Labour Market Analysis with VAR Models[*]

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Abstract

This paper provides an overview of how labour market analysis can be conducted in the context of VAR-based models. The examples presented here show that these methods are quite flexible, and capable of addressing a wide variety of theoretical and policy-related issues. Specifically, the various techniques are illustrated in models which: examine the dynamics of gross job flows; assess the relationship between real wages and unemployment; quantify the contribution of sectoral shocks to the number of people unemployed by duration of unemployment; examine the relative contributions of discouraged worker effects, insider effects, etc. on the persistence of unemployment; and analyse the effects of labour market shocks in the OECD countries.

Introduction

This paper describes the use of vector autoregression (VAR) models in the analysis of labour market issues. VAR models have been used extensively in the twenty years since the pioneering work of Sims (1980), and have become standard tools in empirical macroeconomics.

The plan of this paper is as follows. The next section lays out the general form of a VAR model and discusses the basic analytical techniques, such as impulse responses and forecast error variance decomposition (FEVD). In its most basic (i.e., unrestricted) form, a VAR is essentially a multivariate regression model. Used in this way, a VAR provides a convenient way to summarise the second-moment properties (i.e., means and covariances) of a group of data series. In general, the parameter estimates, impulse responses, FEVDs and other quantities obtained from unrestricted VARs have little or no economic meaning. Section 2 introduces structural VAR models, which were developed in order to introduce some degree of economic content. This is done by *identifying* the model through the imposition of restrictions derived from economic theory. Vector error correction models (VECMs), discussed in section 3, are in one sense a subset of structural VARs, in that the restrictions imposed relate to long-run 'equilibria' based on the presence of cointegration among the model's variables. In this class of models, the short-run dynamic properties can be analysed separately from long-run trend behaviour.

Specific applications relevant to the modelling of Australian unemployment are included in each section. The review contained herein is meant to be illustrative rather than exhaustive. In particular, the literature relating to each of the first three subsections is extensive, to say the least. I also do not provide any more technical detail than absolutely necessary. In addition to Sims (1980), interested readers may wish to consult Hamilton (1994), Enders (1994), or similar texts.

1. Vector Autoregressions

Suppose the *n*-variable vector time series y_t evolves according to the following model:

$$y_t = B(L)y_{t-1} + u_t, \qquad (1)$$

where $B(L)y_{t-1} = B_1 y_{t-1} + B_2 y_{t-2} + \cdots + B_p y_{t-p}$, and u_t is a Normally distributed random error vector with zero mean and covariance matrix Σ. Equation (1) is referred to as a pth-order VAR, or VAR(p), since it includes p lags of each of its variables. In general, deterministic terms such as constants, time trends, or seasonal dummy variables could also be included in (1); they are suppressed for expositional purposes.

If we assume that the VAR is stationary (i.e., that the roots of the equation $|I - B(1)| = 0$ are all less than one in absolute value), then we can write the *moving average representation* of (1) as

$$y_t = D(L)u_t, \quad (2)$$

with $D(L) = (I - B(L))^{-1}$. In (2), y_t is written as a linear combination of the errors, u_t. The moving average representation forms the basis for computing impulse responses and forecast error variance decompositions.

Reduced-form VARs are a convenient way to summarise the second-moment properties (i.e., means, variances and covariances) of a group of data series. One of their most common uses in this regard is testing for so-called *causal orderings* or *Granger causality*. This notion of 'causality' is perhaps more accurately described as 'one-step-ahead predictability.' Suppose that equation (1) has just 2 variables, and that the past values of the first variable do not affect the current value of the second. In that case, we can write (1) as

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{bmatrix} B_{11}(L) & B_{12}(L) \\ 0 & B_{22}(L) \end{bmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}. \quad (3)$$

In this case, we say that y_2 (Granger) causes y_1, but y_1 does not cause y_2; equivalently, y_2 is causally prior to y_1. Tests for the presence of causality are basically likelihood ratio tests of the joint hypothesis that the coefficients in $B_{21}(L)$ are all equal to zero. Notice that if y_1 and y_2 in (3) are multivariate, then the first block of equations can be interpreted as a VAR with exogenous variables. Thus the basic formulation (1) is quite general.

Shan, Morris and Sun (1999) use Granger causality tests to conclude that immigration does not cause unemployment in either Australia or New Zealand. Rather, these authors find that the main causal influences (i.e., predictive factors) of unemployment are changes in industrial structure and other macroeconomic factors.

Impulse response functions (IRFs) and forecast error variance decompositions (FEVDs) are also standard tools for summarising second-moment properties. Both are based on the moving average representation (2), and show how the predicted values of the model's variables at various horizons are influenced by the error terms. They allow one to answer questions such as "if the first variable in the VAR is unexpectedly high in period t, how will this affect the other variables between periods t and $t+k$?" Or, "how important have prediction errors in variable 3 been in explaining the variance in the forecast errors of variable 2?" At this point, these questions may not seem particularly interesting. If we could interpret the errors u_t in terms of economic theory, however, then such questions become central to both academic research and policy debates. Also, it is common to use reduced-form VAR estimates as a baseline or point of departure, in studies that have the primary goal of estimating structural relationships.

To see how these techniques work, consider the moving average representation (2). Write the covariance matrix of the error terms as $\Sigma = PP'$, where P is the lower triangular Choleski factor of Σ. The diagonal elements of the matrix P are the standard deviations of the original residuals. The Choleski factor can be thought of as a type of matrix square root. Now instead of working with the original reduced form errors, u_t, define the *orthogonalised* errors $v_t = P^{-1}u_t$. Because of the structure of P, the orthogonalised errors are uncorrelated with each other.[1] It is important to stress that this orthogonalisation is not unique; in particular, a different ordering of the variables in y_t will produce different impulse responses and variance decompositions. Write the estimated coefficients of D(L) as $\hat{D}(L) = \hat{D}_0 + \hat{D}_1 L + \cdots + \hat{D}_s L^s + \cdots \hat{D}_m L^m$. Then the

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[1] In principle, impulse response functions and variance decompositions could be calculated without orthogonalising the covariance matrix Σ. However, because the original errors, u_t, are correlated, the interpretation of these non-orthogonalised quantities is problematic. In practice, even 'non-structural' VAR analyses report orthogonalised impulse responses.

response of y_{t+s} to an innovation (shock) of one standard deviation in variable j at time t is given by $\hat{D}_s p_j$, where p_j is the jth column of P. That is, the ith row of $\hat{D}_s p_j$ shows how the ith variable in y reacts to an unexpected change in the jth variable. The (orthogonalised) impulse response function (IRF) is a plot of these responses as a function of s. The variance in the s-step ahead forecast error in y_t which is due to variable j is given by

$$p_j p_j' + \hat{D}_1 p_j p_j' \hat{D}_1 + \hat{D}_2 p_j p_j' \hat{D}_2 + \cdots + \hat{D}_{s-1} p_j p_j' \hat{D}_{s-1}.$$

The FEVD is then just this variance component divided by the total forecast error variance of y_t. Further details on the estimation of IRFs and FEVDs can be found in the books by Hamilton or Enders, cited above.

For example, Elias (1998), using an FEVD analysis, finds that sectoral shocks are important determinants of unemployment rate fluctuations in the United States. The idea of converting reduced form errors (regression residuals) into structural innovations (labour demand shocks, real wage shocks, etc.) is the goal of structural VAR analysis, discussed below.

1.1. A VAR-Based Labour Market Analysis

Loungani and Trehan (1997) use a stock market price dispersion index to measure sectoral shocks. It has been suggested that stock market dispersions provide an early signal of shocks that affect sectors differently . Further, dispersion indices put more weight on shocks that investors expect to be permanent, such as structural changes, rather than temporary, such as cyclical shocks. The formula for calculating this index is given as

$$I_t = \left[\sum_{i=1}^{n} W_i \left(R_{it} - R_t \right)^2 \right]^{1/2},$$

where R_{it} is the growth rate of industry i's stock price index, R_t is the growth rate of the aggregate stock market index (the S&P 500 composite index in Loungani and Trehan's analysis), and W_i is the industry's share in total employment.

In addition to the dispersion index, four other variables are included to form a basic five equation VAR model: the unemployment rate; the growth rate of real output; the Federal Funds rate; and the growth rate of the S&P 500 index. The last two variables are included to control for monetary policy effects and business cycle effects, respectively.

The main estimation and variance decomposition results for the basic VAR model are summarised as follows.

- The dispersion index is important in explaining unemployment even after controlling for non-sectoral effects. It is not, however, important in predicting real output.

- A dispersion index shock increases the unemployment rate continuously, for a period of about four to five quarters after the shock to about two years, before a gradual decline. A shock to the federal funds rate yields a similar response.

- Real output also responds negatively (with a lag) to a dispersion index shock but the response is relatively short-lived. The same can be said with regard to a shock to the federal funds rate.

- The dispersion index accounts for 31% of the variance in unemployment three years after the shock. The federal funds rate accounts for about 40%.

- Both the dispersion index and federal funds rate account for around 15% to 18% of the variance in real output two years after the shock.

It is possible that sectoral shocks, representing long-term labour reallocations, may be more important in explaining long-term unemployment than short-term unemployment. To examine this issue, Loungani and Trehan repeated their analysis using the long-duration unemployment rate instead of the aggregate unemployment rate in their basic VAR model. The main estimation and variance decomposition results are summarised as follows:

- Lagged values of the dispersion index play a very significant role in the determination of long-duration unemployment.

- Lags of long-duration unemployment do not influence the level of dispersion.

- The dispersion index accounts for a very high proportion of unemployment variation at the longer horizons; for example, about 45% at the 20-quarter horizon.

The authors also consider four other groups of duration of unemployment: (1) up to 5 weeks; (2) 5 to 14 weeks; (3) 14 to 26 weeks; and (4) more than 26 weeks. The main results from the forecast error variance decomposition are:

- Beyond the first two years, the contribution of sectoral shifts to unemployment fluctuations rises fairly steadily with duration; for example, the fraction of unemployment variance accounted for by shocks to the dispersion index ranges from 9% for the shortest duration to 43% for the longest duration, at the 20-quarter horizon.

- The contribution of innovations to the Federal Funds rate declines as the duration increases; for example, the rate accounts from 60% for the shortest duration to 28% for the longest duration at the 20-quarter horizon.

- The forecast error variance shares of the other variables in the VAR model do not change dramatically as the duration of unemployment changes.

2. Structural VARs (SVARs)

Suppose now that we postulate the following *structural* VAR (SVAR) for y_t:

$$A(0)y_t = A(L)y_{t-1} + \varepsilon_t. \quad (4)$$

In equation (4), the vector ε_t denotes the *structural* shocks impinging on the economy. These could include aggregate supply shocks, labour demand shocks, monetary policy shocks, terms of trade shocks, etc. These shocks are assumed to be uncorrelated both with each other and over time, so that their covariance matrix is diagonal. If $A(0)$ is invertible (which I assume throughout), then the structural and reduced form VAR representations are related in the following way:

$B(L) = A(0)^{-1}A(L); E(u_t u_t') = \Sigma = \left(A(0)^{-1}\right)\Omega\left(A(0)^{-1}\right)'$, where Ω is the covariance matrix of the structural shocks (in much applied work, Ω is taken to be the identity matrix). Therefore, given a particular identification scheme (i.e., an estimate of $A(0)$), the parameters of the structural model (including estimates of the historical values of the various shocks) can be recovered from the reduced form estimates. We can also write the structural moving average representation analogous to (2):

$$y_t = \left(A(0) - A(L)\right)^{-1}\varepsilon_t \equiv C(L)\varepsilon_t. \quad (5)$$

Tools such as IRFs and FEVDs can then be used on the structural model to address issues such as the relative importance of labour supply shocks and monetary policy shocks in explaining fluctuations in unemployment.

The matrix $A(0)$ contains the contemporaneous identifying restrictions. These restrictions can take various forms. A particularly simple identification scheme, discussed earlier in the context of IRFs, assumes that $A(0)$ is a triangular matrix. This implies that shocks to the first variable in the vector y_t can influence the current-period values of all the other variables, shocks to the second can influence all but the first, and so on. In this case, the matrix $A(0)$ can be estimated by the lower-triangular Choleski decomposition of the reduced form error covariance matrix, Σ. To take a simple example, suppose that y_t consists of GDP and a measure of labour supply, say aggregate hours worked h_t. Ordering y_t as $(h_t, GDP_t)'$ would imply that innovations to hours have an immediate influence on current-period output, but that shocks to GDP can affect hours worked only with a lag. This scheme might be attractive if one thought that aggregate employment was a lagging indicator of the business cycle. Notice that this type of ordering can also be deduced on the basis of the Granger causality tests discussed previously. In the present example, the ordering implies that hours are causally prior to GDP. Although computationally convenient, it may be difficult to justify a purely recursive (i.e., triangular) identification scheme, particularly in large systems.

Restrictions on $A(0)$ could also be derived from a particular theoretical model. Alternatively, they could reflect widely held beliefs about the functioning of the economy, beliefs which are consistent with a wide range of theoretical models. For example, monetary policy shocks could be identified in part by the restriction that contemporaneous values of output and prices do not enter the policy reaction function. The argument is informational; current-period data on these variables are typically not available to the monetary authority.

There are other means of identifying the structural shocks. In one of the earliest and most widely cited SVAR papers, Blanchard and Quah (1989) impose the restriction that aggregate demand shocks have no long-run effect on output. This sort of identification involves restricting the long-run impact matrix $C(1) = C_0 + C_1 + \cdots$. Turner (1993)

estimates a SVAR model of the UK business cycle using this sort of identification. Davis and Haltiwanger (1999) also employ such a neutrality restriction, as illustrated below.

2.1. Labour market studies using SVARs

This subsection reviews several SVAR-based labour market studies. See the individual papers for more detail.

Job flows, reallocation and aggregate fluctuations

One of the best-known applications of SVAR methods to labour market analysis is in the work of Davis and Haltiwanger (1990, 1999). These studies model gross job creation and destruction as being driven by a combination of aggregate and allocative (sector-specific) disturbances. The authors use both contemporaneous and long run restrictions to identify these structural shocks from the reduced-form errors. The SVAR setup used in both of these papers can be described as follows.[2] Let gross job creation and destruction at time t be denoted POS_t and NEG_t, respectively. Also, denote the aggregate and allocative shocks to these flows as a and s, respectively. The structural model is given (in moving average form) by

$$\begin{pmatrix} POS_t \\ NEG_t \end{pmatrix} = C(L) \begin{pmatrix} a_t \\ s_t \end{pmatrix}, \tag{6}$$

with $C(0) = C_0$. The structural shocks are assumed to be uncorrelated so that they have a diagonal covariance matrix Ω.[3] In order to recover C_0, Davis and Haltiwanger assume that the reduced form residuals p_t and n_t are related to the structural shocks a_t and s_t in the following way:

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[2] The discussion below relies primarily on Davis and Haltiwanger (1990), but both papers use the same basic technique.

[3] Davis and Haltiwanger (1990) actually estimate a more general model, which allows for serial correlation in the structural shocks. Specifically, they define $z_t = \left(\alpha_t, \sigma_t\right)'$, where σ_t is the intensity of the allocative shock. They then write $z_t = M(L)\varepsilon_t$, where ε is the vector of (iid) disturbances underlying z_t. Their model is thus $y_t = \tilde{C}(L)\varepsilon_t = C(L)M(L)\varepsilon_t$.

$$\begin{pmatrix} p_t \\ n_t \end{pmatrix} = \begin{pmatrix} a_t + c_{0p}s_t \\ s_t - c_{0n}a_t \end{pmatrix}$$

so that

$$C_0 = \begin{pmatrix} 1 & c_{0p} \\ -c_{0n} & 1 \end{pmatrix}.$$

Further assuming that the structural shocks are uncorrelated, the unknown elements of C_0 can be recovered from the reduced form covariance matrix Σ. Since $\Sigma = C(0)\Omega C(0)'$, we have

$$\begin{bmatrix} \sigma_p^2 & \sigma_{pn} \\ \sigma_{pn} & \sigma_n^2 \end{bmatrix} = \begin{bmatrix} \sigma_a^2 + c_{0p}^2\sigma_s^2 & -c_{0n}\sigma_a^2 + c_{0p}\sigma_s^2 \\ -c_{0n}\sigma_a^2 + c_{0p}\sigma_s^2 & c_{0n}^2\sigma_a^2 + \sigma_s^2 \end{bmatrix}. \tag{7}$$

This implies the following set of equations:

$$\begin{aligned} \sigma_p^2 &= \sigma_a^2 + c_{0p}^2\sigma_s^2 \\ \sigma_n^2 &= c_{0n}^2\sigma_a^2 + \sigma_s^2 \\ \sigma_{pn} &= -c_{0n}\sigma_a^2 + c_{0p}\sigma_s^2. \end{aligned} \tag{8}$$

Since there are only three equations but four unknowns, this system is not yet identified. Davis and Haltiwanger note that these equations can be combined to give a one-to-one relationship between c_{0p} and c_{0n}.[4] They then provide qualitative restrictions on c_{0n} (e.g., that it is greater than one), and obtain the corresponding value for c_{0p}. The variances of the structural shocks then follow from the first two equations in (8).

The qualitative restrictions on c_{0n} and c_{0p} are derived from theoretical considerations outlined in Davis and Haltiwanger (1990). For example, an aggregate shock should increase job creation while decreasing job destruction. Furthermore, the contemporaneous reduction in job destruction should be greater than the increase in job creation (i.e., aggregate shocks increase net employment growth), implying that c_{0n} should be positive and greater than one.

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[4] Specifically, $c_{0p} = \left(\sigma_{pn} + c_{0n}\sigma_p^2\right)\left(\sigma_n^2 + c_{0n}\sigma_{pn}\right)^{-1}$.

Davis and Haltiwanger also employ alternative long-run restrictions. In their 1999 paper, for example, a scheme similar to the one described above is combined with an assumption that allocative shocks have no permanent effect on the level of employment. Restrictions such as this constrain the reduced form impulse responses. This particular neutrality assumption implies that over a long enough horizon, the effects of an allocative shock on job creation exactly cancel the effects on job destruction. In the notation above, this means that

$$\sum_{l=1}^{\infty} C_{12}(l) - C_{22}(l) = 0$$

$$\Rightarrow \sum c_{0p} \left[D_{11}(l) - D_{21}(l) \right] + \left[D_{12}(l) - D_{22}(l) \right] = 0.$$

The main result in both the Davis and Haltiwanger papers is that allocative shocks account for a large fraction of the variance of job flows, and in particular the intensity of job reallocation (the sum of job creation and destruction). This result is robust to a number of alternative assumptions used to identify the structural shocks. In addition, results in the authors' 1990 paper suggest that allocative disturbances have played an important role in explaining the variance of the unemployment rate.

An analysis of macro shocks and policies in Italy

In a study of the Italian labour market, Christofides (1996) estimates a SVAR model in three variables: employment, the consumption wage, and the labour force. The author's main objectives are: to quantify the effect of various policies, such as hiring and firing restrictions, wage indexation, and centralisation and domination of the wage bargaining process by national unions and employer associations; to perform historical experiments using an extended version of the basic SVAR model; and to examine the effect of post-1991 labour market liberalisation measures and perform model simulations under certain assumptions. This paper is particularly relevant, as many of the characteristics of the Italian labour market during the sample period have also been present in Australia.

The basic SVAR model consists of an employment equation, a wage-setting equation, and a labour supply equation as shown below.

$$n_t = a_{nn}(L)n_{t-1} + a_{ns}(L)SSC_t + a_{nw}wp_t + a_{nk}k_t + a_{nr}r_t + a_{ny}y_t + \varepsilon_{nt}$$
$$wc_t = a_{wn}(L)n_{t-1} + a_{ww}(L)wc_{t-1} + a_{ws}(L)SSC_t + a_{wp}p_t + \varepsilon_{wt} \qquad (9)$$
$$l_t = a_{ln}(l)n_{t-1} + a_{lw}(L)wc_t + a_{ll}(L)l_{t-1} + a_{la}WAP_t + \varepsilon_{lt}.$$

In (9), n, wc and l represent employment, the consumption wage, and the labour force, respectively. wp is the product wage, k is business sector capital stock, r is the real exchange rate, y is real GDP, p is productivity, WAP is the working age population (14-64), and SSC is the ratio of social security contributions to the wage bill. Unemployment, u, is given by the identity $u_t = l_t - n_t$.

In this model, policy can have three distinct roles. It can affect employment (and unemployment) directly (e.g. through SSC). Alternatively, it can affect unemployment indirectly by influencing the exogenous variables in the extended model (e.g. y, k and r). Finally, it can affect the lags with which both policy and non-policy variables affect unemployment. The article identifies five such effects: an employment adjustment effect; a wage staggering effect; an insider membership effect; a labour force adjustment effect; and a discouraged worker effect.

Christofides conducts simulations to investigate the degree of inertia and sources of lags. The basic SVAR model is augmented by the u and wp identities plus an estimated production function. The model is then subjected to a 1% transitory negative labour demand shock. This results in unemployment settling down to equilibrium after a considerable lag (7-8 years). Employment adjustment costs and the wage staggering effect tended to increase unemployment persistence, while the insider effect tended to reduce it. Unemployment also responded sluggishly to permanent shocks, due to lags from all effects except the discouraged worker effect.

The effects of labour market shocks in the OECD

In a second example, Balmaseda et al (2000) use a set of long-run restrictions to identify structural labour market shocks and analyse the contribution of these shocks to labour market fluctuations in the OECD countries.

The basic theoretical model consists of the following equations:

$$y = \phi(d - p) + a\theta$$
$$y = n + \theta$$
$$p = w - \theta \qquad (10)$$
$$l = \alpha(w - p) - bu + \tau$$
$$w = \arg\left[n^e = \lambda l_{-1} + (1 - \lambda)n_{-1}\right]$$

These equations represent, respectively, aggregate demand, a production function, a price setting (markup) equation, a labour supply function, and a wage-setting equation. The last equation states that nominal wages are set one period in advance so as to equate expected employment with a weighted sum of lagged employment and labour supply. The variables are: y, the log of real output; p, the log of the price level; n, employment; w, nominal wages; d, a shift factor in nominal expenditure; q, a shift factor in productivity; l, the log of the labour force; n^e, the expected value of (log) employment; u, the unemployment rate; τ, a shift factor in labour supply, and ϕ, a, α, b, and λ are parameters. ϕ, a and α are expected to be positive. If the discouraged worker effect is relatively dominant, then $b > 0$. Partial hysteresis occurs when $0 < \lambda < 1$, and full hysteresis when $\lambda = 0$.

To close the model, an identity and three more equations are added. The identity is a direct application of the definition of the unemployment rate, $u = l - n$, and the other three equations specify random walk processes for the shift parameters d, θ, and τ. The innovations in these processes are denoted by ε_{st}, ε_{dt}, and ε_{lt}.

If hysteresis is only partial, the model can be solved to give the following SVAR representation:

$$\begin{pmatrix} \Delta(w - p) \\ \Delta y \\ u \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 + \alpha & 1 & 0 \\ \dfrac{1 + \alpha - \phi - a}{(1 + b)(1 - \rho)} & \dfrac{1}{(1 + b)(1 - \rho)} & \dfrac{-\phi}{(1 + b)(1 - \rho)} \end{pmatrix} \begin{pmatrix} \varepsilon_{st} \\ \varepsilon_{dt} \\ \varepsilon_{lt} \end{pmatrix},$$

where $\rho = (1 + b)^{-1}(1 + b - \lambda)$. On the other hand, if we have full hysteresis, the model is

$$\begin{pmatrix} \Delta(w-p) \\ \Delta y \\ u \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \phi+\alpha & \phi & 0 \\ \dfrac{1+\alpha-\phi-a}{(1+b)} & \dfrac{-\phi}{(1+b)} & \dfrac{1}{(1+b)} \end{pmatrix} \begin{pmatrix} \varepsilon_{st} \\ \varepsilon_{dt} \\ \varepsilon_{lt} \end{pmatrix}.$$

The main results from impulse response analysis are:

- In general, real wages increase both in the short and long run in response to a positive productivity shock and decrease in the short run in response to a positive labour supply shock.

- The response of real wages tends to be counter-cyclical in the short run with respect to a positive aggregate demand shock, save in the US where it is pro-cyclical.

- Real output tends to react positively to both productivity and labour supply shocks at all frequencies, and to demand shocks in the short run.

- Unemployment tends to increase temporarily to a productivity shock. It also increases in response to a labour supply shock but decreases after a positive aggregate demand shock.

FEVD analysis yields the following main results:

- For real wages, productivity shocks account for over 50% in the short run and near 100% in the long run.

- For real output, productivity shocks dominate in the long run in most countries and play an important role in the short run for the UK and US.

- For the unemployment rate, in the short run, demand shocks dominate in nine countries, productivity shocks in five countries, and labour supply shocks in Australia and Ireland.

3. Vector Error Correction Models (VECMs)

Vector error correction models, or VECMs, allow the researcher to model systems in which one or more variables contain stochastic trends. In particular, VECMs provide a convenient means of dealing with cointegrated systems, in which the number of stochastic trends is less than the number of variables. Generally, the cointegrating relationships present in such a system are specified in economically meaningful ways (i.e., equilibrium relationships); for the purposes of this paper, therefore, I will treat VECMs as structural models.

The basic form of a VECM can be derived from the following transformation of (1):

$$\Delta y_t = \Pi y_{t-1} + G(L)\Delta y_{t-1} + \varepsilon_t. \tag{11}$$

In (11), Δ is the first difference operator,[5] G(L) is a $(p\text{-}1)^{\text{th}}$ order matrix lag polynomial, and the matrix $\Pi = -\left(I - B_1 - B_2 - \cdots - B_p\right)$. In a stationary VAR, the Π matrix will have full rank, while if all the components of y_t contain unit roots, Π is zero and (11) reduces to a VAR in the first differences of the series. In the case of cointegration, the rank of Π will be equal to n-k, the number of variables less the number of cointegrating relationships. In this case, $\Pi = \alpha\beta'$ provides a decomposition of the error-correction matrix into the cointegrating vectors β and factor loadings α. This decomposition is not unique, but α and β can be estimated by maximum likelihood, using the methods of Johansen and Juselius (1990), for example.

3.1. A VECM model of real wages and unemployment

Jacobsen et al (1998) analyse the relationship between unemployment and real wages by considering the relative importance of supply and demand factors behind the development of unemployment, the speed at which real wages and employment respond to different types of shocks, and the strength of the long-run relationship between real wages and unemployment.

Identifying assumptions are made so as to present a labour market model that can be written as an economically interpretable common trend model. Output is determined by employment and a stochastic (exogenous) technology variable (all variables are expressed in logarithms). Labour demand is a function of output (y) and the real wage (w), while labour supply is a function of the real wage and a stochastic population variable. Finally, the real wage is a function of unemployment ($l - e$, where l is the labour force and e is employment) and labour productivity. There are thus four structural shocks: to technology, aggregate demand, labour supply, and a wage-setting shock. The first and third of these are assumed to be permanent, while the other two are transitory. Therefore the model contains two common trends and two cointegrating relationships.

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[5] That is, $\Delta y_t = y_t - y_{t-1}$.

The two cointegrating relationships are specified as follows. First, labour demand is assumed to be such that the wage share is stationary:

$$e_t = y_t - w_t + \varepsilon_{dt}.$$

The second vector is obtained from the real wage equation,

$$w_t = -\gamma\left[(l_t - e_t) - \varepsilon_{wt}\right] - \delta(y_t - e_t),$$

where γ and δ are the elasticity of the real wage with respect to unemployment and labour productivity, respectively.

The paper's main results are summarised as follows:

- Much of the medium-run (and all of the long run) fluctuations in real wages can be attributed to permanent shocks to the stochastic labour supply and technology trends.
- Short-run fluctuations are due primarily to transitory shocks.
- The hypothesis that unemployment is stationary is difficult to reject given two cointegration vectors.
- Unemployment appears to be significantly affected only by transitory aggregate demand shocks, thus implying that real wages and unemployment are unrelated in the long run.

4. Conclusions

This paper has provided an overview of how labour market analysis can be conducted in the context of VAR-based models. The examples presented here show that these methods are quite flexible, and capable of addressing a wide variety of theoretical and policy-related issues. Future work in this area will concentrate on applying these models and techniques to Australian data.

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