**		0.007 (1.50)	
b_0		0.19 (6.94)**		0.20 (7.19)**
b_1		0.19 (2.47)**		0.20 (2.49)**
b_2		0.20 (3.18)**		0.22 (3.58)**
b_3		0.01 (1.34)		0.007 (1.12)
Estimates of the CES production function parameters				
λ	0.007 (3.38)**		0.005 (5.04)**	
θ	0.61 (1.70)*		0.38 (1.61)	
α	2.25 (1.65)*		2.81 (1.50)	
β	0.11 (1.19)		0.06 (1.08)	
σ	0.61 (5.48)**		0.56 (5.34)**	
Sample period	1976Q1-2001Q2			
R^2	0.92	0.52	0.90	0.51
D.W.	1.84	1.67	1.97	1.67
Log-likelihood	976.4		968.2	

Note: Asymptotic t-scores are in parentheses.

The risk parameter in p_K is assumed 0.5 per cent per year.

* and **: significant at the 5 per cent and 10 per cent levels, respectively.

inputs. By applying quarterly data and cointegration analysis, Otto and Voss (1996) report a 0.17 public capital output elasticity in a Cobb-Douglas function with constant returns to scale across three inputs. Different sample periods may explain the difference. The sample period employed in this paper is 1976Q1-2001Q2 while Otto and Voss's is from 1959Q3 to 1992Q2. The higher estimates from more recent data indicate a higher marginal product of public capital, and thus may suggest that public capital is not provided in the recent period as sufficiently as in early periods. In a study of 12 OECD countries, Demetriades and Mamuneas (2000) find that the output supply elasticity of public infrastructure capital was about 1.8 for Australia in the 1972-91 period, which is much bigger than the above estimates.

The coefficients (b_0) on the error correction term in the labour demand equation is highly significant and have a correct sign, which would force labour demand back towards its long-run growth path as determined in (14). The estimates of b_0, which provide information on the speed of adjustment, indicate that about 20 per cent of the disequilibrium in labour demand is corrected in a quarter. While it is productive in the long run, it appears that government investment has no effects on labour and investment demand in the short run as the coefficients, a_4 and b_3, are not significant. There is evidence that higher capacity utilisation increases investment demand in the short run (a_5 is significant at the 1 per cent level for the ABS data set and the t-statistic for the TRYM estimate is close to the 10 per cent significance level). The coefficients associated with the Tobin's Q term, however, is not significant for both data sets.

Given the actual levels of employment and capital stocks, production capability, Y^*, can be calculated from the estimated parameters. It is found that for the TRYM data set, actual output is on average about 2 per cent above Y^*, while for the ABS data set actual output has almost reached the theoretical values of the production function in the late 1990s though it is about 8 per cent below the capabilities in the sample period. This may suggest that the ABS data set is preferred as chain volume measures give better indicators of economic growth of productive assets.

Some caveats should be noted. The values of θ imply that the real rate of return of

public capital (i.e., the marginal production of public capital) is about 50 per cent per year. While this is lower than early estimates of the effect of public capital, it is still unreasonably high. The implied Tobin's Q ratio is always well above unity for both data sets for a risk parameter of 0.5 per cent per year as assumed in the estimations. Higher values of the risk parameter up to 4 per cent per year are not be able to lower the Q ratio to close to unity. Moreover, the definition of Tobin's Q in (17) does not take account of tax rules concerning corporate tax rates, investment tax credits and depreciation formulas, which may affect the estimation results of the investment demand equation (see Hayashi, 1982).

6 Conclusion

The purpose of this paper is to extend previous studies on the role of public capital in Australian private production. Two less restrictive approaches were applied and newly constructed data sets were used in the empirical study. In particular, the contribution of public capital services to private factor productivity is modelled to be subject to congestion. The analysis presented here provides further evidence of the nature of private production and the productive role of public capital services.

The estimation results of the flexible Translog cost function of labour and the stocks of private and public capital point out the validity of the restrictions of price homogeneity and constant returns to scale over three inputs. Moreover, the services of public capital appear to complementary to the services of private capital. Imposing the restriction of constant returns to scale over three inputs, a CES production function with congested public capital in terms of total output was postulated and the labour and investment demand equations were derived. The estimates of the production function show a productive role of public capital in private production and the congestion factor is around 0.5, implying a 1 per cent decrease in the public capital output ratio would result in about 0.5 per cent decrease in private output.

The partial equilibrium approach as considered here, however, is not able to indicate

whether the public policy concerning public capital is optimal and the stock of public capital is sufficient. This must be dealt with in a general equilibrium framework. Public capital subjected to congestion could be incorporated into a general equilibrium macroeconometric model, such as TRYM, to assess the level of public investment and its impact on economic growth. A crucial hurdle to do so is a sensible data set of the stocks of private and public capital. As shown in the paper, the ABS chain volume measures of capital stocks are the preferred measures of economic growth but at the cost of lose of additivity. To conduct empirical studies about public investment in a general equilibrium context, the capital stock data satisfying the capital accumulation identities are a must.

Data appendix

This appendix details the methods of constructing the data series used in the paper. The difference between the ABS and TRYM data sets is on capital and investment.

The current dollar value of total private non-dwelling output is derived from the Australian Bureau of Statistics income-based measure of GDP and is the sum of following components: private sector employee compensation, gross operating surplus of non-financial corporations (including both private and public) and financial corporations, taxes less subsidies on production and imports, and statistical discrepancies from ABS National Accounts. The public sector employee compensation is calculated from the number of employees in the general government sector and the defence force multiplying by their average weekly earnings, and these series are obtained from the ABS Modellers' database. The resultant private output series is then converted to constant 1999/00 prices by using the implicit price deflator for the ABS-expenditure-based measure of GDP.

The measure of aggregate labour input is total hours worked in the private sector. The quarterly total employment series (excluding defence personnel) is available from the ABS Modellers' database and private employment is obtained by subtracting employment in the general government sector. The estimate of average hours worked is also obtained from the Modellers' database. The wage rate used in the paper is calculated as private sector compensation of employees divided by private employment.

The following data series are different for the ABS and TRYM data sets.

ABS data set

Capital stock measures are only available on a yearly basis from ABS National Accounts. The private sector capital stock is the sum of the capital stocks of the financial and non-financial corporations and households, adjusted by the stock of dwellings. The capital stock of the general government sector is taken as public capital. The investment series is the chain volume measures of gross fixed capital formation for a certain sector.

The price index of capital goods (p_I) is the ratio of the current to constant value (chain volume measures and the reference period is the 1999/2000 financial year) of the net stock of fixed nonresidential private capital. The price index of private capital services is defined as

$$p_K = p_I(r + \delta) - \Delta p_I \tag{19}$$

where r is the nominal interest rate and δ is the depreciation rate (see Jorgenson, 1963). The nominal interest rate is the yield rate of 10-year Commonwealth bonds plus a 50 basis point adjustment for the private sector. The depreciation rate is the current dollar value of the consumption of fixed capital divided by the current dollar value of the net stock of private capital.

The conversion of the annual series to quarterly makes use of the quarterly series of private non-dwellings investment and public investment, and assumes that changes in investment will affect the changes in the capital stock proportionally. More specifically, the ratio of quarterly investment to the financial-year aggregate of investment forms a quarterly series, which can then be applied to the net annual increase in the capital stock to yield the quarterly series of the capital stock. This procedure is applied to both private and public capital.

TRYM data set

The quarterly data series of the capital stocks and their investment series are obtained from the Modellers' database. Aggregating quarterly figures for each financial year yields annual data series. The price index of investment is the price index of business investment. The implicit price of capital services used in the estimation of the Translog cost function is calculated by (19), where the depreciation rate and the price of investment are from the Modellers' database.

Sources of data

1. Australian National Accounts

(a) *Australian National Accounts: National Income, Expenditure and Product*: ABS Catalog No.5206.0

(b) *Australian System of National Accounts*: ABS Catalog No.5204.0

2. Modellers' Database: ABS Catalog No.1364.0.15.003

3. Web site: `www.abs.gov.au/ausstats/`

Descriptive statistics

The following table reports descriptive statistics of the data sets. The annual data are used in the estimation of the Translog cost function while the quarterly data are used in the estimation of the CES production function with congestion. See the text for notations in the table.

Table 4: Descriptive Statistics

	ABS set		TRYM set	
	Mean	Std. Div.	Mean	Std. Div.
Annual data (1968-2001)				
s_L	0.683	0.073	0.660	0.074
s_G	-0.413	0.120	-0.367	0.135
$\ln Y$	7.992	0.297	7.992	0.297
$\ln G$	12.189	0.221	11.976	0.263
$\ln p_K$	-2.777	0.874	-2.519	0.807
$\ln p_L$	2.450	0.841	2.450	0.841
Quarterly data (1976Q1-2001Q2)				
$\ln Y$	6.722	0.240	6.722	0.240
$\ln H$	8.719	0.143	8.719	0.143
$\ln G$	12.292	0.106	12.094	0.148
$\ln K$	13.259	0.256	13.123	0.289
I_t/K_{t-1}	0.022	0.003	0.026	0.003
$\ln(p_L/p)$	-2.730	0.084	-2.730	0.084
$\ln(p_K/p)$	-7.856	0.171	-7.906	0.125
$\ln(I_{g-1}/G_{t-1})$	-4.433	0.144	-4.235	0.129

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