Knowing the Cycle*

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ABSTRACT

Policy makers are primarily interested in fluctuations in the level of activity — the classical cycle. Academics have in recent times focused their efforts on studying fluctuations and co-movement in aggregate variables that have been rendered stationary after some appropriate transformation. That is academics focus on the growth cycle. One reason for this shift in focus was the impression among academics that Burns and Mitchell's work lacked the precision required in modern macroeconomics. In this paper we show that pattern recognition algorithms which emulate Burns and Mitchell's approach to the cycle can be constructed and used to collect precise information on the classical cycle. The information so marshaled comprises the duration, amplitude, and cumulative movements of output within business cycle phases. We show that this information can be used to assess a range of business cycle models that have been proposed in the literature.

1. Introduction

Judged by the number of papers being published today with “the business cycle” somewhere in their title, we should be starting to have a good understanding of what accounts for that phenomenon. Yet, to paraphrase Christiano and Fitzgerald (1998), the business cycle remains a puzzle. Moreover, if one surveyed policy makers whose concerns are with the business cycle, it seems unlikely that many would agree that this explosion of academic articles has been of much use to them in their decisions. Given that the stimulus for much research on the cycle is supposedly to improve policy actions directed towards it, the latter state of affairs is rather unusual. What could cause it to happen? One possibility, suggested by Lucas (1981, p18), is that the phenomenon is intrinsically difficult to understand.

“The nature of the questions to which we want answers, the level of theorizing at which there seems to be any real hope of obtaining reliable answers, and the equipment at hand for theorizing at this level combine to make genuine progress painfully slow”.

There is undoubtedly some truth in this. Macroeconomic phenomena are inherently complex. But, complexity cannot be the whole story. Anyone who has spent time in both the academic and policy communities knows that there is another problem arising from the two groups tending to have a different perspective on, and way of talking about the cycle. A different perspective because the focus of policymakers is largely upon what has been termed the ‘classical cycle’ in the levels of economic activity, whereas academic research has increasingly moved towards examining cycles in data which have been subject to a rather complex process of trend removal. Thus the attention of academics is fixed on something that does not enter into the policy maker’s calculus. In this transition, information needed to address questions relating to the classical cycle has been lost and potentially valuable information that might be gained from academic research has been sacrificed. A different way of talking about the cycle emerges from the fact that policy makers largely follow Burns and Mitchell (1946) in paying attention to the *turning points* in economic activity, while academics concentrate upon the *moments* of random variables taken to represent economic activity. Perhaps the contrast just made is too stark. In fact, academics are sometimes a little schizophrenic when talking about the cycle and are not averse to motivating their research by reference to published evidence on turning points even if it is never central to their later investigation e.g. Christiano and Fitzgerald (1998).

Section 2 sets out our attitude to some of the issues noted above. It deals with the definition of a cycle, and the business cycle in particular. In it we define a cycle in terms of the turning points of a series, this being the methodology set out in Burns and Mitchell (1946), and we argue that the classical cycle is *the* business cycle. Section 3 has three sub-sections. The first describes methods for locating turning points while the second outlines how such information may be converted into measures that are useful when thinking about the nature of the cycle. The final sub-section takes up the issue of defining the cycle in terms of "co-movements" and discusses how this fits into our schema.

Section 4 constructs some evidence on the cycle that is useful when faced with the need to evaluate theories of the cycle. Some standard theories are examined according to their predictions about such quantities. Theories are stories about the way in which the cycle arises. Many of these stories involve an act of faith, in that they ultimately attribute the cycle to forces that are exogenous to the economic system, and it is natural that new stories are arising which seek to remove such a constraint. We briefly describe some of these and ask how successful they have been in accounting for the evidence.

2. What should we be trying to explain?

Empirical research involves matching phenomena that theory predicts should be found in a population with the sample analog of those phenomena. If many of the phenomena that the theory predicts are ultimately found to be in the sample then our confidence in the theory is increased. Theories that can explain more of the phenomena in the sample than other theories are generally preferred, although, parsimony and rigor of the explanation are also important criteria.

Macroeconomic data show three temporal phenomena that require explanation. The most notable of these is that prosperity and population increase on average, thereby introducing trends into most macroeconomic data. Macroeconomic data also exhibits the somewhat subtler phenomenon of fluctuation. The third and most subtle phenomenon is that macroeconomic series seem to show visual patterns which are referred to as 'cycles'. Actually, although called 'cycles', these patterns are better thought of as recurrences since they involve recurring phases of expansion and contraction that may have no definite periodicity. Ultimately, we seek to jointly explain these three phenomena.

Burns and Mitchell followed a long tradition and defined the cycle in terms of the peaks and troughs in the *level* of variables that measure economic activity i.e. they studied the "classical" cycle. However, a discordance arises between what Burns and Mitchell did, and what much of the academic literature does today, owing to the fact that the emphasis placed upon turning point calculations by Burns and Mitchell has been discarded in favour of computing moments of the data. Indeed, some see this as a crucial distinction. Jacobs (1998, p2), for example, feels that a study of the business cycle always involves an analysis of turning points, while the academic literature focuses upon what he terms "economic fluctuations". The latter is summarized by the moments of the random variables taken to underlie the series representing economic activity. After jettisoning the Burns and Mitchell approach to the business cycle in favour of studying economic fluctuations, academic researchers immediately ran into the difficulty that data on real quantities are almost always non-stationary and moments needed to be computed from data made stationary through some transformation. So the series taken to represent economic activity was transformed and the classical cycle in the levels of activity was lost. Consequently, academic research on fluctuations in the past quarter century has matched theory and evidence using quite a different data set to that which business cycle analysts of much of this century would have used.

Which cycle should we study? Mathematically, three types of cycles might be distinguished. As mentioned earlier the classical cycle pertains to the turning point patterns seen in the log of the level of economic activity, designated by y(t). Other cycles that might be looked at derive from the turning points of y(t) after removing different types of trends. For example, the turning points could be located in either $z_d(t)=y(t)-T_d(t)$, where $T_d(t)$ is a deterministic trend, or $z_{st}(t)=y(t)-T_d(t)-T_s(t)$, where $T_s(t)$ is a stochastic one. As it is possible to extract these different trends in many ways there will be a myriad of cycles. An extensive literature has demonstrated that fact e.g. Canova (1994), (1998). To see the differences between the different cycles consider analysing post-WW2 U.S. data by varying the nature of how trend is accounted for. The classical cycle corresponds to the NBER definition and, according to their datings, there were 8 post-war cycles of average duration of 62 months, with expansions absorbing 51 of these months. If one detrends using the phase-averaging method, thereby producing NBER-type growth cycles, the period 1948-1997 witnessed twelve growth cycles of length 46 months. [1] Expansions and contractions in this growth cycle are of equal size (FIBCR, 1998 p52). Finally, removing both a deterministic and a stochastic trend using the popular HP filter (λ=1600) results in cycles of 30 months duration and such cycles also exhibit expansions and contractions of equal length. Consequently, given the disparity in the characteristics of the cycles produced by varying the trend extraction filter it is somewhat extraordinary to read articles in which the validity of work done with trend adjusted series is justified by reference to classical cycle characteristics. [2] In fact, academic researchers using HP filtered data should be asking whether they are producing a three year cycle, not the six year one that they are familiar with from the NBER data on the classical cycle. In short, which cycle we work with makes a great deal of difference to the business cycle "facts". By far the best-documented cycle is the classical version, and it is certainly the one which gathers most attention in policy and media discussion, so one might expect that it would receive most attention by academics.

We see four arguments in favour of judging business cycle research by how well it explains the classical cycle. First, almost all models of the cycle explain the levels of series through their use of forcing processes that have both a deterministic and a stochastic trend. Since these assumptions are an important part of the model, entering into

[1] Impressed by the fact that countries such as Germany did not have a classical recession in the post War II period until March 1966, NBER researchers themselves - such as Mintz (1969, 1972) - felt that it was productive to investigate *growth cycles* for such countries. These were cycles in the *level of detrended economic activity,* where the trend was measured with the "phase-averaging" method . Until recently, this was also the main focus of attention for research on Asian economies. But the growth cycle has rarely been of much interest in most Western countries. With the exception of those countries whose capital stock was destroyed in WW2, Western countries had generally gone through the "catch-up" phase of high growth many decades before.

[2] For example Christiano and Fitzgerald (1998, p. 58).

both the calibration procedures and the choices of agents, it does not seem unreasonable to ask whether the output of such models can explain the classical cycle. Second, the classical cycle summarizes the interaction between trend and fluctuations and therefore provides a unique opportunity to assess how well particular theories have captured trend and fluctuation and the interaction between them. Third, apart from questions of relevance, gathering information on the phases – expansions and contractions – of the classical cycle involves fewer subjective decisions than for the growth cycle, in that the former is independent of the method of de-trending and thereby of the perspective of the observer. For this reason the classical cycle provides the opportunity to gather statistics that can be unambiguously called business cycle facts. Because of the problem of trend removal the growth cycle does not offer this opportunity. Finally, policy makers and the business community alike seek information on the classical cycle.

3 Gathering evidence on the business cycle

3.1 Finding turning points

The detection and description of the classical and growth cycles is accomplished by first isolating turning points in the series, after which those dates are used to mark off periods of expansions and contractions. Viewed in this light business cycle analysis involves pattern recognition techniques and this fact goes to the heart of how one learns about the business cycle from $y(t)$ and $z(t)$.

As mentioned earlier business cycle features are documented by recognising turning points in a series taken as representing aggregate activity. Location of such points can sometimes be done visually. When performing the datings in this way the eye is also very good at filtering out “false turning points” i.e. movements which are short lived. Translating the ocular judgments into an algorithm has proved to be challenging. At a minimum such an algorithm needs to perform three tasks.

1. Determination of a potential set of turning points i.e. the peaks and troughs in a series.
2. A procedure for ensuring that peaks and troughs alternate.
3. A set of rules that re-combine the turning points established after steps one and two in order to satisfy pre-determined criteria concerning the duration and amplitudes of phases and complete cycles; what we will refer to as "censoring rules".

There are a variety of ways to identify a potential set of peaks and troughs. For example, Wecker (1979) uses the rule $\{\Delta y(t)>0, \Delta y(t+1)<0, \Delta y(t+2)<0\}$ to locate a peak.[3] Essentially, these rules attempt to approximate the standard calculus definition that $dy/dt>0$ to the left of a peak and $dy/dt<0$ to the right of a peak. Harding (1997) introduces the concepts of expansion terminating sequences (ETS) and contraction terminating sequences (CTS) which are patterns that terminate expansions and contractions respectively. Several candidates for these sequences exist. For example, a rule popularized by Arthur Okun is that a recession involves at least two quarters of negative growth so that ETS= $\{ \Delta y(t+1)<0, \Delta y(t+2)<0\}$ signals a peak at time t. This rule is widely used in the media and policy circles to signal a classical recession. An extended version of Okun's procedure involves terminating an expansion when two quarters of negative growth are encountered and terminating a contraction when two periods of positive growth are encountered i.e. the ETS is just that given above while the CTS= $\{ \Delta y(t+1)>0, \Delta y(t+2)>0\}$. This is the rule that we use for dating purposes in this paper.

All these rules emphasise that, even though the classical cycle refers to the behaviour of the level of a variable, the analysis of its turning points is done with a stationary series viz. transformations of the first differenced series, $\Delta y(t)$, such as

[3] A trough would be defined by $\{\Delta y(t-1)<0 , \Delta y(t)<0, \Delta y(t+1)>0\}$. Pagan (1997) uses this rule to compute the average length of a cycle.

sgn($\Delta y(t)$).[4] It cannot be stressed too much that the rules above *are not locating a cycle in $\Delta y(t)$*; rather $\Delta y(t)$ is just an input into the dating process of the classical cycle. Moreover, it will be the characteristics of $\Delta y(t)$ which determine the nature of the classical cycle. To expand further on this theme it is useful to consider the case where GDP follows a random walk with drift μ_y , variance σ^2 and normally distributed innovations. In this instance the probability of obtaining one quarter of negative growth is $\Phi(-\mu_y/\sigma)$, where $\Phi(.)$ represents the cumulative normal distribution. Thus the ratio of drift to standard deviation is important for the cycle. The larger is μ_y/σ the fewer the turning points that will be observed in a series.

Having established the fact that the ratio (μ_y/σ) is likely to be important for the classical cycle, it is natural to estimate that quantity for later use when we seek to explain the observed business cycle. Table 1 summarizes information on μ_y, σ and (μ_y/σ) for a variety of U.S. series, using the sample period 1948q1 to 1997q1. A notable feature is that investment is highly volatile relative to trend growth and thus should exhibit a cycle which is very different to that for GDP and consumption.

Table 1: Moments of Growth Rates in Various Series

	μ	σ	μ/σ
GDP per capita	0.46	1.08	0.42
Consumption per capita	0.49	0.89	0.56
Investment per capita	0.51	5.75	0.09

3.2 Measuring the cycle using turning point information

Given that turning points have been established how should the dating information be used in conjunction with the series from which the dates were derived? Inspection of

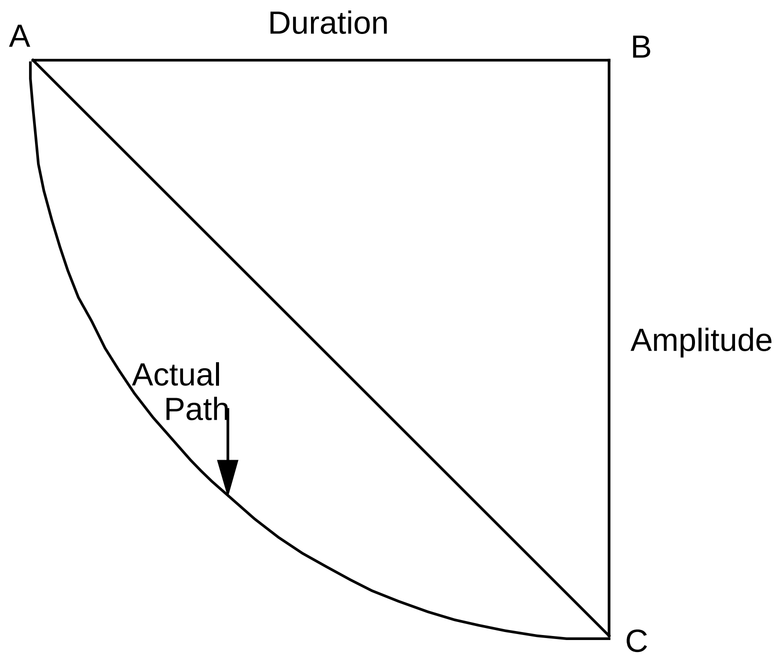
[4] When working with growth cycles it is $\Delta z(t)$ which is analysed. As we explain elsewhere, Harding and Pagan (1999), the Bry-Boschan (1971) algorithm for locating turning points involves the sign of "long differences" y(t)-y(t-k). Of course, these are the sum of first differences.

comments that are frequently made about the cycle suggests that there are four items of interest.

- The duration of the cycle and its phases
- The amplitude of the cycle and its phases
- Any asymmetric behavior of the phases
- Cumulative movements within phases.

In thinking about these measures, it is useful to consider a phase as a triangle. Fig 1 shows a stylized recession with A being the peak and C the trough. The height of the triangle is the amplitude and the base is the duration. Knowledge of these two elements for any cycle enables one to compute the area of the triangle, and thereby an approximation to (say) the cumulated losses in output from peak to trough, relative to the previous peak. Designating the duration of the i'th phase as D_i and the amplitude as A_i , the product $C_{Ti} =.5(D_i A_i)$ will be referred to as the *"triangle approximation" to the cumulative movements*. In practice the *actual cumulative movements* (C_i) may differ from C_{Ti} since the actual path through the phase may not be well approximated by a triangle, and this points to the need for an index of the average *excess cumulative movements;* the natural candidate is $E_i = (C_{Ti} – C_i+0.5*A_i)/D_i$. In this formula D_i is the duration of the phase and the term $0.5*A_i$ removes the bias that arises in using a sum of rectangles (C_i) to approximate a triangle. Although it is C_i which is of fundamental interest to policy makers and historians, the triangle approximation is still likely to be useful in shedding light upon the ability of business cycle models to generate realistic cycles.

Figure 1: Stylized Recession Phase



3.3 Another View of the Cycle

There is no doubt that Burns and Mitchell used a wide range of series to come up with a single reference cycle.[5] This fact has led to an impression in the academic literature that what was important in discussing the business cycle were the inter-relationships (or co-movements) between the specific series used to construct the reference information. For example, Cooley and Prescott (1995, p26) summarize what they feel the implications of Burns and Mitchell's work was in the following way:

"…the one very regular feature of these fluctuations is the way variables move together. It is the co-movements of variables that Burns and Mitchell worked so hard to document and that Robert Lucas emphasized as the defining features of the business cycle".

[5] Epstein (1998) argues that Burns and Mitchell probably valued the specific cycle information as highly as that contained in the reference cycle i.e. the dispersion of the turning points in the specific series around the "central tendency" of the reference cycle was important to them.

Lucas' (1981) statements just invoked are:

"Technically, movements about trend in gross national product in any country can be well described by a stochastically disturbed difference equation of very low order. These movements do not exhibit uniformity of either period or amplitude....Those regularities which are observed are in the co-movements among different aggregative time series". (p 217)

"The central finding, of course, was the similarity of all peacetime cycles with one another, once variation in duration was controlled for, in the sense that each cycle exhibits about the same pattern of co-movements among variables as do the others". (p. 274)

What is strange here is the transformation in the motivation for considering many series. In Burns and Mitchell's case it was simply an instrument used to define *the* business cycle, through the way in which turning points in many series clustered together; in much of the modern literature it has become an end unto itself. In fact the latter's obsession with co-movements between series seems to miss the point of why we are interested in the business cycle. It is an extraordinary feature of much of the modern academic literature that one can find papers which provide extensive accounts of the co-movements of consumption, investment etc. but which make little or no reference to the temporal characteristics of the series that might be taken to be aggregate economic activity, namely output. "Hamlet without the Prince" is the phrase that comes to mind when reading such papers.

Apart from the fact that the modern literature departs from the older one for no apparent reason, what should be of equal concern is the fact that some of the statements quoted in defence of the movement have had a remarkable impact despite being close to contradictory. How exactly could Lucas conclude that there is no uniformity in temporal movements in output and yet be confident that there are uniform co-movements? It is certainly true that it is possible to conclude that individual cycles in activity differ in both duration and depth, through an analysis of their turning points, but then one should apply the same test to the question of co-movements, leading one to enquire into the evidence that such co-movements are stable across individual cycles. As mentioned above, the academic literature has mostly identified co-movements with covariances, and then estimated the latter with a sample period which includes many cycles. Hence, it is *assumed* that the co-movements are the same across cycles.[6] One cannot claim an empirical regularity from an assumption. Indeed, although rarely done, one might try to address the issue of whether the covariances are stable across cycles. An exercise in which an F test is performed to test the hypothesis that the slope coefficient of a regression of HP-detrended consumption upon HP-detrended output (λ=1600) is constant across the eight classical cycles identified in US data is resoundingly rejected (a p value of less than .000).[7]

Nevertheless, even if we are fundamentally interested in explaining the cycle in economic activity, it may be very hard to discriminate between theories simply of the basis of their predictions for this univariate series. Because most theories make predictions about the behavior of sub-aggregates such as consumption and investment, it makes sense to study the cycles in these variables as well. Each variable selected for investigation would have a set of cycle characteristics obtained by applying the dating rules discussed in the preceding section to each of the individual series. Later we use such information to shed light on the ability of models to replicate an actual economy.

4. Theory, Evidence and the Cycle

Textbooks are the obvious places to look in for distilled economic knowledge of the business cycle. A comprehensive text book discussion of the business cycle would involve a description of the cycle, an account of frameworks for analyzing the cycle and identification of the most important shocks. Although there may be more, we found four texts that meet these criteria: Sachs and Larrain (1993), Auerbach and Kotlikoff (1995), Romer (1996), and Blanchard and Fischer (1989). All of these texts use statistics from the NBER dating methodology to describe features of the cycle. The added value of such statistics is very evident on reading these texts.

The search for a theoretical explanation of economic phenomena is an enduring one. Theory endows an analyst with a framework for thinking about an issue; a common

[6] Kim et al (1994) and Fisher et al (1996) examine the stability of covariances across time.

[7] It might be argued that it is the signs of the covariances which are stable across cycles but it still remains true that one cannot ascertain this by pooling all cycles together as the complete sample does. Moreover, one might wonder about the value of economic research which assures us that consumption and income are positively correlated.

language to describe either what has happened or might have happened as a result of an action; the possibility of making quantitative statements about the latter; and the allocation of "names" to the driving forces underlying macroeconomic events. For much of the history of macroeconomics it was the first of these benefits that was perceived of as being paramount and the role of evidence was in fact quite limited. Today, the last two have come into their own. Macroeconomics has an air of precision about it which was not there over much of the past century. Joyful descriptions of the new era, such as the following - taken from Usabiaga's (1998) interview with Larry Christiano - abound.

> " It was exciting and it helped to put an end to the embarrassment that macroeconomists suffered at the hands of other economists who laughed at their primitive ways of doing economics"

Triumphalism like this inevitably breeds strong statements about the ability of the theory to explain macroeconomic phenomena such as the cycle and these too have abounded.[8] In regards to the latter, it is perhaps no accident that the sense of satisfaction expressed about the achievements of this new "quantitative theory" stem from its having paid more attention to the evidence than has been customary with the theories of the past and, indeed, even some of those of the present.

As evident from Christiano's comment above many believe that an important feature of the modern approach to analyzing economic fluctuations is that it shares the same framework as other areas of economics. In fact, the argument could even be strengthened by noting that most policy institutes use models that have as their core a set of principles that Christiano would probably deem to be an important legacy of the modern approach. These include the notions that shocks are the driving forces of the macro economy and that the economy should be viewed as growing along a balanced growth path with forward-looking agents making optimal decisions. These principles have become widely accepted, have been adopted in the conversations of those who follow the cycle, and are embodied in many policy models e.g. the MULTIMOD model used at the IMF, (Laxton et al (1998)). However, these common elements only produce a

[8] Although it is fair to say that Christiano is more reserved in such judgements. In the same interview that the quotation is taken from he sees the developments as important methodologically rather than for the knowledge gleaned from them about specific issues such as the cycle.

skeletal framework; filling in the details leads to heterogeneity, with many different choices being made about the way in which dynamics should be introduced, the naming of the shocks etc.

So we have seen a move towards parametric models featuring stochastic shocks as a primary vehicle for talking about the business cycle. However, accompanying this shift has been an increasing tendency in academia for the business cycle to be spoken of in terms of the parameters of these models, rather than the turning points of the series that they would generate, and this underpins the necessity to tease out the business cycle implications of the models in this alternative dimension. It is important to adopt a turning point perspective as the resulting business cycle statistics combine together the parameters of underlying theoretical models in a natural way. For example, while the knowledge that a model prediction about growth rate volatility differs from its counterpart in the data tells us something about the adequacy of that model in replicating that data, it doesn't tell us much about the importance of the discrepancy for the nature of the cycle. Producing statistics that directly address the ability of a model to replicate business cycle characteristics is an important objective of our work.

Ideally, we would like to map the parameters of any given model into quantities that are more informative about the cycle. To do that, we return to our basic contention that an understanding of the classical cycle derives from a description of what governs the temporal behavior of $\Delta y(t)$, and examine what theoretical work has to say about the latter. One immediately encounters the obstacle that few modern theories of the cycle provide quantitative evidence on this variable. Rather, the information provided is upon the characteristics of $z_{st}(t)$, and these do not readily map into those of $\Delta y(t)$. A simple illustration of the difficulties encountered in effecting the mapping is to be had by making $y(t)$ a pure random walk and constructing $z_{st}(t)$ from it with the HP filter. The serial correlation properties of $z_{st}(t)$ are extremely complex, in that there are negative serial correlation coefficients of very high order, and these serve to produce a spectral density of $z_{st}(t)$ with a peak, something which is certainly not descriptive of $\Delta y(t)$.[9] It is only if $z_d(t)$ is used in place of $z_{st}(t)$ that one might recover the classical cycle properties reasonably easily.[10]

We start with a benchmark model for $\Delta y(t)$ of the form

$$\Delta y(t) = \mu + \sigma e(t). \tag{1}$$

Using values of μ and σ found from US data on per capita GDP over the period 1948/1-1997/1, and assuming that e(t) is n.i.d.(0,1), we simulate business cycle statistics from (1) and compare them to the data. The first column of Table 2 has the latter while the second column contains the former. A simple extension of this model would be to consider what happens if there is serial correlation in growth rates i.e. the model has the form

$$\Delta y(t) = \mu_1 + \rho \Delta y(t-1) + \sigma_1 e(t). \tag{2}$$

In (1) and (2) the shocks are permanent. Instead one might treat them as only being persistent i.e. the data comes from a model that is stationary around trend. Defining z(t)=y(t)-a-bt such a model might be

$$z(t) = \rho_z z(t-1) + \sigma_z \varepsilon(t). \tag{3}$$

One can estimate the parameters of (2) and (3) from the data as well. Doing so, the third column of Table 2 contains simulated statistics pertaining to (2) and the remaining columns –designated as DT - relate to (3), parameterized with two sets of values for ρ_z and σ_z . One of these is from the data, ρ_z=.97; the other has a smaller value of ρ_z=.9. To make these two parameterizations comparable we keep std(z(t)) constant,

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[9] After applying the "Okun" rule to simulated data which has been passed through the HP filter, a cycle of around 30 months emerges, illustrating once again that the information generated from $z_{st}(t)$ is neither about the classical cycle nor the growth cycle appearing in (say) FIBCR publications.

[10] As an aside, even if one thought that the growth cycle was the most important cycle to study, and we don't, it would be $\Delta z(t)$ which one needs information upon.

where std(z(t)) = $\sigma_z/\sqrt{(1-\rho_z^2)}$, by varying σ_z. Thus the objective of the last two simulations is to study the effects of persistence without changing the volatility of z(t).

In Table 2 contractions are designated as PT and expansions as TP. With the exception of duration statistics, all measurements are made in terms of percentage changes. There are some striking results in the table. First the random walk model does an excellent job of matching classical cycle statistics on most dimensions. Its deficiencies are twofold. It fails to produce contractions that are deep and the shape of an average expansion is much closer to a triangle than that in reality. Rapid recovery in the early part of an expansion has been documented in Sichel (1994) and this is most likely the origin of the magnitude of the "excess" computations for expansions in table 2.[11]

Table 2: Actual and Simulated Business Cycle Characteristics

	Data	RW eqn 1	RW eqn 2	DT (ρ_z=.97)	DT (ρ_z=.9)
Mean Duration (quarters)					
PT	4.4	3.7	4.2	4.0	4.5
TP	13.9	11.8	10.3	9.8	7.5**
Mean Amplitude (%)					
PT	-3.4	-1.8**	-2.6	-2.00**	-4.1
TP	12.4	8.9	9.2	6.8**	8.3**
Cumulation (%)					
PT	-8.0	-4.3	-7.8	-5.0	-11.3**
TP	153	91	83	52	44.2**
Excess					
PT	-0.0	-0.1	-0.1	-0.1	-0.2
TP	1.4	0.1**	0.1**	0.1**	0.2**

** Indicates that less than 5% of simulations were further out in the tail relative to the data estimate.

[11] Note that the excess measure in the tables is found by averaging the E_i over all cycles rather than constructing it from the average values of C_i and C_{Ti}.

Adding in serial correlation to growth rates tends to improve the explanation of the cycle on some dimensions and worsen it on others. Attempts to replicate this feature are the motivating force in many studies of US GDP e.g. Ramey and Watson (1997). Comparing the results of columns two and three shows that the presence of positive serial correlation in growth rates makes for shorter cycles. The origin of this result is most clearly understood by thinking about the Okun rule. When there is positive serial correlation between adjacent growth rates the probability of getting two negative outcomes is greater than when they were independent, and this should have the effect of producing more turning points and a shorter cycle; just what is observed. Column four shows that one can do quite well with a model of output that does not have a stochastic trend but whose shocks are still quite persistent, while column five indicates that the degree of persistence is very important in getting the duration of the cycle and the length of expansions correct. Generally, a model without very persistent shocks will fail to match many of the characteristics.

As a companion to the statistical models simulated in Table 2 we present equivalent simulations of some popular theoretical models in Table 3. A difficulty one faces in moving to theoretical models is that such models ensure a steady state growth path of per capita output through a deterministic trend term within the forcing processes. Because the magnitude of this trend is unknown it is chosen to be that which replicates the observed deterministic trend in per capita output. Accordingly, we also force the simulated output to agree with the data on this dimension by adding on a trend term to ensure that all variables grow at the .456 % per quarter observed over 1948-1997. That rate is a little higher than the .4% used in King et al (1998). There are then four models to be investigated. First, there is the basic real business cycle (RBC) model set out in King, Plosser and Rebelo (1988), where technology is a unit root process. This configuration is termed RBC1. RBC2 is a variant of RBC1 wherein the standard deviation of the shocks to technology is set to 1.19% per quarter versus the 1.48% of RBC1; the latter was the level needed to reproduce the observed standard deviation of per capita quarterly GDP growth of 1.08%. It is interesting to observe that, with the unit root in technology, the variance of technology shocks needs to be larger than that for the growth in GDP. RBC3 is the model of Christiano and Eichenbaum (1992), as interpreted by Cogley and Nason (1995); this has a unit root in technology and a stationary government expenditure process. Finally, the last model, ENDO, is the endogenous growth model set out and calibrated in Collard (1999).

Table 3: Actual and Simulated Business Cycle Characteristics

	Data	RBC1	RBC2	RBC3	ENDO
Mean Duration (quarters)					
PT	4.4	3.9	3.6	3.8	3.9
TP	13.9	11.6	13.7	11.2	11.0
Mean Amplitude (%)					
PT	-3.4	-1.9**	-1.4**	-2.0**	-2.7
TP	12.4	9.0	9.2	8.9	11.3
Cumulation (%)					
PT	-8.0	-5.0	-3.1**	-5.0	-6.8
TP	153	95	117	86	103
Excess					
PT	-0.0	-0.1	-0.1	-0.1	-0.1
TP	1.4	0.1**	0.1**	0.1**	0.1**

** Indicates that less than 5% of simulations were further out in the tail relative to the data.

Table 3 shows that all models generate cycles that roughly accord with the data, although none can reproduce the shape of expansions. RBC2 manages to get the cumulated gains in output during the average expansion closest to reality but does so by having expansions that last too long. In many ways the endogenous growth model does best, albeit it understates the duration of expansions. Comparing RBC1 and RBC3 there is clearly little to be gained by adding in government expenditure shocks.

The business cycle characteristics just found for various statistical and theoretical models can serve as the foundation for investigating how theoretical models would generate a classical cycle. A perusal of the results from Table 2 points to the need for theories of the cycle to provide accounts of the origin of the following three elements.

(i) Trend growth of the right magnitude.
(ii) Persistent shocks.
(iii) Volatility in growth rates of the right magnitude.

How do theoretical models incorporate these features? Generally (i) has been handled with words somewhat like those used by Schmitt-Grohe (1998)

> "..I follow the literature by assuming that the drift in the technology process… is equal to 1.6% per year",

and we have already adopted the strategy in the simulations of Table 3 of equating it to what is seen in the data. Trend growth is very important to cycle outcomes and is the source of the "business cycle asymmetry" often referred to. Since drift in the technology process is the only source of trend growth in the majority of theoretical models it is clear that a major determinant of the classical cycle is thereby made exogenous. As the "new paradigm" debate of the past few years in the U.S. has emphasized, being unable to explain the determinants of trend growth is a major limitation when it comes to eliciting the business cycle implications of any such developments. From our earlier analysis, it is clear that a rise in trend growth, with the volatility of output growth being held constant, will lead to a rise in both the length of the cycle and the duration of its expansions, thereby providing a possible explanation for the length of the current classical expansion in the U.S. [12]

How is persistence of shocks to be introduced? The three real business cycle models in Table 3 have it as being the concomitant of exogenous processes that are highly persistent e.g. as Cogley and Nason (1995) show, most RBC type models tend to preserve the nature of the process that has been specified for technology shocks. Three methods of endogenously generating persistence seem to have had some success. One is through allowing for endogenous growth e.g. Collard (1999). Such models produce a unit root in output, independently of whether the technology process has a stochastic trend, and the ability to generate persistence endogenously is a point in its favor. It is also the case that it produces some serial correlation in growth rates – see Collard (1999, p.479) – and, as the simulations of an ARIMA(1,1,0) model in Table 2 (equation (2)) showed, this

[12] Another explanation derives from the argument that σ has fallen, see McConnell and Quiros (1998)

tends to reduce the length of the cycle. Clearly, it doesn't produce the same amount of serial correlation as the ARIMA model does, so the cycle is not as short.[13]

The second method of inducing persistence is through "belief shocks". Belief shocks stem from indeterminancies in models which allow the addition of such shocks to the Euler equation describing consumption choices. Schmitt-Grohe (1998) investigates the ability of such shocks to produce serial correlation in $\Delta y(t)$ and finds that the models in which they are imbedded are more successful on this dimension than are RBC models featuring only technology shocks. What is not directly answered in her work is whether such shocks can make y(t) a process that is persistent enough. Simulations of her calibrated "belief shocks only" model suggests that not enough persistence eventuates in those circumstances, since the average durations are 8.2/8.3 quarters, making for expansions that are much too short and contractions that are too long.

The last approach involves price and wage rigidities. These seem to provide a promising alternative mechanism, although there is little quantitative evidence on their effectiveness. Work by Chari et al (1996) suggested that, in an optimizing model, it was hard to get much persistence of demand shocks from price rigidities, but Huang and Liu (1998) note that this would not be true of models with staggered wage contracts. Effectively, this result makes the slope of the Phillips curve the key parameter; if it is low then output will tend to have a unit root. Kiley (1999) shows that the way in which one models sticky prices is very important to any conclusions on persistence. He demonstrates that partial adjustment models of prices tend to have much more persistence over a range of values of the elasticity of labor supply than does the staggered price model investigated by Chari et al (1996). Each of these models effectively involve a price-cost cycle; a mechanism that has had a long history in empirical business cycle work, see Pagan (1997).

The final item in the trio of issues listed above involves how one produces volatility in growth rates that matches the data. In many instances the parameters of

[13] In the notation of Collard's paper γ=.117 is selected as this seems to be his preferred value (p 477). He notes later (p 479) that γ=.5 would be needed to get the correct serial correlation in growth rates.

exogenous forcing processes have been selected to produce this outcome.[14] In other cases Solow residual type computations are used to estimate the volatility of productivity shocks for use in a model. This is a little closer to being an independent measure of volatility, although it is not entirely satisfactory, since it uses a series, output, that one is seeking to explain –see Hartley et al (1997).

Now let us look at the models according to how they account for the trio of issues just raised. Model RBC1 of Table 3 satisfies all of the requirements by construction; both trend growth in output and the volatility of growth rates are fixed at what is observed, and the stochastic trend in output derives from the assumptions on technological change. Given that it is known that RBC1 generates little serial correlation in output growth rates, it is scarcely surprising that its statistics are essentially those of the random walk with drift model in Table 2. Consequently, although it does produce a realistic cycle, it does so because the crucial elements in the explanation have been *assumed* and very little, if anything, derives from the economics underlying RBC theory. The RBC2 simulations illustrate just how important it is to get the volatility in growth rates right. As we observed earlier, having a unit root in the technology process in the RBC1 model leads to the volatility of output growth being below that of technology, unlike the situation when technology is an AR(1). Ideally one wants a small volatility in technology in order to avoid "technological regress". Consequently, if the standard deviation of technology shocks was set to a realistic value, the output cycles would be unrealistic. This can be seen from the RBC2 simulations in which that parameter has been reduced by 20%. Recent work by King and Rebelo (1998), in the situation where technology is a stationary process, has managed to amplify technology shocks by allowing for the impact of variable capacity utilization, and this seems a step in the right direction. The same set of problems arises in regard to RBC3, since Cogley and Nason (1995) calibrated the

[14] There seems to be an emerging trend in this vein e.g. Schmitt-Grohe (1998) and Christiano and Harrison (1998) . No defence of the practice is given but it seems to be rooted in the idea that the business cycle is about co-movements and so it doesn't matter if σ is set so as to match the data on output. One might also point out that estimation of parameters is often done in a GMM framework in which the second moment of output is one of those used to define the estimator. If the number of moments and parameters were identical then one has simply re-parameterized the problem, and the volatility of output would be perfectly reproduced, regardless of the validity of the theoretical model. Even if the number of moments exceeds the number of parameters the number that are effectively used in the construction of the estimator may be the same i.e. the weights attached to the excess moments by GMM may be very small.

variance of both of the shocks in that model to reproduce the variance of output. Nevertheless, in this situation the standard deviation of technology shocks could be set at a much lower value than in RBC1; effectively "amplification" comes about due to the presence of a second shock. To summarize, getting a standard deviation for output growth of the right magnitude from standard RBC models has been quite a challenge

In gauging the success of any theoretical model it is obviously unwise to restrict one's attention to the output cycle, even if this is the business cycle, particularly if the first two moments of output growth predicted by theoretical models are largely designed to match the data. An important insight of the DSGE literature is that one needs to look at other implications of a model when judging theories. Accordingly, an analysis of specific cycles is useful in this task. One advantage of proceeding in this way is that one needs to be confident that the cycle in aggregate economic activity is not being reproduced at the expense of inducing implausible cycles in some other macroeconomic series. The endogenous growth model provides a good illustration of this point. In Collard's (1999) version the levels of consumption and investment are a fixed ratio to output and so the turning points must be exactly the same as for output, something which is incompatible with the data.

A more subtle demonstration of the value of specific cycle information is available from the RBC1 model. Table 4 records specific cycles in consumption and investment from the actual data as well as artificial series simulated from that model. It is apparent that the model does not perform as well in terms of capturing specific cycles as it does the business cycle. Concentrating first on consumption, and thinking in terms of our earlier analysis, a possible reason for the discrepancy between the predicted and actual durations of an expansion is that the predicted volatility of consumption growth is incorrect. This conjecture is borne out by the data: volatility of consumption growth implied by RBC1 is .56% per quarter, while the data has a value of .89% . The smaller volatility acts to produce a longer cycle in consumption. Investment has a quite different difficulty. Durations are well matched but the amplitudes are problematic. To explore this further we simulated data from a random walk with the same mean and variance for investment growth as in the data. The resulting statistics were quite close to those from

RBC1 in Table 4. Thus the failure to match the amplitudes of expansions and contractions may point to a problem with models driven solely by technology in explaining the specific cycle in that series.[15] One factor which is accorded importance in much public discussion of investment movements is “sentiment”. Studies of asset price fluctuations have introduced fluctuations in sentiment by making the risk aversion coefficient in the CRRA utility function be realizations of a two state Markov process – see Gordon and St. Amour (1998) - and this may be useful for explanations of the investment cycle as well.

Table 4: Actual and Simulated Specific Cycle Characteristics

	Consumption		Investment	
	Data	RBC1	Data	RBC1
Mean Duration (quarters)				
PT	3.7	3.4	5.1	4.7
TP	21.3	24.5	7.7	7.8
Mean Amplitude (%)				
PT	-2.2	-0.8**	-19.1	-5.4**
TP	14.2	13.6	26.3	11.0**
Cumulated (%)				
PT	4.2	-1.9	-46.6	-16.7**
TP	262	394	161.2	65**
Excess				
PT	0.2	-0.0**	1.0	-0.2**
TP	0.6	0.1	2.9	0.2**

** Indicates that less than 5% of simulations were further out in the tail relative to the data estimate.

[15] The conclusion we reach about the ability of the basic RBC model to explain the specific cycles for investment and consumption differs from that in King and Plosser (1994). The difference seems to stem from two sources. First, King and Plosser did not look at specific cycles but “cycle relatives” i.e. measurements of the amplitude of a specific cycle were done in relation to the turning points of the reference cycle and not the specific cycle. Second, the volatility of investment in the RBC1 model is about 2/3 of what it is in the model used by King and Plosser, which had technology following an AR(1) process with parameter .95. The parameters we use for RBC1 are taken directly from King, Plosser and Rebelo (1988).

All of the above has been predicated upon the idea that it is the evidence on the average cycle which theories should be confronted with. Using the framework outlined earlier, computer simulations reveal something about the nature of the shocks which are needed to realize realistic cycles and the types of models that will produce the latter. For example, it seems fair to conclude that linear models are more than capable of accounting for the cycle. But perhaps the emphasis upon averages misses something. It is possible that items like monetary shocks are important to specific contractions but may appear very unimportant when averaged over many expansions and contractions. This was the experience in Dungey and Pagan (1997) when accounting for movements in $z_d(t)$ in Australia. Moreover, there can be no denying that many policy makers are very influenced by particular cycles, and lessons are drawn from them that are often meant to apply to the average cycle. A very good example of the latter phenomenon is the cycle of the early 1990s, which did seem to depart significantly from the average cycle on some dimensions, at least in countries such as the U.K. and Australia. Specifically, the contraction phase was of lengthy duration and, accordingly, this resulted in very large cumulated output losses. Since the cycle was also accompanied by strong asset price movements it has been argued that investment was inhibited through constraints on the amount of collateral that firms could provide to the banking sector for loans. Some models have emerged to capture such feedbacks, e.g. Kiyotaki and Moore (1997), although it is currently unclear whether in accounting for a particular cycle they have produced a model of output dynamics that fails to reproduce the average cycle. It may be in accounting for particular cycles that non-linearities become important, with the simple dynamics characterizing output being temporarily modified to quite complex ones depending on whether (say) some threshold is exceeded. When looked at over many cycles, it is unlikely that this threshold would have been attained very often, so that estimated equations would reveal only simple dynamics.

The final element in textbooks is the naming of the important shocks. A long list of these is generally entertained and sometimes they are located out of the realm of economics e.g. Sachs and Larrain (1993, p530) give credence to policy shocks designed for political outcomes as a factor driving the cycle with their claim:

"Thus, when Democrats are elected president, monetary policy is generally eased, while when Republican presidents are elected, monetary policy is generally tightened. As a result, Democratic presidents tend to have booms early in their tenure, while Republican presidents tend to preside over recessions in the first half of a new term".[16]

After providing a list of shocks, textbook discussion frequently founders on the question of the relative importance of the listed shocks in explaining the cycle. It would seem that we are better at agreeing on the statistical nature of the important shocks, as exemplified by their persistence and volatility, than we are at agreeing over what we should call them. To some extent this reflects the fact that much of our information about such questions has traditionally come from VAR models in which the definition of the shocks relies on very little theory. That has to change. Only then can we really begin to sort out the relative contribution of a nominated list of shocks, and only then will we know the cycle.

[16] Table 17-4 of Sachs and Larrain provides growth rates in GDP for each year of the presidency from 1948 to 1984 in support of this contention.

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