Dissecting the Cycle*

Don Harding
Melbourne Institute of Applied Economic and Social Research
The University of Melbourne

and

Adrian Pagan
The Australian National University and The University of Melbourne

Melbourne Institute Working Paper No. 13/99

ISSN 1328-4991

ISBN 0 7340 1465 1

May 1999

* Don Harding is Senior Research Fellow and Assistant Director at the Melbourne Institute at the University of Melbourne. Adrian Pagan is at the Australian National University and is a Professorial Fellow at the University of Melbourne. Research for this paper was supported by ARC Grant No A79802751. We are grateful for the comments of Ern Boehm, Michael Boldin, Jan Jacobs, Lou Maccini, Jim Nason, Graeme Wells and participants at various seminars where earlier versions of the paper were given: the Theory and Evidence in Macroeconomics Conference held at the University of Bergamo, and seminars at CEMFI, The University of Melbourne and the Reserve Bank of New Zealand.

Melbourne Institute of Applied Economic and Social Research
The University of Melbourne
Parkville, Victoria 3052 Australia
Telephone (03) 9344 5330
Fax (03) 9344 5630

Email **melb.inst@iaesr.unimelb.edu.au**
WWW Address **http://www.ecom.unimelb.edu.au/iaesrwww/home.html**

ABSTRACT

Macroeconomics has a long tradition of inspecting and interpreting patterns in graphs of aggregate data. However, the move towards more precise quantification of macroeconomic phenomena has seen academics shift away from a study of turning points, which are a natural and obvious way of summarizing business cycles, towards measures of co-movement in detrended series. This shift arise from several developments, but an important one was the belief among academics that Burns and Mitchell's methods lacked the statistical basis and, hence, the precision required in modern macroeconomics.

We adopt the older perspective that business cycles are to be defined in terms of the turning points in the level of economic activity. We show that such turning points can be associated with a well defined sequence of outcomes and can therefore be precisely analyzed. In turn this enables us to explore how various parametric models of aggregate output generate a cycle through the interaction of trend movements in activity with the volatility and serial correlation in growth rates.

One of the strongest points in the rhetoric of modern business cycle theory is that trend and cycles should not be divorced. Consequently, any definition of the business cycle in terms of the co-movement of detrended data has to find the task of integration a difficult one. In contrast, we show that a return to the older tradition of studying the classical cycle in the level of economic activity produces a natural interpretation of the origin of the cycle in terms of the interaction of trend and the second moments of growth rates. This seems a critical advantage for the approach taken in this paper.

An important issue that has also been debated in the literature is whether non-linear models are required to make a business cycle. Using the techniques developed in this paper we dissect the cycle of a number of countries and find little evidence that non-linearities, of the type investigated in the literature, are important in accounting for the broad features of the average cycle.

1. Introduction

From the beginning of macroeconomics investigators have been fascinated by patterns in graphs of data on production, employment and prices. Such patterns were termed cycles. Many theories have been advanced to explain these cycles and there has been constant debate over whether particular series, such as prices, are "pro or counter cyclical". Amongst academics, evidence on the nature of cycles has changed from a graphical orientation towards quantitative measures extracted from parametric models; a shift that is vividly captured by comparing the books by Burns and Mitchell (1946) and Cooley and Prescott (1995). Such a movement is not as apparent amongst policy makers and the business community. Indeed these latter groups have shown little interest in the debates which have engaged academics and continue to emphasize graphical methods when describing the cycle. This suggests that there are some disadvantages to conducting discussion of the cycle in the mode favored by academics and that a good deal of useful information about the likely causes of cycles is not reaching its intended recipients. In this paper we argue a brief that much of what is referred to as "modern business cycle research" is less valuable than it might be because of the way in which discussion of the cycle has been conducted. We see little reason for the shift in emphasis by academics and point out that the older tradition can be given a formal treatment that clarifies what the connections between the two approaches are. After doing this we suggest that the older one might be preferred.

Section 2 of the paper considers definitions of the cycle framed in terms of the turning points of a series, this being the methodology set out in Burns and Mitchell (1946). Looking at the cycle in this way is the obvious method of summarizing what we see in any graph of a macro-economic time series. We suggest some new measures that might be useful when thinking about the nature of the cycle and take up the central issue of whether one wants to detrend a series before its cycle is investigated. Section 3 explores the connections between a definition of the cycle derived from the turning points of a series and the moments of the random variables taken to represent that series, as well as other issues involving "co-movements" between series across cycles. We embark on this latter investigation since a common criticism of Burns and Mitchell's work was that it did not have a statistical foundation, and recent writers such as Stock and Watson (1998) have repeated such a criticism when adopting the academic approach.

Sections 2 and 3 are largely methodological in canvassing different ways of describing and measuring the cycle. They do not directly address the issue of how to describe the temporal behaviour of a series. For that task, researchers have always resorted to parametric models, either statistical or economic, and a large body of evidence has now been accumulated in this format. Section 4 asks what type of business cycles these parametric models generate, where the cycle is defined through the set of measures built up in section 2. Our aim here is to try to learn what are the most important ingredients in "making a cycle" with a parametric economic model. Section 5 concludes.

2. What Should We be Trying to Explain?

2.1 Definitions of the Cycle

The business cycle is a pattern seen in any series y_t taken to represent aggregate economic activity. Clearly such a statement lacks precision on two counts; it does not say how one can measure aggregate economic activity and it does not indicate how one is to describe the patterns in it. In their classic work on business cycles, Burns and Mitchell gave an answer to the second question through a description of how they located *turning points* in many series, each of which was a partial reflection of "economic activity". Such turning points defined *specific cycles* and the information in these specific cycles was distilled into a single set of turning points that identified the *reference cycle*.[1] It was the latter which was called the business cycle, and the tradition continues today in the publication of a single set of turning points by the National Bureau of Economic Research.

[1] This description is too simple. Reading their text one is struck by the amount of iteration they engaged in. Tentative reference cycle dates were found from a variety of measures of general business conditions and these were then refined with specific cycle information. There is also a problem with terminology as the "reference cycle" actually describes many cycles in time. Later, when comparing specific cycles and the reference cycle, we need to bear in mind the latter fact.

Given how natural it is to think of a cycle through its turning points, it is somewhat odd that academic work has largely departed from this emphasis.[2] The origin of the transition seems to be Koopmans' (1947) attack on Burns and Mitchell's work. One way to interpret Koopmans' stance is to observe that, if one could completely describe the characteristics of y_t, then it must be true that its turning points could be extracted, since they are functions of the y_t. Hence Koopmans' recommendation was that one should model y_t. Of course this still leaves the question of how the measured characteristics of the process generating y_t map into the business cycle. Moreover, there are suggestions in the literature that it was much easier to think about how to parametrically model y_t than to analyse turning points, because the procedures followed by Burns and Mitchell were more an art than a science. Stock and Watson (1998), for example, comment on Burns and Mitchell's procedures in the following way:

"… the methods of business cycle analysis have been criticized for lacking a statistical foundation".;

Of course, this is not an argument against working with the Burns and Mitchell methodology but rather one for developing a statistical foundation for the latter and it is a major objective of this paper to do just that.[3]

As should be clear from the way the reference cycle was constructed Burns and Mitchell also had reservations about whether any of the series that were available to them were suitable measures of aggregate economic activity. They comment (p 72)

"Aggregate [economic] activity can be given a definite meaning and made conceptually measurable by identifying it with gross national product"

but

[2] Not completely though. King and Plosser (1994) and Simkins (1994) look at the cycle in this way and it is also very common to see articles in which reference is made to turning points before the analysis proceeds in quite a different way e.g. Christiano and Fitzgerald (1998).

[3] It is true that there is judgement in what Burns and Mitchell did, so that one could never exactly replicate their thinking unless one knew exactly what steps they followed. But one can get close enough for most purposes.

"Unfortunately, no satisfactory series of any of these types is available by months or quarters for periods approximately those we seek to cover" (p. 73)

Accordingly they used a wide range of series to come up with a single reference cycle.[4] It is not surprising then that an application of Koopmans' philosophy would point towards the need to first find representations of the joint behaviour of a number of series $y_{1t}, \ldots y_{Kt}$ and to then determine a method of combining them into a single measure of activity. Curiously, whilst the first of these steps was enthusiastically endorsed in the academic literature, the second was largely discarded.[5] Somehow the impression has been created in the academic literature that what was important in discussing the business cycle were the inter-relationships (or co-movements) between the specific series used to construct the reference information. For example, Cooley and Prescott (1995, p26) summarize what they feel the implications of Burns and Mitchell's work was in the following way:

"…the one very regular feature of these fluctuations is the way variables move together. It is the co-movements of variables that Burns and Mitchell worked so hard to document and that Robert Lucas emphasized as the defining features of the business cycle".

What is strange here is the transformation in the motivation for considering many series. In Burns and Mitchell's case it was simply an instrument used to define *the* business cycle, through the way in which turning points in many series clustered together; in much of the modern literature it has become an end unto itself. In fact the latter's obsession with co-movements between series seems to miss the point of why we are interested in the business cycle. It is an extraordinary feature of much of the modern academic literature that one can find papers which provide extensive accounts of the co-movements of consumption, investment etc. but which make little or no reference to the temporal characteristics of the series that might be taken to be aggregate economic

---

[4] The arguments in favor of using a variety of measures of activity rather than a single one are reviewed in Boehm (1998).

[5] It has returned recently in papers such as Stock and Watson (1991) and Diebold and Rudebusch (1996) where a common factor is taken to be present among the y_t and this may be defined as the equivalent of the reference cycle. Epstein (1998) observes that Burns maintained that he did not subscribe to this common factor view and we return to this issue later.

activity, namely output. “Hamlet without the Prince” is the phrase that comes to mind when reading such papers.

The discordance between what Burns and Mitchell did, and what much of the modern literature on business cycles does, widens when one notes that the latter has even ceased to study the cycle in y_t and has replaced it with a “detrended” series, z_t. Unfortunately, there are many different ways of removing trends. Official agencies generally use Henderson filters e.g. Australian Bureau of Statistics (1987); NBER type researchers use “phase averaging” as described in Boschan and Ebanks (1978); and academics either remove a low order polynomial in time (a deterministic trend) or both stochastic and deterministic trends either with a “band-pass” or Hodrick- Prescott (HP) filter.[6] The question that obviously arises is why one wants to do this? A number of reasons might be given.

1. It is the quantity that is of most interest for policymakers. There are times when it is important to possess a detrended series e.g. “output gaps” are a key ingredient in some versions of the Phillips curve. But most policy and public discussion is concerned with cycles in the *level* of y_t i.e. the classical cycle. For example, using a series on post-war US GDP, and detrending it with the HP filter (λ=1600), one finds that there was a peak in 1994/4 and a trough in 1996/1 i.e. there was a “recession” over these years.[7] It is very hard to square this with the current “longest peace time expansion” rhetoric unless one adopts a classical view of the cycle. The period 1994/4-1996/1 did show a slowdown in growth rates, and there is no doubt that policymakers sometimes compare growth rates to what is believed to be their potential values, but the latter tend to be constant for long periods of time, so that such comparisons are akin to removing a low order deterministic trend.

2. What determines trends in data is unknown and so modelers should stick to modeling quantities that they have something to say about. There is some truth to this

---

[6] As Phillips (1998) argues the distinction between a deterministic and a stochastic trend is largely one of degree. Here we conceive of a deterministic trend as one that is *linear* in time,

argument. Most dynamic stochastic general equilibrium (DSGE) models exhibit a deterministic long run balanced growth path whose determinants are unknown and whose magnitude is generally set at what is seen in the data. Thus linear detrending of the log of the data makes some sense for modelers. Moreover, even though one is now looking at a "deviation" or *growth cycle* rather than the classical cycle, the latter can be recovered relatively easily by adding back in a known quantity.[8] However, this argument does not extend to rationalizing the removal of a stochastic trend, as is done by the HP filter. It is a central theme of DSGE models that the characteristics of the cycle are bound up with the stochastic trend, so that removing it is not a neutral operation for cyclical analysis. Moreover, one cannot recover the classical cycle by any simple operation, as one can do with deterministically de-trended data. In fact, the cycle is changed to something that is quite unrecognisable. For example, using HP detrended data on post-WW2 US GDP, one finds some 19 peaks and troughs, implying a 30 month cycle. That cycle can be contrasted with the 72-96 months of the classical cycle (as dated by the NBER). Consequently, it is always rather odd to see articles which work with HP filtered data justifying their results by reference to classical cycle attributes.

3. One sometimes suspects that detrending was also attractive to academics because it seemed to offer a different definition of the cycle that put them into the scientific mainstream. Specifically, once the process had been rendered stationary, its spectral density could be computed, and the existence of cycles could be taken as corresponding to a peak in the spectral density of z_t. As the experiment in Slutsky (1937) suggests however there may be no connection between these two ways of talking about the cycle. An excellent illustration of this point is the discussion in Kydland and Prescott (1990) about the cycle in data simulated from an AR(1) in z_t , where they concentrate on turning points when describing the cycle. Of course, an AR(1) has no peaks in the spectral density, except at the origin. Thus conclusions about cycles drawn from spectral density arguments, as in Burnside (1998), have very little relevance to the cycle that we

---

[7] We use the BBQ program described later to find these turning points although one could have done it visually.
[8] See Mintz (1972) for probably the earliest formalization of this concept.

speak most about. Any series has a cycle, in the sense of possessing turning points, and so the shape of its spectral density is irrelevant, and contains no direct information about cycle characteristics. Later we demonstrate that one can reproduce published cycle characteristics using series that have no peaks in the spectrum of Δy_t.

4. Although rarely expressed, we think there is a feeling that detrending is necessary owing to statistical difficulties when working with the levels of variables, particularly if they have a stochastic trend. Certainly, the latter create problems for many traditional parts of estimation and inference. However, as we will show, this is not an issue in this instance, as the location of turning points is done with the differences of a series rather than their levels, making the presence of a stochastic trend in y_t unimportant.

2.2 Methodologies For Describing a Cycle

The detection and description of any cycle is accomplished by first isolating turning points in the series, after which those dates are used to mark off periods of expansions and contractions. Viewed in this light business cycle analysis involves pattern recognition techniques, and this fact goes to the heart of how one learns about the business cycle, regardless of whether we work with the *log* levels of series y_t or their detrended form z_t. Location of turning points can sometimes be done visually. When performing the datings in this way the eye is also very good at filtering out "false turning points" i.e. movements which are either short lived or of insufficient amplitude. Translating the ocular judgments into an algorithm has proved to be challenging. At a minimum such an algorithm needs to perform three tasks.

1. Determination of a potential set of turning points i.e. the peaks and troughs in a series.
2. A procedure for ensuring that peaks and troughs alternate.
3. A set of rules that re-combine the turning points established after steps one and two in order to satisfy pre-determined criteria concerning the duration and amplitudes of phases and complete cycles; what we will refer to as "censoring rules".

The best known algorithm for performing these tasks is that associated with the NBER and set out in Bry and Boschan (BB) (1971) for monthly observations on a series. Although there are many sub-stages involved in it, the heart of the first step is a definition of a local peak (trough) as occurring at time t whenever $\{y_t >(<) y_{t\pm k}\}$, k=1,…,K, where K is generally set to five.[9] The main criteria relating to the third step are that a phase must last at least six months and a complete cycle should have a minimum duration of fifteen months. Growth cycles have also been dated with the same set of rules, with the inputs now being z_t rather than y_t ; clearly the turning point definitions should be invariant to this change but it may not be so reasonable to adopt the same censoring rules.

When the data is measured at the quarterly frequency an analogue to the first step of the BB algorithm would be to put K=2 i.e. $\{\Delta_2 y_t>0 , \Delta y_t>0, \Delta y_{t+1}<0, \Delta_2 y_{t+2}<0\}$, as this ensures that y_t is a local maximum relative to the two quarters (six months) on either side of y_t. Later, this quarterly version of the BB algorithm, combined with some censoring rules, is described as BBQ. An even simpler rule, based on the idea that the derivative changes sign at peaks and troughs, would be to treat Δy_t as a measure of the derivative of y(t) with respect to t, producing$\{\Delta y_t>0, \Delta y_{t+1}<0\}$as the criterion. The problem with the latter is that it would conflict with the requirement that a phase must be at least six months in length.

Instead of attempting to locate local maxima directly a class of "sequence" rules have also been suggested for identifying peaks and troughs. A good example of one of these for locating a peak is $\{\Delta y_t>0, \Delta y_{t+1}<0, \Delta y_{t+2}<0\}$ (Wecker (1979)).[10] In this form it replicates a rule popularized by Arthur Okun that a recession involves at least two quarters of negative growth. The latter rule is widely used in the media and policy circles

---

[9] King and Plosser (1994) give a description of the steps. One of these involves smoothing of the data and this has led to a belief that the data is therefore being detrended. In fact the smoothing is simply aiding in the process of identifying peaks and troughs through the removal of some idiosyncratic variation. The utility of smoothing is much reduced if the dating is being done with quarterly data, and for that reason we ignore it in the algorithm developed later.

[10] A trough would be defined by $\{\Delta y_t<0 , \Delta y_{t+1}>0, \Delta y_{t+2}>0\}$. Pagan (1997a,b) uses this rule to compute the average length of a cycle.

to signal a classical recession and has also been used by Canova (1994) to establish growth cycle dates, with y_t replaced by z_t.

All of these methods for locating a turning point can be thought of as defining a turning point as an *event* to which probabilities can be attached, and recognition of that fact enables a formal statistical analysis to be performed. Notice however that sequence rules used in the literature only correspond to step one of the BB algorithm and, most importantly, ignore the third step i.e. they impose no censoring upon the basic set of turning points and therefore cannot be directly compared to growth cycle information published by the Foundation for International Business Cycle Research (FIBCR).[11] Nevertheless, they all emphasise that, even though the classical cycle refers to the behaviour of the level of a variable, the analysis of its turning points is done with a stationary series viz. functions of the first differenced series, Δy_t, such as $sgn(\Delta y_t)$.[12] It cannot be stressed too much that the rules above *are not locating a cycle in* Δy_t; rather Δy_t is just an input into the dating process of the classical cycle. Most of the literature sees the action of taking differences as a "detrending filter", see Canova (1998a) for example. The importance of Δy_t for us, however, is that it points to the need to develop models of that quantity so as to account for the classical cycle. As the models of Δy_t are varied, so also will the probabilities of the sequences which define the classical cycle.

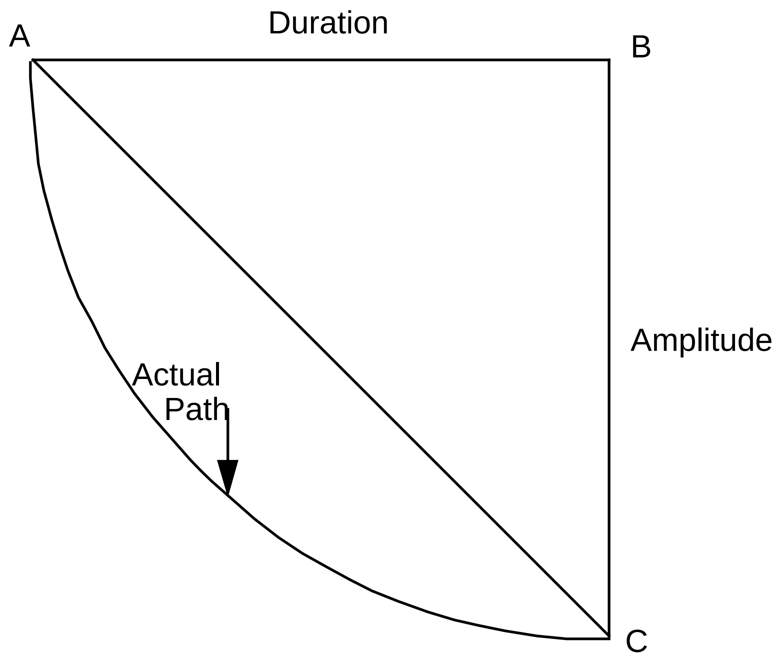
Given that turning points have been established how should the dating information be used in conjuction with the series from which the dates were derived? Burns and Mitchell provided an elaborate classification of the cycle into nine stages. We suspect that this level of detail is of marginal interest to most of those who think about and observe the business cycle. Much more useful would be some general summaries of what one sees in a graph of data. Inspection of comments that are frequently made about the cycle suggest that there are four items of interest.

[11] It is also the case that none of the trend adjustment methods Canova uses in Canova (1994), (1998a) and (1999) corresponds to that used by FIBCR.
[12] When working with growth cycles it is Δz_t which is analysed. With the Bry-Boschan definition of a turning point it is "long differences" y_t-y_{t-k} that need to be examined, but these are the sum of first differences.

- The duration of the cycle and its phases
- The amplitude of the cycle and its phases
- Any asymmetric behavior of the phases
- Cumulative movements within phases.

In thinking about these measures it is useful to consider a phase as a triangle. Fig 1 shows a stylized recession, with A being the peak and C the trough. The height of the triangle is the amplitude and the base is the duration. Knowledge of these two elements for any cycle enables one to compute the area of the triangle, and thereby an approximation to (say) the cumulated losses in output from peak to trough, relative to the previous peak. Designating the duration of the i'th phase as D_i and the amplitude as A_i , the product $C_{Ti} = .5(D_i * A_i)$ will be referred to as the *"triangle approximation" to the cumulative movements*. In practice the *actual cumulative movements* (C_i) may differ from C_{Ti} since the actual path through the phase may not be well approximated by a triangle, and this points to the need for an index of the *excess cumulated movements;* the natural candidate is, $E_i = (C_{Ti} - C_i + 0.5 * A_i)/D_i$. In this formula D_i is the duration of the phase and the term $0.5*A_i$ removes the bias that arises in using a sum of rectangles (C_i) to approximate a triangle. Although it is C_i which is of fundamental interest to policy makers and historians, the triangle approximation is still likely to be useful in shedding light upon the ability of business cycle models to generate realistic cycles.

Figure 1: Stylized Recession Phase



Most of the characteristics just described can be obtained through regression analysis using data on $\{y_t,S_t\}$, where S_t is a dichotomous variable taking the value unity when the economy is in expansion at time t and zero if it is in contraction. Two examples can be given. First a regression of Δy_t against S_t gives the average amplitude of an expansion. Second, taking the regression equation $S_t = \alpha+\beta S_{t-1}$, and designating estimates of α and β by a and b, the average duration of an expansion is given by 1/(1-a-b). Other uses of S_t , such as checking for duration dependence, are given in Pagan(1998).

Although the reference cycle was the key element in Burns and Mitchell's business cycle analysis, any understanding of the business cycle was also seen to involve a knowledge of the nature of specific cycles and how they behaved in relation to reference cycles.[13] To summarize the information provided by specific cycles one could use the same set of characteristics as was used for the reference cycle viz. the average amplitude, duration etc. To describe relationships with the reference cycle they devised an "index of

[13] Epstein (1998) argues that Burns and Mitchell valued the specific cycle information as highly as that contained in the reference cycle i.e. the dispersion of the turning points in the specific series around the "central tendency" of the reference cycle was important to them.

conformity". Defining A_{ij} now as the change in a specific series y_{jt} over a phase (say expansion) of the i'th reference cycle (out of a total of k), their index of conformity was $k^{-1}\Sigma_{i=1,k}$ sgn(A_{ij}) *100. King and Plosser (1994) and Simkins (1994) used this index in their work. In some ways it is not an ideal measure of the extent of coherence between cycles. Consider for example the i'th expansion of the business (reference) cycle dated between time t and t+m. Then sgn(A_{ij}) = sgn ($y_{j,t+m}$-y_{jt}), and this could be positive even if observations t+n,…,t+m corresponded to a recession in the specific variable y_{jt}. This suggests that another measure of the concordance between cycles would be useful. To this end we propose that the *degree of concordance* between the specific cycle for y_{jt} and the reference cycle (based on (say) the variable y_{rt}) can be measured by the fraction of time they are both in the same state. Mathematically this is

$$I_{jr} = n^{-1} [\#\{S_{jt}=1, S_{rt}=1\}] + n^{-1} [\#\{S_{jt}=0, S_{rt}=0\}]$$

$$= n^{-1} \{ \Sigma S_{jt}S_{rt} + (1-S_{jt})(1-S_{rt}) \}$$

This index might be used in a number of ways. First, it can capture the notion of whether a variable y_{jt} is pro or counter-cyclical. If it is exactly pro-cyclical then the index would be unity, while a value of zero marks it down as being exactly counter-cyclical. An advantage of the index is that it is a well defined quantity even if the variables y_{jt} and y_{rt} are non-stationary. Most of the literature relating to the nature of cyclicality of a series has proceeded as if detrending was necessary, so as to define the concept in terms of a covariance. An unresolved problem with the concordance index is to define a cross over point. If the two series underlying the cycles were independent then $E[I_{jr}] = E[S_{jt}]E[S_{rt}] + (1-E[S_{jt}])(1-E[S_{rt}\})$ and $E[S_{jt}]= prob(S_{jt} =1)$. The latter can be estimated by the fraction of time spent in an expansion for y_{jt} while $E[S_{rt}]$ can be measured in the same way using the reference cycle. The concordance index also represents a way to summarize information on the clustering of turning points. If the turning points of a specific and reference cycle are coincident then the index would equal one. Hence one would wish any of the series that are used to set up the reference cycle to have high values for this index.

2.3 Characteristics of Actual Cycles

For later reference it is worth establishing some of the characteristics of the classical cycle in a number of countries. We choose three of these — the US, the UK and Australia. Most modern business cycle research has been conducted upon data relating to the former but it is important not to get into the habit of thinking that this is "the" business cycle. The UK and Australia represent economies that, historically, have been more open than the US. They differ both in terms of size and in population growth. Australian population growth tends to be about 1% p.a. more than the UK, and this translates into a higher trend growth rate in GDP. Turning points were established for each country using the quarterly analogue of the Bry-Boschan program applied to series on the log of GDP.[14] Data was over 1947/1-1997/1 (US), 1955/1-1997/1 (UK), and 1959/1-1997/1 (Australia). A difficulty was encountered in adapting the censoring rules in BB to BBQ, in that a decision has to be made on the appropriate minimal length of the phases in terms of quarters. A fifteen month minimal length to a complete cycle would be compatible with either a four or five quarter restriction, depending on the month in which the turning point occurred, and the relative magnitudes of the monthly values in the quarter. US and Australian turning points were invariant to using either a four or five quarter rule, but this was not so for the UK. With a five quarter rule the cycle in 1974 was ignored. Since the latter was quite a major event in terms of magnitude the four quarter minimum was therefore adopted to effect the UK dating.

Table 1 gives some of the business cycle information; contractions are designated as PT and expansions as TP. With the exception of duration statistics, all measurements are made in terms of percentage changes. For the US and UK the statistics are very close to those established with monthly dates, in that the average cycle length is recognised to

[14] There is an issue of what series should be used to measure economic activity. As Burns and Mitchell (1946, p 72) noted it is difficult to go past GDP as the single index. Through the expenditure side of the national accounts it captures all aspects of demand; through the production side it aggregrates all industry sectors; and through the income side it captures hourly wages, employment, hours worked and profits. Moreover, through the production function, output can be related to labour, capital and material inputs. A problem with GDP is that in some cases there are different measures of it from the income, production and expenditure sides, and these may need to be combined together in some optimal way so as to produce a single measure of it.

be around 62 months for the US and 60 for the UK.[15] The Australian cycle dated this way is shorter than the eighty months that is the standard. What is striking about the table is the similarity of the contraction phases and the divergences from a triangle in the expansion phase. Rapid recovery in the early part of an expansion has been documented in Sichel (1994) and this is consistent with the results of the “excess” computations in Table 1 which indicate that the shape of expansions deviate substantially from a triangle.[16] It is also apparent that the cycle and its expansions tend to be longer in Australia. The indices of concordance between UK and Australian GDP (relative to the US) are .82 and .86 respectively. Based on the probabilities of expansions and contractions revealed in Table 1, the expected values would be .72 and .76 respectively if there was no relationship. The latter values point to the fact that graphical methods might suggest a close connection between the cycles of different countries, even though activity evolves independently in each case.

[15] In the appendix we compare the peaks and troughs in the cycle found using BBQ with those given by FIBCR (converted to a quarterly basis) which combine together a variety of series in the way that Burns and Mitchell did in order to come up with reference cycle dates. Hence this comparison yields some insight into how effective it is to use a single series such as GDP to represent aggregate economic activity. Unfortunately, the comparison is clouded by the different frequency of observations in the series used to accomplish the dating. Nevertheless, with one exception, the two sets of dates are close. The exception is FIBCR’s identification of a trough in 1970/4 which is dated as 1970/1 by BBQ. Inspection of GDP shows that the level of GDP in 1970/1 is slightly below that in 1970/4.

[16] Note that the excess measure in the tables is found by averaging the E_i over all cycles rather than constructing it from the average values of C_i and C_{Ti}.

Table 1

Business Cycle Characteristics for the US, UK and Australia

	U.S	UK	Aust
Mean Duration (quarters)			
PT	3	3.75	3.3
TP	17.8	16.14	20.6
Mean Amplitude (%)			
PT	-2.5	-2.5	-2.2
TP	20.2	14.5	24.7
Cumulation (%)			
PT	-4.1	-6.1	-4.0
TP	256	196	320
Excess			
PT	-0.1	0.0	0.1
TP	1.1	0.7	1.0

Some specific cycles may also be of interest e.g. the cycles in hours, asset prices, consumption, investment etc. Table 2 looks at the cycles in US consumption and investment. Taking GDP data as establishing the reference cycle, there are some marked differences between the two specific cycles and the reference cycle; expansions in consumption are much longer and stronger than those in GDP while investment has strong expansions but with an average duration that is much shorter than that in GDP. Contractions in investment are also relatively long. Again there is some asymmetry in the shapes of the two phases. The indexes of concordance of US Consumption and US investment (relative to US GDP) are .89 and .78 respectively, compared with their expected values of .81 and .61 (under independence).

Table 2

Business Cycle Characteristics for US GDP, Consumption and Investment

	GDP	Con	Inv
Mean Duration (quarters)			
PT	3	2.8	5.3
TP	17.8	37.0	10.3
Mean Amplitude (%)			
PT	-2.5	-2.0	-22.6
TP	20.2	36.0	35.2
Cumulation (%)			
PT	-4.1	-2.4	-53.2
TP	256	1012	233
Excess			
PT	-0.1	0.2	1.6
TP	1.1	0.1	2.4

3. Understanding the Burns and Mitchell Methodology

Although it is true that business cycle characteristics can be established whenever a series representing aggregate economic activity is available, simply by passing observations on the latter, y_t , through a dating algorithm, there are advantages to be had from relating the cycle characteristics to the nature of y_t. Following the tradition of business cycle research established by Tinbergen (1939) the nature of y_t has generally been described with parametric statistical models. Indeed, for many years econometricians have sought statistical models that would adequately describe the temporal behaviour of y_t. Consequently, it seems a useful exercise to consider a range of statistical models that have been proposed for y_t and to relate the parameters of these models to the classical cycle characteristics described in section 2.

Before embarking on such a task we need to establish some of the relationships that are critical to the dating of cycles.[17] Using definitions of conditional probability,

[17] Some of this material appears in Harding (1997).

$$\Pr(\text{peak at t-1}) = \Pr(S_t=0 \mid S_{t-1}=1)\Pr(S_{t-1}=1) \qquad (1)$$

$$\Pr(\text{trough at t-1}) = \Pr(S_t=1 \mid S_{t-1}=0)\Pr(S_{t-1}=0). \qquad (2)$$

Making the additional assumption that the random variable, S_t , is strictly stationary, implies that $\Pr(S_t=1)=\Pr(S_{t-1}=1)$. Because the number of peaks and troughs are the same, and $\Pr(S_t=1)=1-\Pr(S_t=0)$, it follows that there are four unknowns in (1) and (2). Once two of these are fixed the other two follow. NBER dating rules effectively fix Pr(peak) and $\Pr(S_t=1)$, since the latter follows once the turning points are located. Harding (1997) analyses the cycle by setting the two exit probabilities; to do this he introduces the concepts of an expansion terminating sequence (ETS) and a contraction terminating sequence (CTS), so that $\Pr(ETS|S_{t-1}=1) = \Pr(S_t=0 |S_{t-1}=1)$ and $\Pr(CTS|S_{t-1}=0)= \Pr(S_t=1 |S_{t-1}=0)$. His preferred ETS = $\{\Delta y_t <0 , \Delta y_{t+1} <0\}$ with CTS=$\{\Delta y_t >0, \Delta y_{t+1} >0\}$when dating a classical cycle. Making the peaks and troughs alternate and using Harding's descriptors leads to

$$\Pr(S_t=1)= \Pr(CTS|S_{t-1}=0)/[\Pr(CTS|S_{t-1}=0) + \Pr(ETS|S_{t-1}=1)]. \qquad (3)$$

(3) may be substituted back into (1) to find the probability of a turning point that satisfies steps one and two of the dating algorithm. If the data is quarterly, the average length of a cycle will then be 3/Pr(peak) months and the average length of an expansion in months would be $3*\Pr(S_t=1)/\Pr(\text{peak})$. Of course, these computations ignore the censoring performed in the final step of an algorithm like BBQ.

The analysis of dating algorithms through the concepts of terminating sequences is very useful for many purposes. One is to relate classical cycle characteristics to the nature of $\Delta y(t)$. It will prove hard to do this solely with analytical methods, although it can be done for some simple models that are close to reality, and the results thereby established should provide insights into more complex cases. Taking the frequency of observation to be a quarter, a very simple model for y(t) is that of a random walk with drift,

$$\Delta y_t = \mu_y + \sigma e_t, \qquad (4)$$

where e(t) is n.i.d. (0,1). The absence of serial correlation in quarterly growth rates of GDP is characteristic of quite a few countries, for example the UK and Australia. Moreover, what correlation there is tends to be generally rather weak e.g. on data over 1961/1-1997/4 both the US and Canadian GDP growth rates have first order serial correlation coefficients around .3. As Cogley and Nason (1995) observe the output processs in a variety of RBC models have the structure of (4) and so a study of (4) sheds light on the ability of those models to generate a cycle.[18] After such a study, the case where e_t is correlated can be examined.

When e_t is n.i.d. (0,1) there are only two parameters, μ_y and σ, and these completely describe the temporal behaviour of Δy_t. Consequently, it follows that all the business cycle characteristics that are derivative from dating algorithms based on y_t must also be a function of these parameters. In fact, given that dating methods for the classical cycle have as their basis a study of events such as $\{ \Delta y_t < 0\}$, and the $\Pr(\Delta y_t<0)= \phi =$ $\Pr[e_t<-(\mu_y/\sigma)]$, it is the *ratio* $-(\mu_y/\sigma)$ which will be critical. The exact mapping between (say) the duration of the cycle and the single index (μ_y/σ) will however depend upon the nature of the dating algorithm employed. For example, if the CTS is $\{\Delta y_t > 0\}$ and the ETS=$\{ \Delta y_t < 0\}$, $\Pr(CTS|S_{t-1}=0)=1-\phi$, $\Pr(ETS|S_{t-1}=1)=\phi$ and (3) shows that $\Pr(S_t=1)= 1-\phi$, making $\Pr(peak)= \phi(1-\phi)$ and the average duration of the cycle will be $3/[\phi(1-\phi)]$ months.

Another dating rule, referred to as "Okun's Rule" in Harding and Pagan (1998), puts ETS = $\{\Delta y_t<0 , \Delta y_{t+1}<0\}$ and CTS=$\{\Delta y_t>0, \Delta y_{t+1}>0)$. This rule is slightly more complex than the previous example, because conditioning on S_{t-1} restricts the set $=\{\Delta y_{t-1}, \Delta y_t\}$. Conditioning on $S_{t-1}=1$ means that it cannot be the case that $\{\Delta y_{t-1}<0 , \Delta y_t<0\}$, while conditioning on $S_{t-1}=0$ means that it cannot be the case that $\{\Delta y_{t-1}>0 , \Delta y_t>0\}$. These pairs are ruled out because, if observed, they would have terminated the assumed state one period earlier. Thus,

[18] Harding and Pagan (1998) explore the type of classical cycle generated by a variety of theoretical models that appear in the literature.

$$Pr(ETS|S_{t-1}=1)= Pr(\{\Delta y_t<0 , \Delta y_{t+1}<0\}|S_{t-1}=1)$$
$$=Pr(\Delta y_t<0|S_{t-1}=1). Pr(\Delta y_{t+1}<0|S_{t-1}=1) \qquad (5)$$

and $S_{t-1}=1$ implies that $\{\Delta y_{t-1} , \Delta y_t\}$ is drawn from the set $\{(\Delta y_{t-1}<0 , \Delta y_t>0); (\Delta y_{t-1}>0 , \Delta y_t<0); (\Delta y_{t-1}>0 , \Delta y_t>0)\}$. Thus $Pr(\Delta y_t<0| S_{t-1}=1)= \phi(1+\phi)$ and $Pr(\Delta y_t<0| S_{t-1}=1)= (1-\phi)/(2-\phi)$ making $Pr(ETS|S_{t-1}=1)= \phi^2(1+\phi)$ and $Pr(ETS|S_{t-1}=0)= (1-\phi)^2(2-\phi)$. Use of equation (3) then shows that $Pr(S_{t-1}=1)= (1-\phi)^2(1+\phi)/(1-\phi+\phi^2)$ making $Pr(peak)= (1-\phi)^2\phi^2/(1-\phi+\phi^2)$ and the average duration of the cycle will be
$3(1-\phi+\phi^2)/ (1-\phi)^2\phi^2$.

More generally, the $Pr(ETS|S_{t-1}=1)$ and $Pr(CTS|S_{t-1}=0)$ will depend upon other features of Δy_t, such as serial correlation in it. Because this is so it is useful to examine many questions relating to the modeling of Δy_t through their effects upon these terms. One example is to study the relationship between the states S_t and the latent states ξ_t found in popular statistical models of the cycle such as Hamilton (1989). Clearly a terminating sequence involves a switch in the state S_t and a statement of the rules governing termination is equivalent to specifying a set of transition probabilities for the S states. Take the simple case that ETS = $\{\Delta y_t<0 \}$ and CTS=$\{\Delta y_t>0\}$. Then

$$Pr[S_t= 1 | S_{t-1}=1]=1-Pr(\Delta y_t<0)=p , \qquad (6)$$

$$Pr[S_t= 0 | S_{t-1}=0]=1-Pr(\Delta y_t>0)=q . \qquad (7)$$

If an extra assumption is made that the growth rate and its volatility varies with the state as

$$\Delta y_t = a_1S_t + a_0(1- S_t) + [S_t\sigma_1+(1-S_t)\sigma_0]\varepsilon_t , \qquad (8)$$

where ε_t is n.i.d. (0, 1), we would have Hamilton's (1989) model of the cycle when S_t are replaced by the latent states ξ_t. Given that the same format (6)-(8) is involved in both the dating rules and Hamilton's model, there is clearly some connection between the two approaches. Differences might simply reside in the values of p and q, but they might also be located in the auxiliary assumption in (8). In the simple dating rule being examined, $S_t=1$ if $\Delta y_t>0$ and so $E[\Delta y_t|S_t=1]= E[\Delta y_t|\Delta y_t>0]$. Consequently, it cannot be the case that ε_t is n.i.d., as the situation is equivalent to that which occurs with sample selection. Therefore, the states S_t generated by a set of rules such as BBQ will differ from the ξ_t and it is unwise to treat statistics derived from the S_t as also pertaining to the ξ_t (and conversely). In particular, the lengths of phases corresponding to ξ_t , and the possibility of duration dependence within the ξ_t states, cannot be answered by examining these issues with the S_t, unless some allowance is made for the selection bias. These questions are further investigated in Pagan(1998).

The terminating rules can also be useful when looking at issues of "co-movement". The index of coherence introduced earlier is a useful way of thinking about whether variables "co-move", in the sense that they have common cycles. Because a fundamental determinant of the length of a classical cycle is the ratio μ_y/σ, to get common cycles i.e. for turning points to cluster together, it will be necessary that the ratios for any two series being compared – y_{jt} and y_{rt} - must be of similar magnitude. Moreover, having a common "serial correlation feature" in Δy_{jt} and Δy_{rt} will also be useful, as serial correlation in growth rates will be a determinant of the length of a cycle through its effect upon the probability of the event defining a turning point. But this cannot be the end of the story. Having the same univariate processes for Δy_{jt} and Δy_{rt} tends to make for the same cycle lengths but does not ensure that the turning points will occur together. To examine the forces acting on the latter phenomenon, take the simplest termination rule. A peak in both series occurs at the same point if the event $T = \{\Delta y_{jt}<0, \Delta y_{rt}<0\}$occurs. Hence to get turning points to coincide one would wish to maximize the probability of this event occurring. Obviously, a crucial element will be the $cov(\Delta y_{jt}, \Delta y_{rt})$ and one would normally expect that, as this rises, the prob (T) will rise. It is here that a common factor in the two growth rates proves useful since it will tend to raise the correlation between them. It is not necessary though; one could still have a high correlation between the two series without a common factor being present. This line of thought suggests that co-integration between y_{jt} and y_{rt} would also maximize the chances of having common turning points in y_{jt} and y_{rt}, since that introduces a common factor and, if the permanent shocks have much higher variance than the transitory ones, Δy_{jt} and Δy_{rt} will tend to have the same serial correlation structure and variances.[19]

4. Dissecting the Cycle

Having established the fact that the ratio (μ_y/σ) is important for the cycle, it is natural to estimate that quantity and to see if it provides an explanation of the observed business cycle characteristics established earlier. Table 3 summarizes information on μ_y,σ and (μ_y/σ) using the same sample periods as underlie Tables 1 and 2.

Table 3

Moments of Growth Rates in Various Series

	μ_y(%)	σ(%)	(μ_y/σ)
US GDP	.81	1.07	.76
UK GDP	.58	1.04	.56
Aust GDP	.93	1.36	.68
US Cons	.83	.74	1.12
US Inv	.98	5.0	.20

We will use Table 3 later but, for purposes of comparison with some published work, it will also be useful to use the period 1952/1-1984/4 for estimating μ_y and σ for the US. These are .82 and 1.15. The difference between these estimates and those in Table 3 is largely a reduction in volatility, see McConnell and Perez-Quiros (1998). It might also be thought that the implied trend growth rate seems high; today estimates of the long-run growth potential of the US economy are closer to .65% per quarter rather than .8

Adopting the stance that it is the ratio (μ_y/σ) which controls much of the nature of the cycle, Table 3 might be used in order to interpret the outcomes in Tables 1 and 2. There are some clear successes – the relative length of the cycles of consumption, investment and US GDP for example, but in other instances the mapping seems to be

[19] In this discussion we have assumed that μ was the same for both series i.e. the series need to co-trend as well as to co-integrate. If the μ's are very disparate then it would be improbable that a bunching of turning points would occur.

much more imperfect. However, one has to exercise some care, since Tables 1 and 2 involve censoring of durations, while the simple sequence rules, from which the importance of (μ_y/σ) was derived, do not incorporate it. Table 4 below shows the cycle characteristics one would get from simulating data from (4) with μ_y and σ set to their 1947/1-1997/1 sample estimates. A comparison of the second and third columns shows the impact of censoring; clearly such actions have a major impact and cannot be ignored if one wishes to compare the business cycle characteristics of some model to those established by institutions such as the NBER. A comparison of columns one and two also shows that the pure random walk model of the log of GDP does pretty well at capturing the main features of the cycle, in particular the durations of the phases, the cycle length, and the asymmetry between expansions and contractions. To a first approximation then, one could provide an analysis of the business cycle simply by asking what it is that determines μ_y and σ. If there is a discrepency between the simulated and actual features found from the random walk simulations it lies in the inability to fully capture the shapes of the expansion phases. Table 5 provides the same statistics for the UK and Australia using the values of μ and σ in Table 3, and employing the censored version of BBQ as the dating algorithm.

Table 4

Simulated Business Cycle Characteristics, Various Models for US GDP

Mean Duration (quarters)	Data	RW(Cen)	RW(Unc en).	RW+Ser Corr
PT	3.0	2.8	2.3	3.3
TP	17.8	23.6	16.4	17.9
Mean Amplitude (%)				
PT	-2.5	-1.5*	-1.5*	-2.0
TP	20.2	22.9	16.6	19.0
Cumulation (%)				
PT	-4.1	-2.4*	-1.8*	-4.0
TP	256	471	254	297
Excess (%)				
PT	-0.1	-0.0	0.0	0.0
TP	1.1	-0.0*	-0.0*	-0.0*

* Indicates that less than 5% of simulations were further out in the tail relative to the data estimate.

Table 5

Simulated Business Cycle Characteristics for the UK and Australian GDP

Mean Duration (quarters)	UK Data	UK Sim	Aust Data	Aust Sim
PT	3.75	3.1	3.33	2.9
TP	16.1	13.9	20.6	19.7
Mean Amplitude (%)				
PT	-2.5	-1.7*	-2.2	-2.1
TP	14.5	11.5	24.7	23.0
Cumulation (%)				
PT	-6.1	-3.1*	-4.0	-3.4
TP	196	137	320	386
Excess (%)				
PT	0.0	-0.0	0.1	0.0
TP	0.7	0.0*	1.0	0.0*

* Indicates that less than 5% of simulations were further out in the tail relative to the data estimate.

Suppose we focus upon the prediction of cycle length for each country based on the random walk with drift model. Both the UK and Australian predictions are quite close to reality. In contrast, the predicted length of the US cycle is closer to that of Australia's rather then the UK one it matches in reality. This suggests that there is another factor at work which is missing in the description of output growth in the US. Indeed, as we observed earlier, the UK and Australian growth rates show no serial correlation, whereas the US does, leading to the suspicion that this difference plays a role in explaining the evidence on cycle length. Consequently, let us examine what happens when there is serial correlation in growth rates of the form

$$\Delta y_t = \mu_y (1-\rho) + \rho \Delta y_{t-1} + \sigma_\varepsilon \varepsilon(t), \qquad (9)$$

where $\varepsilon(t)$ is n.i.d. (0,1). This model was fitted to the quarterly growth rate in US GDP over the period 1947/1-1997/1 and simulations were then performed with the resulting estimates of μ_y , σ_ε and ρ (.81, 1.01 and .34 respectively). These results are presented in column 4 of Table 4. Comparing these results to column two shows that the presence of positive serial correlation in growth rates makes for shorter cycles. The origin of this result is most clearly understood by thinking about what would happen were one of the sequence rules given earlier to be used for dating the turning points. When there is positive serial correlation between adjacent growth rates the probability of getting two negative outcomes is greater than when they were independent, and this should have the effect of producing more turning points, a shorter cycle, and shorter expansions. Accordingly, the prediction of a lengthy cycle coming from a high value of (μ_y/σ) needs to be revised downwards. With serial correlation present there is quite a good match with US data. Consequently, some positive serial correlation in Δy_t is useful in explaining the cycle, but its main features can be explained without such an effect, and that fact enables one to appreciate why King and Plosser (1994) found that RBC models replicated business cycle features pretty well. Conclusions such as Wen (1998, p 186)

> "This indicates that standard RBC models lack the necessary propagation mechanism to generate movements around business cycle frequencies. In other words, business cycles do not really exist in current real-business-cycle models",

are correct in pointing to the need for serial correlation in growth rates to get a cycle that is close to reality, but quite incorrect in saying that a model without such a feature does not have a cycle. The invalid conclusion stems from a failure to define a business cycle in the appropriate manner. As we have emphasized earlier, the concentration in that article upon whether there is a peak in the spectral density of output movements "at the business cycle frequencies" is an irrelevant one for whether there is a business cycle; the latter is produced even with no peaks in the spectral density of $\Delta y(t)$ at any frequency.

If there is any notable deficiency in all the models discussed above, it relates to the shapes of expansions and the ability to reproduce cumulated movements. Given that these models are linear it is natural to ask whether some non-linearity might be helpful in explaining such "fine details" of the cycle. A problem which arises in any such inquiry is how to choose suitable representatives of the non-linear approach. There is a huge literature on this. One stream stems from chaotic phenomena and is best classified as "gee whiz" research, with little or no attempt to bring evidence to bear upon the utility of the models. It is hard to know what to say about such work. A second stream concentrates upon non-linear stochastic models of GDP growth and does have some foundation in evidence. Hess and Iwata (1997) provide a collection of these.[20]

An appealing class of models is that of Hamilton (1989); this class has been used in many applied studies describing economic fluctuations and it sometimes forms the foundation for theoretical models as well. In his model the state of the economy ξ_t is a two-state Markov Chain process with transition probabilities $\Pr[\xi_t =1|\xi_{t-1} =1]=p$ and $\Pr[\xi_t =0|\xi_{t-1} =0] = q$. The basic model produces a statistical representation of Δy_t which exhibits serial correlation and also makes $E[\Delta y_t| \Delta y_{t-1}]$ a non-linear function of Δy_{t-1}. Timmermann (1998) derives $E[\Delta y_t \Delta y_{t-1}]$ as $c_1^2\pi_1 (1-\pi_1)(p+q-1)$, where π_1 is the unconditional probability of being in the first state. Consequently, such a model can account for some observed serial correlation in growth patterns, and the implied non-linear conditional mean may create some extra movement that is useful in replicating certain elements of the business cycle. Extensions of the basic model have also been proposed. For example, Durland and McCurdy (1994) suggested that the transition probabilities p and q be dependent upon how long the economy has been in a given state i.e. the $\Pr[\xi_t =1|\xi_{t-1} =1]$ might depend upon how long the expansion had been in existence at time t (d_{et}). They assume that the modified p has the logit form

$$\Pr[\xi_t =1|\xi_{t-1} =1] =\exp(a+bd_{et})/(1+\exp(a+bd_{et})), \qquad (10)$$

with a similar expression for q.

Both the basic Hamilton model and one exhibiting duration dependence have been fitted by Durland and McCurdy over the period 1952/2-1984/4, and the parameter estimates in their paper may be used to determine the nature of the non-linearity in Δy_t through a computation of $E[\Delta y_t|\Delta y_{t-1}]$ at a range of values for Δy_{t-1}.[21] 15,000 observations

[20] Hess and Iwata also study the ability of these models to produce selected characteristics but their definitions of turning points are not standard and do not produce cycle characteristics that are familiar. Nevertheless, given their definitions, it seemed as if the non-linear models they studied did not improve much on linear models when it came to explaining the cycle.

[21] Durland and McCurdy actually fitted a non-linear AR(4) process. Since none of the AR coefficients were significantly different from zero we set them to that value. Note that the latent state process implies that there is serial correlation in Δy_t so that it may not be surprising that extra linear terms are not needed.

were generated from these two models and $E[\Delta y_t|\Delta y_{t-1}]$ was then non-parametrically estimated by kernel methods for each model. Fig 2 shows a plot of both conditional means against values of Δy_{t-1}, along with the linear forecast function obtained from a regression of Δy_t against Δy_{t-1}. There is little doubt that predictions made of the following period's growth rate using current period information will differ between the two latent state models, and that the model featuring duration dependence produces markedly different outcomes when the economy is in contraction. Even more striking is the gap between those two functions and that of the linear model. Given the latter feature, it is interesting to investigate whether these differences show up as business cycle characteristics. To assess that aspect, the simulated output from each of the models was passed through the BBQ dating algorithm and the results are given in Table 6.

Figure 2: Estimates of Conditional Mean of Growth Rate in US GDP, Various Models

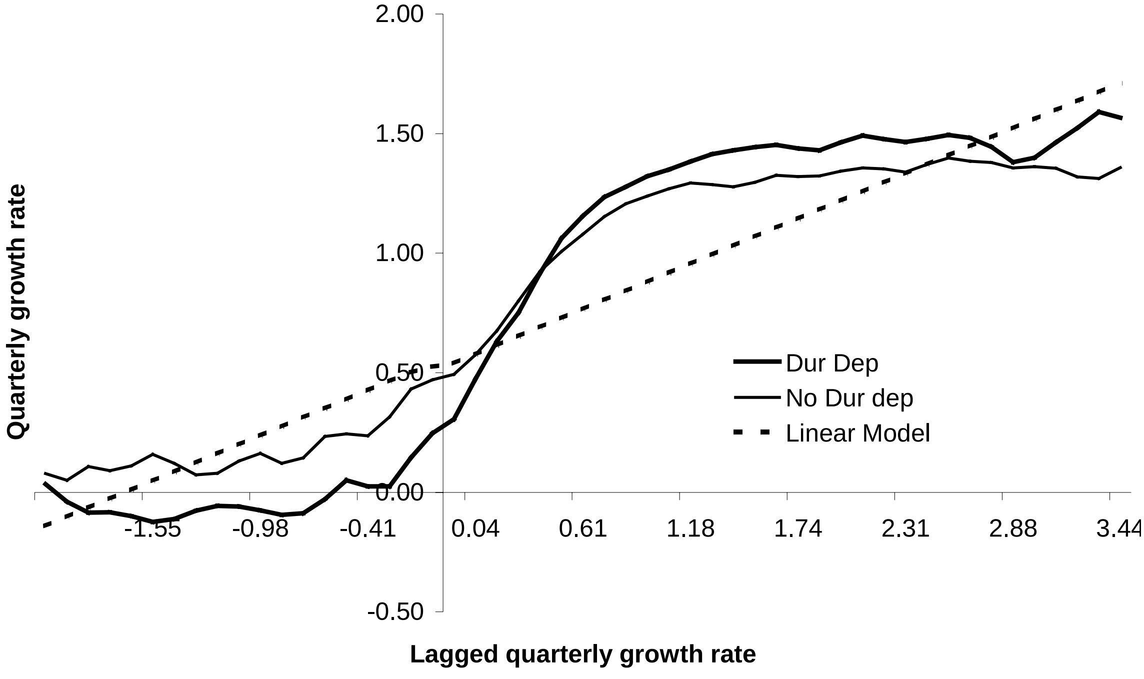


Table 6

Business Cycle Characteristics, Various Models

	Data	Hamilton	Dur Dep
Mean Duration (quarters)			
PT	3.0	4.4	4.8*
TP	17.8	20.0	16.9
Mean Amplitude (%)			
PT	-2.5	-2.8	-3.3*
TP	20.2	27.3	25.0
Cumulation (%)			
PT	-4.1	-8.2	-8.5*
TP	256	496	293
Excess (%)			
PT	-0.1	0.0	0.0
TP	1.1	-0.0*	0.0*

* Indicates that less than 5% of simulations were further out in the tail relative to the data estimate.

Introducing duration dependence certainly has an impact. In particular, contractions tend to last longer and the cumulated losses are higher than with the random walk plus drift model. One might have expected this, given the shape of the non-parametric means for both models. However, the models seem to go too far, producing cycles that are too extreme, particularly in relation to cumulated movements, and do little to get the shape of expansions right.[22] Given that these models are chosen over random walk models by statistical tests, such an outcome was a little unexpected, but it does serve to show that adding non-linear structure to the conditional moments can have a powerful effect upon cycle characteristics, albeit they may be undesirable.

So far it is the question of whether univariate statistical models have the ability to generate the reference cycle which has held center-stage. By and large our conclusion has been that very simple models can produce realistic cycles. It is inevitable then that the information in specific cycles will need to be examined if cycle theories are to be differentiated. Following the same strategy as adopted for univariate models, it is necessary to establish a relationship between multivariate statistical models and multivariate measures of the cycle. The obvious multivariate statistical model to start

[22] Hess and Iwata's (1997) results can also be interpreted in this way.

with would be a VAR in the differences of the extended set of variables. A popular set is the trio in Table 2. Our strategy involved fitting a VAR(2) to a sample of observations on these and then simulating data from the fitted VAR. The simulated series can then be used to describe the cycle characteristics for US GDP and investment. An argument that can be advanced to justify this choice is that many theoretical models of fluctuations have as their objective the reproduction of the VAR parameters found from the data. Consequently, the simulations we are performing can be taken as representing what would be found from such models if they were completely successful in their objective.[23]

As for the analysis conducted with univariate models, the determinants of the statistics in Table 7 will be the means and variances of the growth rates, along with any multivariate serial correlation in them i.e. the turning points in specific cycles will depend upon the joint moments of the random variables underlying them. The extent of clustering in the turning points will be reflected in the values of the indices of concordance and, in turn, these will depend upon the covariances between the VAR errors, since such indices fundamentally depend upon the joint probability, $\Pr(\Delta y_{jt}<0, \Delta y_{rt}<0)$. It is interesting to note that fitting an AR(1) to the simulated VAR output on GDP growth, and averaging the parameters over 1000 replications, produced estimates for a univariate model of output growth that were extremely close to those from the data sample. However, this does not mean that the cycles generated by a univariate model will be the same as from a multivariate one, since the dating algorithms are functionals of the joint *probabilities* of sequences of events, and the latter may well differ between univariate and multivariate models, due to the fact that the building blocks of such computations are conditional densities with differing information sets. Table 7 gives the results. By and large one can conclude that any theoretical model that was capable of replicating the VAR in growth rates would do well in explaining specific cycles. Moreover, there is a good match between the extent to which cycles cohere, as the average indices of concordance between consumption and investment relative to output are .85 and .73, versus the .87 and .76 of the US data.

[23] Harding and Pagan (1998) perform the simulations with some well-known theoretical models.

Table 7

Simulated Business Cycle Characteristics for US GDP and Investment from VAR

Mean Duration (quarters)	GDP Data	GDP Sim	Inv Data	Inv Sim
PT	3	3.2	5.3	4.8
TP	17.8	23.2	10.3	7.7
Mean Amplitude (%)				
PT	-2.5	-1.8	-22.6	-16.8
TP	20.2	23.3	35.2	30.2
Cumulation (%)				
PT	-4.1	-3.8	-53.2	-58.8
TP	256	549	233	198
Excess (%)				
PT	-0.1	0.0	1.6	0.0*
TP	1.1	-0.0*	2.4	-0.1*

* Indicates that less than 5% of simulations were further out in the tail relative to the data estimate.

5. Conclusion

To dissect a cycle one first needs to define it. In this paper we adopt the stance that the business cycle is defined by the turning points in aggregate economic activity, so that any statistical analysis of it requires that one be able to define such events. Although there is no unique way of doing this, we adopted a method that corresponds quite closely to that used by the NBER when dating cycles in the level of activity. Once this definition has been made mathematically precise, it emerges that the statistical behavior of the growth rate of output determines the nature of the business cycle. Accordingly, by examining different parametric statistical models for output growth, we are able to dissect observed cycles according to the contributions made by trend growth, volatility and serial correlation in growth rates, and non-linear effects. Regarding the latter we find little evidence that non-linear effects are important to the nature of business cycles. The perspective on the business cycle which we expound was also used to discuss specific cycles in the components of output, and to suggest ways of defining concepts such as “pro-cyclicality” between the levels of two series.

Appendix

Classical Cycle Turning Point Comparisons, US Data
FIBCR and Quarterly Bry-Boschan Dating

P		T	
FIBCR	BBQ	FIBCR	BBQ
48/4	48/4	49/3	49/2
53/2	53/3	54/2	54/2
57/3	57/3	58/2	58/1
60/2	60/1	61/1	60/4
69/4	69/3	70/4	70/1
73/4	73/4	75/1	75/1
80/1	80/1	80/3	80/3
81/3	81/3	82/3	82/4
90/3	90/2	91/1	91/1

References

Australian Bureau of Statistics (1987), *A Guide to Smoothing Time Series- Estimates of "Trend"* , Catalogue No1316.0, 1987.

Boehm, E.A. (1998), "A Review of Some Methodological Issues in Identifying and Analysing Business Cycles", *Melbourne Institute Working Paper*, No. 26/98.

Boschan,C. and W.W. Ebanks (1978), "The Phase-Average Trend: A New Way of Measuring Growth", *Proceedings of the Business and Economic Statistics Section of the American Statistical Association*

Bry, G. and C. Boschan (1971*), Cyclical Analysis of Time Series: Selected Procedures and Computer Programs,* New York, NBER.

Burns, A.F. and W.C. Mitchell (1946), *Measuring Business Cycles,* New York, NBER

Burnside, C. (1998), "Detrending and Business Cycle Facts: A Comment", *Journal of Monetary Economics*, 41, 513-532.

Canova, F. (1994), "Detrending and Turning Points", *European Economic Review*, 38, 614-623.

Canova, F. (1998a), "Detrending and Business Cycle Facts", *Journal of Monetary Economics*, 41, 475-512.

Canova, F. (1998b), "Detrending and Business Cycle Facts: A User's Guide*", Journal of Monetary Economics*, 41, 533-540.

Canova (1999), "Does Detrending Matter for the Determination of the Reference Cycle and the Selection of Turning Points", *Economic Journal*, 109, 126-149.

Christiano L., T., and Fitzgerald, (1998) “The Business Cycle: Its still a Puzzle”, Federal-Reserve-Bank-of-Chicago-Economic-Perspectives, 4th Quarter, p56-83.

Cogley,T. and J. Nason (1995), “Output Dynamics in Real-Business-Cycle-Models”, *American Economic Review*, 85, 492-511.

Cooley, T.F. and E.C. Prescott (1995) "Economic Growth and Business Cycles", ch 1 of

Cooley, T.F. and E.C. Prescott (eds) *Frontiers of Business Cycle Research* Princeton University Press.

Diebold, F.X. and G.D. Rudebusch (1996), “Measuring Business Cycles: A Modern Perspective”, *Review of Economics and Statistics*, 78, 67-77.

Durland, J.M and T.H. McCurdy (1994), “Duration-Dependent Transitions in A Markov Model of US GNP Growth”, *Journal of Business and Economic Statistics*, 12, 279-288.

Epstein, P. (1998), “Wesley Mitchell’s Grand Design and its Critics”, *Journal of Economic Issues* (forthcoming)

FIBCR, (1998), International Economic Indicators – A Monthly Outlook, January/February, Vol. 21, Nos.1-2.

Hamilton, J.D. (1989), “A New Approach to the Economic Analysis of Non-Stationary Times Series and the Business Cycle”, *Econometrica*, 57, 357-384.

Harding, D. (1997), “The Definition, Dating and Duration of Cycles”, Paper presented to the Melbourne Institute conference on *Business Cycles: Policy and Analysis*.

Harding, D. and A.R. Pagan (1998), “Knowing the Cycle”, paper presented at the conference on *Theory and Evidence in Macroeconomics*, University of Bergamo.

Hess, G.D. and S. Iwata (1997), “Measuring and Comparing Business-Cycle Features”, *Journal of Business and Economic Statistics*, 15, 432-444.

King, K.H. , R.A, Buckle and V. Hall (1994), “Key Features of New Zealand Business Cycles”, *Economic Record*, 70, 56-72.

King, R.G. and C.I. Plosser (1994), “Real Business Cycles and the Test of the Adelmans”, *Journal of Monetary Economics*, 33, 405-438.

Koopmans. T.J. (1947), “Measurement without Theory”, *Review of Economics and Statistics*, 29, 161-172.

Kydland. F.E. and E.C. Prescott (1990), “Business Cycles: Real Facts and a Monetary Myth”, *Federal Reserve Bank of Minnesota Quarterly Review*, Spring, 383-398.

Lucas (1981), “Methods and Problems in Business Cycle Theory in R.E. Lucas *Studies in Business Cycle Theory*, 1981, M.I.T. Press.

McConnell, M. and G.P. Quiros (1998), “Output Fluctuations in the United States: What Has Changed Since the Early 1980’s”, *Staff Report no. 41*, Federal Reserve Bank of New York.

Mintz, I (1972), "Dating American Growth Cycles", in V. Zarnowitz (ed) *The Business Cycle Today*, NBER, New York, p 39-88

Pagan, A.R. (1997a), Towards an Understanding of Some Business Cycle Characteristics", *Australian Economic Review*, 30, 1-15.

Pagan, A. R. (1997b), "Policy, Theory and the Cycle", *Oxford Review of Economic Policy*, 13, 19-33.

Pagan, A.R. (1998), "Bulls and Bears", Walras-Bowley Lecture given at the North American Summer Meetings of the Econometric Society, Montreal, June.

Phillips, P.C.B. (1998), "New Tools for Understanding Spurious Regressions" *Econometrica* (forthcoming)

Scutella, R. (1996), "Obtaining an Efficient Estimate of the True Level of GDP", Honours Research Essay, Melbourne University

Sichel, D.E. (1994), "Inventories and the Three Phases of the Business Cycle", *Journal of Business and Economic Statistics*, 12, 269-277.

Simkins, S.P. (1994), "Do Real Business Cycle Models Really Exhibit Business Cycle Behavior", *Journal of Monetary Economics*, 33, 381-404.

Slutsky, E.E. (1937), "The Summation of Random Causes as the Source of Cyclic Processes", Econometrica, 5, 105-146.

Stock, J.H. and M.W. Watson (1991), "A Probability Model of the Coincident Economic Indicators", in K. Lahiri and G.H. Moore,(eds) *Leading Economic Indicators*, Cambridge University Press, Cambridge.

Stock, J.H. and M.W. Watson (1998), "Business Cycle Fluctuations in US Macroeconomic Time Series", *Working Paper No. 6528*, National Bureau of Economic Research.

Timmermann, A. (1998), "Moments of Markov Switching Models", mimeo, University of California at San Diego

Tinbergen, J (1939), *Statistical Testing of Business Cycle Theories*, 2 vols, Geneva, League of Nations.

Wecker, W. (1979), "Predicting the Turning Points of a Time Series", *Journal of Business*, 52, 35-50.

Wen, Y. (1998), "Can a Real Business Cycle Model Pass the Watson Test", *Journal of Monetary Economics*, 42, 185-203.