Costs of Children and Living Standards in Australian Households

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Abstract

Measuring the costs of children is of immense practical importance in a whole range of economic and social policy areas. In this paper, a new econometric procedure that improves on existing methods for obtaining estimates of such costs using the demand system approach is introduced. The study is based on the use of an extended linear expenditure system and develops an iterative maximum likelihood estimator that overcomes possible estimation problems that arise from the 2-step estimation procedures employed by the earlier authors. We also allow for a more general assumption about the equation "errors", that of non-zero correlation between the errors for different commodities in the same household. Another important contribution is the development of an estimation procedure for sets of seemingly unrelated regressions where the different sets of equations are linked by some common parameters. The proposed procedure is applied to the 1984, 1988-89 and 1993-94 Australian Household Expenditure Survey and results obtained update estimates of both the commodity-specific and general scales previously obtained for Australia.

1. Introduction

Measuring the costs of children is of immense practical importance in a whole range of economic and social policy areas. In assessing the distribution of income, the progressiveness and effectiveness of tax and social security systems and the impact of government policies on living standards of households, it is necessary to examine the nature and level of these costs. In the economic literature, a conventional approach is to estimate child costs through the use of micro unit record data within the context of a utility framework. This approach yields child cost estimates (otherwise known as equivalence scales) that allows one to make direct comparisons between households of different sizes and composition. For example, a comparison of equivalence scales for households with and without children is a popular means of obtaining some representation of the costs that raising children imposes on a household. Indeed, it is the use of equivalence scales in income maintenance programs that results in larger benefits accruing to families with more and older children compared to families with fewer and younger children.

The calculation of household equivalence scales has a long and controversial history beginning with the pioneering work of Engel (1895) on Belgian working class expenditure data. The focus of the debate in more recent times centers on the legitimacy of making welfare comparisons based on “conditional” equivalence scales, which are scales derived from demand data and are computed “conditioned” on a predetermined demographic composition. It is argued that household welfare should be thought of as depending on a household composition directly as well as through the effects of household composition on commodity demands. “The expenditure level required to make a three-child family as well off as it would be with two children and $12,000 depends on how the family feels about children”, wrote Pollak and Wales (1979). This argument led some authors to conclude that such scales are not useful for welfare comparisons. (See Browning (1992) and Nelson (1993) for a detailed overview of the identification problems of equivalence scales). Other authors, however, regard this as an overly negative assessment and counter claim that estimation of equivalence scales based on conditional preferences has a purposeful role in welfare comparisons (e.g. Deaton and Muellbauer (1986), Blundell and Lewbel (1991), Nelson (1993)).

It is not the purpose of this study to contribute to this on-going debate. Given that unconditional scales are not estimable at the present time, and that equivalence scales are in great demand for policy and welfare analysis, this study is developed based on the premise that equivalence scales from demand data are the best practicable approach to estimating costs of children. In this context, we introduce a new econometric procedure that improves on existing methods for estimating commodity-specific and general scales from an extended linear expenditure system. Specifically, we develop an iterative maximum likelihood estimator that overcomes possible estimation problems that arise from the 2-step estimation procedures employed by earlier authors. We also allow for a more general assumption about the equation “errors”, that of non-zero correlation between the errors for different commodities in the same household. This assumption is more in line with that usually made for Engel functions and other systems of demand equations. Another important contribution is the development of an estimation procedure for sets of seemingly unrelated regressions where the different sets of equations are linked by some common parameters.

The proposed procedure is applied to the 1984, 1988-89 and 1993-94 Australian Household Expenditure Survey and results obtained update estimates of both the commodity-specific and general scales previously obtained for Australia.

The paper is structured as follows. Section 2 describes the model and sets the notation. Section 3 details the stochastic assumptions and outlines the estimation methodology. Section 4 discusses the data and results from the empirical application and a final section concludes.

2. The Model

The demand system employed here is the extended linear expenditure system (ELES) of Lluch (1973). A feature of this system is linearity, an assumption that is often questioned. Also, the utility function from which it is derived is directly additive - a restrictive assumption, particularly in studies using a detailed disaggregation of commodities. In spite of these disadvantages, the ELES was chosen for this study for a number of important reasons. First, the ELES is a convenient vehicle for carrying out relatively sophisticated research on consumer behaviour even when available data on private consumption are limited. Because time series data on private consumption are not disaggregated over various commodity groups, one can only use cross-section information for estimating demand parameters. Since purely cross-section data generally give no price variation, inference about price effects requires strong theoretical specifications. Second, the ELES is chosen for its historical significance in equivalence scale research. ELES-based equivalence scales have been repeatedly estimated in the past and are particularly popular among researchers using Australian data. See, for example, Kakwani (1980), Binh and Whiteford (1990), Bradbury (1994), Valenzuela (1996) and Lancaster and Ray (1998). Using the ELES in this study facilitates comparison of results to these earlier ones. Thirdly, the ELES is used because of its simplicity. In this work, a new method of estimation of equivalence scales is derived. The ELES is ideal for this purpose because the system remains mathematically tractable but is still sufficiently complicated to warrant a number of econometric innovations. Once these innovations have been developed, they can be more readily applied to more complicated models at a later time. Special care was taken to split the sample into groups with few parametric restrictions on the scales and estimation was restricted to just eleven broad commodity groups, thereby mitigating the assumption of additive utility. The exercise is a natural starting point for demonstrating a new iterative procedure using maximum likelihood techniques. The study as a whole provides a useful addition to the available empirical evidence on equivalence scales in Australia.

2.1 The Extended Linear Expenditure System

To describe the model, consider n commodity groups indexed by $i=1,2,\ldots,n$ and H types of households indexed by $h=1,2,.,H$ where household types are defined according to the number of adults and the number of children in the household. Define q_{ih} as the quantity of the i^{th} commodity consumed by the h-type household and s_{ih} is the i^{th} commodity-specific scale for the h-type household. The s_{ih} are factors used to adjust quantities q_{ih} in utility functions to show the effect of a change in the household's demographic composition on household utility and on specific commodity expenditures. On a per unit basis, a given q_{ih} provides less utility if it is shared with more people. How q_{ih} should be deflated to give the same per unit utility will depend on the commodity i and on the household type h. Thus the scale is subscripted with i and h. Utility is specified relative to a reference unit which is a household with two adults and no children; in this case we set s_{ih}=1. The q_{ih} in the utility functions for other household types are scaled by s_{ih} to give a comparable two-adult-zero-children utility function for all households.

Given this background, consider now the Klein-Rubin utility function where the consumption quantities q_{ih} are scaled as follows:

$$u_h = \sum_{i=1}^{n} b_i \ \ln\left\{\frac{q_{ih}}{s_{ih}} - c_i\right\} \tag{1}$$

where

b_i = is the marginal contribution to utility of the i^{th} commodity and satisfies the constraints $0 < b_i < 1$ and $\sum_{i=1}^{n} b_i = 1$;

c_i = is a parameter which, if interpreted as the subsistence quantity of the i^{th} commodity, satisfies the constraint $c_i > 0$.[1]

Let p_i be the price of the i^{th} commodity and v_h be the total expenditure for the h-type household. Maximising the utility function (1) subject to the budget constraint $\sum_{i=1}^{n} p_i q_{ih} = v_h$ leads to the linear expenditure system (LES)

$$p_i q_{ih} = p_i s_{ih} c_i + b_i \left(v_h - \sum_{j=1}^{n} p_j s_{jh} c_j \right) \tag{2}$$

A household whose demand system is LES is often described as first purchasing “necessary”, “subsistence” or “committed” quantities for each good ($s_{1h}c_1,\ldots,s_{nh}c_n$) and then dividing its

[1] Pollak and Wales (1992) prefer not to give c_i a strict subsistence interpretation letting negative values be a possibility.

remaining or "supernumerary" expenditure (v_h $-\sum_{i=1}^{n} p_i s_{ih} c_i$), among the goods in fixed proportions ($b_1,\ldots,b_n$). The system in (2) can be more compactly expressed as

$$v_{ih} = a_{ih} + b_i (v_h - a_h) \qquad (3)$$

where

v_{ih} = $p_i q_{ih}$ is expenditure on the i^{th} commodity by the h-type household;

a_{ih} = $p_i s_{ih} c_i$ is subsistence expenditure for the i^{th} commodity and h-type household; and,

a_h = $\sum_{i=1}^{n} a_{ih}$ is the total subsistence expenditure for the h-type household.

The objective is to estimate a_{ih} and b_i with these estimates later being used to estimate the scales s_{ih}. Specifically, if s_{ir} = 1 denotes the scale for the reference household type, then

$$s_{ih} = \frac{p_i s_{ih} c_i}{p_i s_{ir} c_i} = \frac{a_{ih}}{a_{ir}} \qquad (4)$$

However, without further information, not all of the a_{ih} are identified. The identification problem arises because, for a given household type, one of the n equations in (4) is redundant, redundancy being illustrated by summing both sides of (4) to yield

$$\sum_{ih}^{n} v_{ih} = \sum_{i=1}^{n} a_h + \sum_{i=1}^{n} b_i (v_h - a_h) \qquad (5)$$

or

$$v_h = a_h + (v_h - a_h) \qquad (6)$$

The redundancy of one equation means that separate information is only available from n-1 equations. The problem is to estimate n intercept terms with only n-1 equations.

One solution to this identification problem is to include in the linear expenditure system in (4) a micro-consumption function given by

$$v_h = a_h + b(x_h - a_h) \qquad (7)$$

where v_h is the total expenditure, x_h is net income, b is a common marginal propensity to consume for all households. This function shows that total expenditure v_h is composed of "committed" or "subsistence" expenditure a_h and a proportion b of "uncommitted" expenditure (x_h-a_h). The extended linear expenditure system or ELES is thus comprised of equations (3) and (7).

To estimate the parameters in ELES, Kakwani (1980) appended errors to these equations, and assumed the error variances can be different for each household type and for each commodity. He suggested first estimating a_h and b from (7) and then replacing a_h with its estimated equation in each of the commodity equations in (3). Then, to estimate a_h and b_i in (3), weighted least squares which allows for heteroskedasticity across different household types was applied to each of these equations. Using an external estimate of a_h identifies the remaining parameters.

The estimation procedure that is developed in this paper attempts to improve on Kakwani's procedure in two ways. First, because Kakwani estimated each of the commodity equations separately, he ignored any correlation that might exist between the errors that correspond to different commodity equations for a given household. Second, the '2-step' nature of the procedure ignored the effect of using estimates from one equation on the properties of the estimates from a second equation. An estimator which allows for error correlation across different commodity equations and which estimates all parameters simultaneously would seem more desirable.

2.2 Expressions for the Commodity Specific Scales

To investigate how all the parameters might be jointly estimated, (7) is substituted into (3) to obtain

$$\begin{aligned} v_{ih} &= a_{ih} + b_i[(a_h + b(x_h - a_h)) - a_h] \\ &= a_{ih} + b_i b(x_h - a_h) \\ &= a_{ih} - b_i b a_h + b_i b x_h \\ &= \theta_{ih} + \eta_i x_h \end{aligned} \tag{8}$$

where $\theta_{ih} = a_{ih} - b_i b a_h$ and $\eta_i = b_i b$.

Consider now the estimation of θ_{ih} and the η_i and how estimates of the structural parameters a_{ih}, b_i, b and a_h can be retrieved from these estimates. Given θ_{ih} and η_i, estimates of the structural parameters can be obtained using the expressions

$$b = \sum_{i=1}^{n} \eta_i \tag{9}$$

$$a_h = \frac{\sum_{i=1}^{n} \theta_{ih}}{1 - \sum_{i=1}^{n} \eta_i} \tag{10}$$

$$b_i = \frac{\eta_i}{\sum_{j=1}^{n} \eta_j} \tag{11}$$

$$a_{ih} = \theta_{ih} + \frac{\eta_i \sum_{j=1}^{n} \theta_{jh}}{1 - \sum_{j=1}^{n} \eta_j} \tag{12}$$

The system in (8) does not suffer from an identification problem because there are no redundant equations. All the n commodity equations for a given household type can be utilised.

2.3 Expressions for General Scales

A general equivalence scale s_h for the h-type household is defined as the ratio of income for that type of household to income of the reference household such that the indirect utility functions of the two household types are the same.

To obtain an expression for the general scales, we first consider the demand equations in (2). Dividing through by $p_i s_{ih}$ and using the result in equation (7), we get

$$\frac{q_{ih}}{s_{ih}} = c_i + \frac{b_i b}{p_i s_{ih}} \left(x_h - \sum_{j=1}^{n} p_j s_{jh} c_j \right) \tag{13}$$

Equation (13) is then substituted into the direct utility function in (1) and noting that $\sum_{i=1}^{n} b_i = 1$, we get

$$\begin{aligned} u_h &= \sum_{i=1}^{n} b_i \ \ln\left[\frac{b_i b}{p_i s_{ih}} \left(x_h - \sum_{j=1}^{n} p_j s_{jh} c_j \right) \right] \\ &= \sum_{i=1}^{n} b_i \left[\ln\ b_i - \ln\ p_i - \ln s_{ih} + \ln b + \ln\left(x_h - \sum_{j=1}^{n} p_j s_{jh} c_j \right) \right] \qquad (14) \\ &= \ln b + \ln\left(x_h - \sum_{i=1}^{n} p_i s_{ih} c_i \right) + \sum_{i=1}^{n} b_i \ln b_i - \sum_{i=1}^{n} b_i \ln p_i - \sum_{i=1}^{n} b_i \ln s_{ih} \end{aligned}$$

For the standard reference household where $s_{ir} = 1$, the indirect utility function is thus expressed as

$$u_r = \ln b + \ln(x_r - \sum_{i=1}^{n} p_i c_i) + \sum_{i=1}^{n} b_i \ln b_i - \sum_{i=1}^{n} b_i \ln p_i \qquad (15)$$

The general scale for the h-type household is given by the ratio of incomes $s_h=x_h/x_r$ that equates the two indirect utility functions. Working in this direction, we set $u_r=u_h$ to obtain

$$\begin{aligned} &\ln b + \ln(x_r - \sum_{i=1}^{n} p_i c_i) + \sum_{i=1}^{n} b_i \ln b_i - \sum_{i=1}^{n} b_i \ln p_i \\ &= \ln b + \ln\left(x_h - \sum_{i=1}^{n} p_i s_{ih} c_i \right) + \sum_{i=1}^{n} b_i \ln b_i - \sum_{i=1}^{n} b_i \ln p_i - \sum_{i=1}^{n} b_i \ln s_{ih} \end{aligned} \qquad (16)$$

Simplifying (16), we have

$$\ln\left(x_r - \sum_{i=1}^{n} p_i c_i \right) = \ln\left(x_h - \sum_{i=1}^{n} p_i s_{ih} c_i \right) - \sum_{i=1}^{n} b_i \ln s_{ih}$$

$$\left(x_h - \sum_{i=1}^{n} p_i s_{ih} c_i \right) = \left(x_r - \sum_{i=1}^{n} p_i c_i \right) \left(\prod_{i=1}^{n} s_{ih}^{b_i} \right)$$

$$\begin{aligned} x_h &= x_r \prod_{i=1}^{n} s_{ih}^{b_i} + \sum_{i=1}^{n} p_i s_{ih} c_i - \left(\prod_{i=1}^{n} s_{ih}^{b_i}\right)\left(\sum_{i=1}^{n} p_i c_i\right) \\ \frac{x_h}{x_r} &= \prod_{i=1}^{n} s_{ih}^{b_i} + \frac{1}{x_r}\left[\sum_{i=1}^{n} p_i s_{ih} c_i - \left(\prod_{i=1}^{n} s_{ih}^{b_i}\right)\left(\sum_{i=1}^{n} p_i c_i\right)\right] \end{aligned} \tag{17}$$

Noting that $a_{ih}=p_i s_{ih} c_i$ and $a_h = \sum_{i=1}^{n} a_{ih}$ for all i, (16) can be equivalently written as

$$\begin{aligned} s_h = \frac{x_h}{x_r} &= \prod_{i=1}^{n} s_{ih}^{b_i} + \frac{1}{x_r}\left[\sum_{i=1}^{n} a_{ih} - \prod_{i=1}^{n} s_{ih}^{b_i} a_r\right] \\ &= \frac{a_h}{x_r} + \prod_{i=1}^{n} s_{ih}^{b_i}\left[1 - \frac{a_r}{x_r}\right] \end{aligned} \tag{18}$$

Along with the commodity specific scales s_{ih}, these general scales are the final quantities of interest. They capture the overall effect of a change in demographic composition on the total expenditure of the household. From (18), they are shown to be a function of the commodity specific scales s_{ih}'s and are calculated based on a chosen reference income level of the reference household. If we write the first term in the second line of (18) as $\frac{a_h}{a_r}\frac{a_r}{x_r}$ we can see that a general scale is a weighted average of the $\prod_{i=1}^{n} s_{ih}^{b_i}$ term and $\frac{a_h}{a_r}$, where the former is a weighted geometric mean of the s_{ih}'s and the latter is a ratio of relative subsistence costs.

3. Estimation Methodology

Suppose now that there are M_h observations (households) with demographic composition type h. In the notation that follows, the symbols v_{ih} and x_h which previously represented scalar quantities for a given household, will become ($M_h \times 1$) vectors containing all observations on households of type h. Returning to equation (8), adding stochastic terms, the system we wish to estimate can be written as

$$\begin{bmatrix} v_{1h} \\ v_{2h} \\ \vdots \\ v_{nh} \end{bmatrix} = \begin{bmatrix} z_h & & & \\ & z_h & & \\ & & \ddots & \\ & & & z_h \end{bmatrix} \begin{bmatrix} \theta_{1h} \\ \theta_{2h} \\ \vdots \\ \theta_{nh} \end{bmatrix} + \begin{bmatrix} x_h & & & \\ & x_h & & \\ & & \ddots & \\ & & & x_h \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_n \end{bmatrix} + \begin{bmatrix} e_{1h} \\ e_{2h} \\ \vdots \\ e_{nh} \end{bmatrix} \tag{19}$$

or

$$\mathbf{V}_h = \mathbf{Z}_h \Theta_h + \mathbf{X}_h \eta + \mathbf{E}_h \tag{20}$$

where

h = 1,2,…, H refers to household composition type h;

n = refers to the number of commodity groups;

v_{ih} is an $(M_h \times 1)$ vector of observations on expenditure for the i^{th} commodity and the h-type household;

z_h is an $(M_h \times 1)$ vector of ones;

x_h is an $(M_h \times 1)$ vector of observations on income for household type h;

e_{ih} is an $(M_h \times 1)$ vector or errors;

$\mathbf{V}_h$ is of dimension $(nM_h \times 1)$;

$\mathbf{Z}_h = I_n \otimes z_h$ is an $(nM_h \times n)$ matrix of dummy variables;

$\mathbf{X}_h = I_n \otimes x_h$ is an $(nM_h \times n)$ matrix matrix of household incomes;

Θ_h, η are $(n \times 1)$ vectors of unknown parameters; and

$\mathbf{E}_h$ is an $(nM_h \times 1)$ vector of errors which are assumed to be distributed as

$$\mathrm{E}_h \sim N\left[0,\ \Omega_h \otimes I_{M_h}\right] \tag{21}$$

Thus, the error covariance matrix Ω_h is allowed to be different for different household types. Because Ω_h is not diagonal, correlation between errors from equations for different commodities and the same household, is permitted. Zero error correlation is assumed across different households[2]. Thus, in addition to (21), $E(\mathbf{E}_h \mathbf{E}_k') = 0$ for $h \neq k$.

[2] The sample is assumed to be random.

The task is to derive expressions for the maximum likelihood estimators of Θ_h, Ω_h, and η, as well as asymptotic covariance matrices for these estimates and asymptotic covariance matrices for the consequent maximum likelihood estimates for the parameters in equations (9) – (12). In the econometrics literature, this model can be viewed as a set of H independent seemingly unrelated regressions which have a common slope vector, but different intercept vectors.

A maximum likelihood estimator for Θ_h, Ω_h, and η are derived in the Appendix. The results from this derivation exercise lead to the following convenient iterative procedure for computing these parameters:

[1] Express v_{ih} and x_h in terms of deviations from their household-type means. That is, compute $v_{ih}^* = v_{ih} - \bar{v}_{ih} z_h$ and $x_h^* = x_h - \bar{x}_h z_h$ where $\bar{v}_{ih} = M_h^{-1} z_h' v_{ih}$ and $\bar{x}_h = M_h^{-1} z_h' x_h$.

[2] Find the least squares estimates $\hat{\eta}_i^h = \left(x_h^{*\prime} x_h^* \right)^{-1} x_h^{*\prime} v_{ih}^*$

[3] Find an initial estimate of Ω_h as[3]

$$\left[\hat{\Omega}_h \right]_{ij} = \left(v_{ih}^* - x_h^* \hat{\eta}_i^h \right)' \left(v_{jh}^* - x_h^* \hat{\eta}_j^h \right) / M_h \tag{22}$$

[4] Compute a pooled estimate for η as

$$\hat{\eta} = \left[\sum_{h=1}^{H} \left(x_h^{*\prime} x_h^* \right) \hat{\Omega}_h^{-1} \right]^{-1} \sum_{h=1}^{H} \left(x_h^{*\prime} x_h^* \right) \Omega_h^{-1} \hat{\eta}^h \tag{23}$$

where $\hat{\eta}^h = \left(\hat{\eta}_1^h, \hat{\eta}_2^h, \ldots, \hat{\eta}_n^h \right)'$.

[5] Repeat step [3] with $\hat{\eta}_i^h$ replaced by $\hat{\eta}_i$ that is computed from (23).

[6] Repeat steps [4] and [5] until convergence.

[7] Compute estimates of the θ_{ih} from $\hat{\theta}_{ih} = \bar{v}_{ih} - \bar{x}_{ih} \hat{\eta}_i$.

In the estimation exercise, convergence of steps [4] and [5] took less than 6 iterations.

[3] Note that steps [2] and [3] can be computed at the same time with a seemingly unrelated regression of each of $\left(v_{1h}^*, v_{2h}^*, \ldots, v_{nh}^* \right)$ on x_h^*, with no constant.

4. Data and Results

The data used in this study are derived from the 1984, 1988-89 and 1993-94 Household Expenditure Survey (HES) conducted by the Australian Bureau of Statistics (ABS). These surveys are the third, fourth and fifth, respectively, of a series of surveys designed to obtain details of expenditure, income and a wide range of demographic characteristics of Australian private households on a nationwide basis. The public-use tapes contain a total of 4492 (1984 HES), 7225 (1988-89 HES) and 8390 (1993-94 HES) households representing between 3.8 and 5.4 million Australian households from all over the country for each year the surveys were conducted.

The estimation procedure used for the estimation of the cost of children here required the use of information from households composed of related persons with one or two adults and at most three children only. This resulted in eight household types. Adults are all persons aged 17 or older and children refer to all those aged 16 or younger. Households not belonging to any of these types are excluded. Information from some 300 households from each year of data was also discarded because of reported negative expenditures on certain items[4]. These observations were not consistent with the economic model set up for this purpose.

The number and characteristics of household types from the three data set are given in Table 1. Of the 6752 households considered from the 1993-94 HES, 37 percent were of the type (2,0) where the first number in the bracket refers to the number of adults, and the second number refers to the number of children. Further, 25 percent were of the type (1,0). This implies that 62 percent of the total households in the sample are without children. Effectively, 62 percent of the total households in the sample were households without children. These households were mostly headed by persons in the older age groups (average age of household head is 48 years for couples and 53 years for singles) and are inferred to have children who are already moved out and are financially independent. A number of then are retired couples or individuals. In contrast, the 38 percent of sample households that have children had household heads aged between 25 and 40.

[4] For example, of the households which reported negative expenditures in the 1988-89 dataset, 72 percent of the negative expenditures were on transport while 27 percent on recreation and entertainment.

Table 1. Sample Characteristics

1984 HES	*Household Type (no. of adults, no. of children)* (1,0)	(1,1)	(1,2)	(1,3)	(2,0)	(2,1)	(2,2)	(2,3)
Sample Size	777	79	82	38	1272	406	643	275
Age of HH Head	53.33	26.16	30.80	30.58	49.41	32.07	33.16	36.48
Average Weekly HH Income	234.76 (195.11)	195.39 (106.82)	242.68 (141.89)	264.13 (143.89)	435.60 (293.19)	513.66 (289.94)	497.32 (254.02)	553.55 (352.12)
Average Weekly HH Expenditure	241.71 (200.67)	246.26 (163.98)	297.71 (248.58)	330.44 (244.02)	445.11 (307.26)	557.37 (292.10)	556.30 (304.67)	615.21 (460.34)

1988-89 HES	*Household Type (no. of adults, no. of children)* (1,0)	(1,1)	(1,2)	(1,3)	(2,0)	(2,1)	(2,2)	(2,3)
Sample Size	1372	132	103	42	2074	532	889	388
Age of HH Head	52.68	33.52	30.28	28.76	48.38	32.68	33.84	35.12
Average Weekly HH Income	306.73 (246.53)	274.54 (172.51)	315.38 (166.09)	313.64 (159.76)	595.93 (417.50)	697.00 (579.69)	767.83 (493.50)	720.57 (378.47)
Average Weekly HH Expenditure	255.04 (194.60)	281.20 (162.12)	315.15 (153.57)	310.44 (142.05)	461.62 (285.24)	555.94 (285.82)	603.90 (348.75)	623.19 (321.88)

1993-94 HES	*Household Type (no. of adults, no. of children)* (1,0)	(1,1)	(1,2)	(1,3)	(2,0)	(2,1)	(2,2)	(2,3)
Sample Size	1702	192	149	60	2509	690	845	425
Age of HH Head	53.03	34.08	33.69	30.58	48.12	33.1	35.46	35.04
Average Weekly HH Income	281.89 (227.89)	339.99 (162.32)	377.83 (151.71)	346.83 (100.60)	547.18 (345.90)	612.54 (333.63)	634.78 (349.07)	632.85 (365.70)
Average Weekly HH Expenditure	319.23 (232.00)	404.52 (218.53)	468.86 (278.42)	440.3 (253.24)	593.52 (347.72)	695.37 (345.04)	739.92 (370.39)	762.18 (390.11)

Note: Standard errors are in parentheses

The table further shows that two-adult households have higher weekly incomes compared to one-adult households, and households with children have higher incomes than those without. Large variances associated with these averages indicate that the absolute differences in the reported incomes are not significant among households with two adults and among households with one adult, whether they have children or not. As may be expected, however, there are significant differences in income levels between one-adult and two-adult households. Reported levels of total expenditures were, on average, consistently higher than reported total income but the large variances indicate no significant differences in the values.

The households covered in the 1984 and 1988-89 HES were very similarly distributed, though income and expenditure levels were progressively lower in these earlier years. In 1984, single adult households with one child were younger and poorer compared to those in similar situations in the later years.

In both Tables 2a, 2b and 2c, the 2nd columns provide the marginal budget shares b_i and the 3rd through the 10th columns give the estimates of subsistence expenditures a_{ih} for each expenditure category. In general, the subsistence expenditures increase with household size, with wider differentials occurring across two-adult households compared to one-adult households. For all household types, expenditure on Food was on top of the shopping list, followed closely by Housing, then Transport, and then Household furnishings. Together, these items make up between 60 to 73 per cent of subsistence expenditures of a typical Australian household.

Tables 3a, 3b and 3c present the estimates of commodity-specific scales. A two-adult household with no children is chosen to be the reference household for which s_{ih} is set to 1. The 1993-94 scale value of 1.38 for Housing of a (2,1) household (see Table 3c) means that this type of household needs a housing budget 38 percent more than the typical 2 adult, no children

Table 2a Parameter Estimates of Marginal Propensities and Subsistence Expenditures. Australia 1984.

		Subsistence Expenditures (a_{ih}'s) Household Type (no. of adults, no. of children)							
Commodity Type	b_i	*(1,0)*	*(1,1)*	*(1,2)*	*(1,3)*	*(2,0)*	*(2,1)*	*(2,2)*	*(2,3)*
Housing	0.0809	32.8516	53.0603	54.9843	52.4823	44.8173	69.8950	70.6669	71.7905
	(0.0165)	(1.7937)	(4.4948)	(7.7882)	(9.7435)	(2.2827)	(3.6503)	(3.2700)	(7.1395)
Fuel & Power	0.0065	6.1167	10.4388	10.6784	12.6145	9.8482	12.9185	13.9918	15.6523
	(0.0020)	(0.1676)	(0.9881)	(0.9453)	(1.2242)	(0.2981)	(0.5278)	(0.4331)	(0.8156)
Food	0.0512	32.1867	47.1748	59.9861	72.5586	62.9764	77.1668	91.3443	104.1488
	(0.0099)	(0.9910)	(3.2068)	(4.9182)	(8.2481)	(1.3484)	(2.4135)	(2.3673)	(4.1632)
Alcohol & Tobacco	0.0247	8.7058	11.7255	13.8010	11.7258	17.4612	21.5562	20.5659	22.3741
	(0.0056)	(0.6118)	(1.6749)	(2.5392)	(3.2138)	(0.8014)	(1.2240)	(1.0735)	(2.1468)
Clothing & Footwear	0.0330	9.0226	17.7694	25.8131	24.7771	20.4158	26.1354	29.2710	34.8324
	(0.0081)	(0.8550)	(2.8014)	(4.0070)	(9.8763)	(1.0986)	(1.7714)	(1.8363)	(3.1203)
Household Furnishings	0.0688	23.8504	38.3620	44.9380	47.1475	45.2752	57.1834	61.0771	65.6387
& Equipment	(0.0181)	(1.9377)	(5.9027)	(8.4555)	(11.4133)	(2.4011)	(3.9003)	(4.1461)	(6.9692)
Medical & Health Care	0.0159	6.6053	6.0384	7.6707	8.7500	13.5153	17.5943	18.5922	20.1649
	(0.0035)	(0.4041)	(1.1251)	(1.5632)	(2.0255)	(0.4724)	(0.8864)	(0.6496)	(1.3068)
Transport	0.0813	28.1529	39.5010	48.0318	61.8413	57.9589	78.6268	74.4979	80.2571
	(0.0205)	(2.2948)	(7.5193)	(9.8911)	(16.0141)	(2.7226)	(5.6363)	(3.8978)	(7.0483)
Recreation	0.0903	25.6312	33.6411	38.1619	38.5464	48.3659	52.2621	61.7254	65.9825
& Entertainment	(0.0184)	(2.0737)	(5.3111)	(7.4730)	(11.6776)	(2.5947)	(4.0700)	(3.7903)	(6.9492)
Personal Care	0.0094	3.8541	4.8124	7.4534	7.2261	6.6789	8.0158	8.2586	8.7448
	(0.0024)	(0.3515)	(0.6751)	(1.6974)	(1.4564)	(0.3199)	(0.5388)	(0.4375)	(0.8265)
Others	0.5381	82.9906	117.2381	130.6038	166.8160	142.7404	250.7325	261.0984	287.4648
	(0.0909)	(9.0168)	(33.3144)	(47.2060)	(75.8134)	(11.9973)	(22.5628)	(19.2897)	(51.2205)
Total	1.0000	259.9678	379.7618	442.1225	504.4856	470.0535	672.0867	711.0894	777.0510

Note: The estimated standard errors are in parentheses.

Table 2b Parameters Estimates of Marginal Propensities and Subsistence Expenditures. Australia 1988-89.

		Subsistence Expenditures (a_{ih}'s) Household Type (no. of adults, no. of children)							
Commodity Type	b_i	*(1,0)*	*(1,1)*	*(1,2)*	*(1,3)*	*(2,0)*	*(2,1)*	*(2,2)*	*(2,3)*
Housing	0.1721	44.6461	55.6935	62.2498	69.3904	54.3326	80.8232	82.4249	89.4976
	(0.0041)	(1.6292)	(3.9026)	(4.4800)	(9.9430)	(1.8154)	(4.1615)	(3.4164)	(4.4639)
Fuel & Power	0.0097	7.5720	10.3483	11.9601	12.4822	11.2492	13.6669	15.1027	16.1827
	(0.0004)	(0.1717)	(0.5242)	(0.6003)	(1.2544)	(0.1994)	(0.3794)	(0.3524)	(0.4757)
Food	0.1035	40.4086	55.4349	72.0089	80.7954	76.2837	94.2910	108.5187	120.1607
	(0.0023)	(0.8475)	(2.8826)	(3.3039)	(7.6824)	(1.0129)	(2.1782)	(2.0471)	(2.8829)
Alcohol & Tobacco	0.0342	12.6534	10.2387	8.5610	7.6153	22.1017	20.9678	19.0782	16.8621
	(0.0014)	(0.6188)	(1.0512)	(1.2132)	(1.4246)	(0.7419)	(1.2963)	(1.2491)	(1.0574)
Clothing & Footwear	0.0701	10.6531	18.1682	18.4018	28.1081	20.0279	25.5854	27.9937	32.8947
	(0.0022)	(0.8219)	(2.7734)	(2.7713)	(5.7923)	(1.0271)	(1.9432)	(1.7799)	(2.5240)
Household Furnishings	0.1322	27.0147	32.7264	38.0498	40.1224	49.4040	71.6005	56.8178	65.3866
& Equipment	(0.0047)	(1.5508)	(3.5949)	(5.0889)	(9.7593)	(2.2763)	(5.9201)	(3.9197)	(4.5354)
Medical & Health Care	0.0376	9.4661	8.2428	11.8985	8.9702	17.6169	22.2135	22.5436	23.1035
	(0.0012)	(0.6868)	(1.3382)	(1.9893)	(2.4079)	(0.5132)	(1.3274)	(0.7944)	(1.0619)
Transport	0.1358	33.5432	36.5327	39.7687	49.8903	64.0183	65.0505	76.2271	87.4452
	(0.0051)	(2.1086)	(4.4341)	(5.5586)	(14.7220)	(2.4982)	(4.2747)	(3.9894)	(7.4524)
Recreation	0.1745	25.9738	27.8446	24.7377	39.8047	48.3563	49.6488	61.9587	66.1164
& Entertainment	(0.0047)	(1.4963)	(4.3098)	(3.1839)	(9.7877)	(2.4763)	(4.3307)	(4.6053)	(6.0698)
Personal Care	0.0156	4.1486	5.9926	7.4373	5.6231	7.6723	9.1436	9.8783	9.0123
	(0.0006)	(0.2291)	(0.6766)	(1.0986)	(1.0526)	(0.3116)	(0.6180)	(0.4776)	(0.5997)
Others	0.1150	12.9057	23.3362	20.2497	18.4044	22.8208	31.8058	40.8624	47.4238
	(0.0031)	(1.2465)	(3.4371)	(2.2069)	(3.1771)	(1.4203)	(2.5751)	(3.5113)	(5.3230)
Total	1.0001	228.9851	284.5588	315.3233	361.2065	393.8838	484.7969	521.4062	574.0855

Note: The estimated standard errors are in parentheses.

Table 2c. Parameters Estimates of Marginal Propensities and Subsistence Expenditures. Australia 1993-94.

Commodity Type		*Subsistence Expenditures (a_{ih}'s) Household Type (no. of adults, no. of children)*							
	b_i	*(1,0)*	*(1,1)*	*(1,2)*	*(1,3)*	*(2,0)*	*(2,1)*	*(2,2)*	*(2,3)*
Housing	0.1742	58.9867	92.4664	86.0663	93.7371	72.0887	99.4455	94.3581	91.7331
	(0.0033)	1.5850	5.0680	5.5520	7.9753	1.7958	4.0119	3.4111	4.4120
Fuel & Power	0.0089	10.5317	14.5945	17.3831	19.8280	15.5426	19.1783	20.8313	21.2057
	(0.0004)	0.1935	0.6168	0.8775	1.6085	0.2185	0.4735	0.4056	0.5296
Food	0.1144	49.2828	70.0878	90.9117	93.8108	94.3576	114.7908	136.1056	153.2327
	(0.0021)	0.9224	2.7902	3.6088	4.9965	1.1158	2.3024	2.3963	3.5558
Alcohol & Tobacco	0.0267	15.0098	15.0217	13.0949	15.6954	27.0314	26.2622	22.2245	22.1742
	(0.0012)	0.6296	1.3700	1.3927	2.1010	0.7533	1.2903	1.0745	1.5231
Clothing & Footwear	0.0798	11.5402	19.3199	27.9234	27.2097	24.4825	27.2869	38.0946	39.6102
	(0.0020)	0.8750	2.2997	3.5633	5.1879	1.2075	1.8204	2.4502	4.2270
Household Furnishings	0.1118	32.0272	52.0984	58.1885	56.8986	66.3583	81.3064	82.7068	76.0864
& Equipment	(0.0033)	1.2990	3.7832	5.1105	9.7928	2.2263	4.1964	4.0728	4.4750
Medical & Health Care	0.0404	12.5725	13.0920	16.7215	12.9669	25.2899	27.9016	32.1832	31.0671
	(0.0011)	0.5075	1.3643	1.9228	2.7447	0.6022	1.4535	1.1503	1.5508
Transport	0.1382	44.5204	54.7218	61.2014	68.1301	84.1265	101.4519	95.0724	102.5578
	(0.0051)	2.5005	8.5809	6.7731	14.8432	2.8015	5.7070	5.2991	7.3800
Recreation	0.1770	38.3862	43.7980	56.1104	50.0688	75.6770	71.7307	77.2948	92.9126
& Entertainment	(0.0041)	1.8603	5.0096	6.5329	10.7139	2.6125	4.2536	3.8692	6.3789
Personal Care	1.37E-02	5.2558	7.3174	7.5513	5.5752	10.3200	9.8865	11.8390	12.2997
	(0.0006)	0.2605	0.7996	0.7734	0.9157	0.3362	0.6148	0.5695	0.8062
Others	0.1149632	18.3025	25.4807	50.3898	29.9269	33.3312	43.2878	51.5687	63.9146
	(0.0029)	1.2940	2.5045	14.6067	6.1602	1.9183	3.4159	2.8845	4.5032
Total	1.0000	296.4158	407.9986	485.5424	473.8474	528.6056	622.5285	662.2790	706.7942

Note: The estimated standard errors are in parentheses.

Table 3a. Estimates of Commodity-Specific Scales, Australia 1984.

	Commodity Specific Scales							
	Household Type (no. of adults, no. of children)							
Commodity Type	*(1,0)*	*(1,1)*	*(1,2)*	*(1,3)*	*(2,0)*	*(2,1)*	*(2,2)*	*(2,3)*
Housing	0.73	1.18	1.23	1.17	1.00	1.56	1.58	1.60
	(0.04)	(0.08)	(0.09)	(0.19)	(0.00)	(0.09)	(0.08)	(0.10)
Fuel & Power	0.62	1.06	1.08	1.28	1.00	1.31	1.42	1.59
	(0.02)	(0.05)	(0.06)	(0.11)	(0.00)	(0.04)	(0.04)	(0.05)
Food	0.51	0.75	0.95	1.15	1.00	1.23	1.45	1.65
	(0.01)	(0.04)	(0.05)	(0.10)	(0.00)	(0.03)	(0.03)	(0.04)
Alcohol & Tobacco	0.50	0.67	0.79	0.67	1.00	1.23	1.18	1.28
	(0.03)	(0.05)	(0.06)	(0.07)	(0.00)	(0.07)	(0.06)	(0.05)
Clothing & Footwear	0.44	0.87	1.26	1.21	1.00	1.28	1.43	1.71
	(0.05)	(0.15)	(0.15)	(0.30)	(0.00)	(0.11)	(0.11)	(0.15)
Household Furnishings	0.53	0.85	0.99	1.04	1.00	1.26	1.35	1.45
& Equipment	(0.04)	(0.08)	(0.11)	(0.20)	(0.00)	(0.14)	(0.09)	(0.11)
Medical & Health Care	0.49	0.45	0.57	0.65	1.00	1.30	1.38	1.49
	(0.04)	(0.08)	(0.11)	(0.14)	(0.00)	(0.08)	(0.06)	(0.07)
Transport	0.49	0.68	0.83	1.07	1.00	1.36	1.29	1.38
	(0.04)	(0.07)	(0.09)	(0.23)	(0.00)	(0.08)	(0.08)	(0.13)
Recreation	0.53	0.70	0.79	0.80	1.00	1.08	1.28	1.36
& Entertainment	(0.04)	(0.09)	(0.07)	(0.21)	(0.00)	(0.10)	(0.11)	(0.14)
Personal Care	0.58	0.72	1.12	1.08	1.00	1.20	1.24	1.31
	(0.04)	(0.09)	(0.15)	(0.14)	(0.00)	(0.09)	(0.08)	(0.09)
Others	0.58	0.82	0.91	1.17	1.00	1.76	1.83	2.01
	(0.06)	(0.16)	(0.11)	(0.15)	(0.00)	(0.14)	(0.19)	(0.27)

Note: The estimated standard errors are in parentheses.

Table 3b. Estimates of Commodity-Specific Scales, Australia 1988-89.

	Commodity Specific Scales Household Type (no. of adults, no. of children)							
Commodity Type	*(1,0)*	*(1,1)*	*(1,2)*	*(1,3)*	*(2,0)*	*(2,1)*	*(2,2)*	*(2,3)*
Housing	0.82	1.03	1.15	1.28	1.00	1.49	1.52	1.65
	(0.04)	(0.08)	(0.09)	(0.19)	(0.00)	(0.09)	(0.08)	(0.10)
Fuel & Power	0.67	0.92	1.06	1.11	1.00	1.21	1.34	1.44
	(0.02)	(0.05)	(0.06)	(0.11)	(0.00)	(0.04)	(0.04)	(0.05)
Food	0.53	0.73	0.94	1.06	1.00	1.24	1.42	1.58
	(0.01)	(0.04)	(0.05)	(0.10)	(0.00)	(0.03)	(0.03)	(0.04)
Alcohol & Tobacco	0.57	0.46	0.39	0.34	1.00	0.95	0.86	0.76
	(0.03)	(0.05)	(0.06)	(0.07)	(0.00)	(0.07)	(0.06)	(0.05)
Clothing & Footwear	0.53	0.91	0.92	1.40	1.00	1.28	1.40	1.64
	(0.05)	(0.15)	(0.15)	(0.30)	(0.00)	(0.11)	(0.11)	(0.15)
Household Furnishings	0.55	0.66	0.77	0.81	1.00	1.45	1.15	1.32
& Equipment	(0.04)	(0.08)	(0.11)	(0.20)	(0.00)	(0.14)	(0.09)	(0.11)
Medical & Health Care	0.54	0.47	0.68	0.51	1.00	1.26	1.28	1.31
	(0.04)	(0.08)	(0.11)	(0.14)	(0.00)	(0.08)	(0.06)	(0.07)
Transport	0.52	0.57	0.62	0.78	1.00	1.02	1.19	1.37
	(0.04)	(0.07)	(0.09)	(0.23)	(0.00)	(0.08)	(0.08)	(0.13)
Recreation	0.54	0.58	0.51	0.82	1.00	1.03	1.28	1.37
& Entertainment	(0.04)	(0.09)	(0.07)	(0.21)	(0.00)	(0.10)	(0.11)	(0.14)
Personal Care	0.54	0.78	0.97	0.73	1.00	1.19	1.29	1.17
	(0.04)	(0.09)	(0.15)	(0.14)	(0.00)	(0.09)	(0.08)	(0.09)
Others	0.57	1.02	0.89	0.81	1.00	1.39	1.79	2.08
	(0.06)	(0.16)	(0.11)	(0.15)	(0.00)	(0.14)	(0.19)	(0.27)

Note: The estimated standard errors are in parentheses.

Table 3c. Estimates of Commodity-Specific Scales, Australia 1993-94.

Commodity Type	*Commodity Specific Scales Household Type (no. of adults, no. of children)* (1,0)	(1,1)	(1,2)	(1,3)	(2,0)	(2,1)	(2,2)	(2,3)
Housing	0.82	1.28	1.19	1.30	1.00	1.38	1.31	1.27
	(0.04)	(0.10)	(0.10)	(0.14)	(0.04)	(0.08)	(0.07)	(0.09)
Fuel & Power	0.68	0.94	1.12	1.28	1.00	1.23	1.34	1.36
	(0.03)	(0.06)	(0.09)	(0.16)	(0.03)	(0.06)	(0.05)	(0.06)
Food	0.52	0.74	0.96	0.99	1.00	1.22	1.44	1.62
	(0.03)	(0.06)	(0.08)	(0.11)	(0.04)	(0.06)	(0.07)	(0.09)
Alcohol & Tobacco	0.56	0.56	0.48	0.58	1.00	0.97	0.82	0.82
	(0.06)	(0.10)	(0.10)	(0.15)	(0.09)	(0.11)	(0.09)	(0.12)
Clothing & Footwear	0.47	0.79	1.14	1.11	1.00	1.11	1.56	1.62
	(0.11)	(0.24)	(0.36)	(0.48)	(0.19)	(0.24)	(0.33)	(0.45)
Household Furnishings	0.48	0.79	0.88	0.86	1.00	1.23	1.25	1.15
& Equipment	(0.06)	(0.14)	(0.18)	(0.31)	(0.11)	(0.17)	(0.16)	(0.17)
Medical & Health Care	0.50	0.52	0.66	0.51	1.00	1.10	1.27	1.23
	(0.06)	(0.12)	(0.16)	(0.22)	(0.09)	(0.15)	(0.14)	(0.16)
Transport	0.53	0.65	0.73	0.81	1.00	1.21	1.13	1.22
	(0.08)	(0.20)	(0.17)	(0.34)	(0.12)	(0.18)	(0.17)	(0.21)
Recreation	0.51	0.58	0.74	0.66	1.00	0.95	1.02	1.23
& Entertainment	(0.07)	(0.14)	(0.18)	(0.29)	(0.12)	(0.14)	(0.14)	(0.20)
Personal Care	0.51	0.71	0.73	0.54	1.00	0.96	1.15	1.19
	(0.07)	(0.17)	(0.16)	(0.18)	(0.12)	(0.15)	(0.15)	(0.19)
Others	0.55	0.76	1.51	0.90	1.00	1.30	1.55	1.92
	(0.10)	(0.17)	(0.82)	(0.36)	(0.16)	(0.25)	(0.25)	(0.35)

Note: The estimated standard errors are in parentheses.

household to maintain the same standard of living as the latter. Similarly, the Fuel and power scale of 0.68 for the (1,0) type household implies that a childless couple would require 9 percent less (in per capita terms) in Fuel expenditures than a single member householder to be on comparable standards of living – a clear demonstration of the economies of scale advantage for multiperson households. The equivalent 1988-89 scale values for these commodities and household types are 1.49 and 0.67 respectively (in Table 4a) and can be similarly interpreted. The equivalent scale values for 1988-89 are 1.49 for Housing and 0.67 for Fuel and power, and for 1984 are 1.56 for Housing and 0.62 for Fuel and Power (in Table 3a) and can be similarly interpreted.

For most expenditure items, the commodity scales increase with the increase with the increase in the number of children. These increases are observed to occur at a decreasing rate indicating economies of scale for additional children. After the first child, there exists strong economies of scale for additional children, particularly for expenditures towards Housing, Fuel & Power, Household furnishings and Transport. For two-adult households in 1993-94, the decline in the Housing scale after the first child is unexpected. One interpretation is that this could reflect the growing practice of many young families to get children to share a room, rather than provide for a room per child. The scales in 1984 and 1988-89 clearly show that economies of scale in housing were achieved with every additional child.

The 1984 scales for Alcohol and Tobacco tend to increase with the number children but this trend is clearly reversed in 1988-89 and 1993-94. Also, the scales for Medical and health care, Recreation and entertainment, and Others exhibit no defined trend for one-adult households. A more thorough investigation of expenditure patterns of households may be required for us to provide definitive explanations for such deviations but one possibility is that the presence of children in the household tends to influence expenses away from ‘adult goods’ under which alcohol, tobacco and other miscellaneous goods are classified.

Has there been a significant change in the scale relativities over time? Information from Tables 4 and 5, which compare some estimates for the survey years 1984, 1988-89 and 1993-94 provide some answers. In Table 4, commodity-specific scale estimates are presented in such a way that comparisons over the three survey years is facilitated. There are noticeable changes in the scales over time. For two-adult households with children, the scales for Housing, Fuel & power, Alcohol & tobacco, Medical & health care, Transport and Recreation and Entertainment have decreased consistently in the last 10 years. Such decrease in the scale values indicates that the cost requirements for the additional family member (to maintain standard of living) are not as much as it used to be. In other words, children are more affordable these days than before! The scale for Food remained more or less the same over time, implying then that the implied affordability of having children in more recent years do not come from Food expenditures. This analysis however holds only for a two-parent family. For single parent households, the typical trend of the scale values for most commodities is a decline between 1984 and 1988-89 and then an increase between 1988-89 and 1993-94.

It is further noted here that the largest differences in the scale estimates occurred in the one-adult, three-children household groups. Since the number of households in this group is relatively small, and the standard errors of the estimated scales are relatively high, these differences may reflect sampling error.

The general scales computed from equation (18) are presented in Table 5. Because these scales depend on income x_r, they are computed for three income levels[5]. First noted is that the estimated general scales are stable over different reference income levels. Also, for two-adult households, the 1993-94 scales are less than both the 1984 and 1988-89 scales. The 1988-89 and 1993-94 estimates are generally quite similar. Further, the conclusion by Binh and Whiteford (1990) that "there is strong evidence of economies of scale in the second child but adding the third child increased these households' needs considerably" no longer holds for the later data sets. The 1993-94 scales, in particular, suggest equal cost requirements for the 2nd and 3rd children.

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[5] Levels were made comparable to those used by Binh and Whiteford (1990) to facilitate comparison.

Table 4 Estimates of Commodity-Specific Scales

		Commodity-Specific Scales (S_{ih}) Household Type (no. of adults, no. of children)							
Commodity Type	***Year***	***(1,0)***	***(1,1)***	***(1,2)***	***(1,3)***	***(2,0)***	***(2,1)***	***(2,2)***	***(2,3)***
Housing	1984	0.73	1.18	1.23	1.17	1.00	1.56	1.58	1.60
	1988-89	0.82	1.03	1.15	1.28	1.00	1.49	1.52	1.65
	1993-94	0.82	1.28	1.19	1.30	1.00	1.38	1.31	1.27
Fuel & Power	1984	0.62	1.06	1.08	1.28	1.00	1.31	1.42	1.59
	1988-89	0.67	0.92	1.06	1.11	1.00	1.21	1.34	1.44
	1993-94	0.68	0.94	1.12	1.28	1.00	1.23	1.34	1.36
Food	1984	0.51	0.75	0.95	1.15	1.00	1.23	1.45	1.65
	1988-89	0.53	0.73	0.94	1.06	1.00	1.24	1.42	1.58
	1993-94	0.52	0.74	0.96	0.99	1.00	1.22	1.44	1.62
Alcohol & Tobacco	1984	0.50	0.67	0.79	0.67	1.00	1.23	1.18	1.28
	1988-89	0.57	0.46	0.39	0.34	1.00	0.95	0.86	0.76
	1993-94	0.56	0.56	0.48	0.58	1.00	0.97	0.82	0.82
Clothing & Footwear	1984	0.44	0.87	1.26	1.21	1.00	1.28	1.43	1.71
	1988-89	0.53	0.91	0.92	1.40	1.00	1.28	1.40	1.64
	1993-94	0.47	0.79	1.14	1.11	1.00	1.11	1.56	1.62
Household Furnishings	1984	0.53	0.85	0.99	1.04	1.00	1.26	1.35	1.45
& Equipment	1988-89	0.55	0.66	0.77	0.81	1.00	1.45	1.15	1.32
	1993-94	0.48	0.79	0.88	0.86	1.00	1.23	1.25	1.15
Medical & Health Care	1984	0.49	0.45	0.57	0.65	1.00	1.30	1.38	1.49
	1988-89	0.54	0.47	0.68	0.51	1.00	1.26	1.28	1.31
	1993-94	0.50	0.52	0.66	0.51	1.00	1.10	1.27	1.23
Transport	1984	0.49	0.68	0.83	1.07	1.00	1.36	1.29	1.38
	1988-89	0.52	0.57	0.62	0.78	1.00	1.02	1.19	1.37
	1993-94	0.53	0.65	0.73	0.81	1.00	1.21	1.13	1.22
Recreation	1984	0.53	0.70	0.79	0.80	1.00	1.08	1.28	1.36
& Entertainment	1988-89	0.54	0.58	0.51	0.82	1.00	1.03	1.28	1.37
	1993-94	0.51	0.58	0.74	0.66	1.00	0.95	1.02	1.23
Personal Care	1984	0.58	0.72	1.12	1.08	1.00	1.20	1.24	1.31
	1988-89	0.54	0.78	0.97	0.73	1.00	1.19	1.29	1.17
	1993-94	0.51	0.71	0.73	0.54	1.00	0.96	1.15	1.19
Others	1984	0.58	0.82	0.91	1.17	1.00	1.76	1.83	2.01
	1988-89	0.57	1.02	0.89	0.81	1.00	1.39	1.79	2.08
	1993-94	0.55	0.76	1.51	0.90	1.00	1.30	1.55	1.92

6 Conclusion

This paper introduced a maximum likelihood estimation procedure for an extended linear expenditure system that has different intercepts and different error covariance matrices for groups of households with different composition. Cost estimates of children in the form of household equivalence scales, both commodity specific and general scales, were obtained from this system.

Table 5 Estimates of General Scales

Reference Income	*Year*	*General Scales (S_h)* *Household Type (no. of adults, no. of children)* (1,0)	(1,1)	(1,2)	(1,3)	(2,0)	(2,1)	(2,2)	(2,3)
Low Income	**1984**	0.58	0.70	0.77	0.88	1.00	1.17	1.26	1.40
($325 p.w.)	**1988-89**	0.58	0.72	0.81	0.92	1.00	1.23	1.32	1.45
	1993-94	0.56	0.78	0.91	0.91	1.00	1.18	1.25	1.33
Medium Income	**1984**	0.58	0.73	0.82	0.94	1.00	1.27	1.36	1.49
($450 p.w.)	**1988-89**	0.58	0.72	0.80	0.91	1.00	1.23	1.33	1.46
	1993-94	0.56	0.77	0.92	0.90	1.00	1.18	1.25	1.34
High Income	**1984**	0.57	0.76	0.86	0.99	1.00	1.36	1.44	1.58
($700 p.w.)	**1988-89**	0.58	0.72	0.79	0.91	1.00	1.23	1.33	1.47
	1993-94	0.56	0.77	0.92	0.89	1.00	1.18	1.25	1.34

In general, the results show that to be able to maintain the living standards of the household before the addition of children, the family budget will have to be increased by about 18 per cent for the first child for two parent families, and by about 22 percent for single parent families. The additional budget requirements for the 2nd and 3rd children will still be positive but not as much as that of the first child. Larger adjustments are required by single parent families compared to two-parent families.

The commodity specific scales show that the budget requirements vary across the various commodity groups. For the first child, there is a significantly high budget requirement (38 per cent) to meet Housing needs of the child while in terms of Food, the budget needs to be adjusted by only 20 percent. The results show that single parent households needs more assistance to meet Housing adjustment needs for than two-parent families. After the 1st child, large gains in economies of scale were observed for Housing, Fuel & Power, Household Goods and Transport items. Also, the findings seem to suggest that the presence of children induces adult family members to consume less of the "adult goods" such as Alcohol & Tobacco.

Appendix 1. Derivation of the ML Estimation Procedure and Expressions for the Covariance Matrices

This Appendix outlines details of the derivation of the maximum likelihood estimators which is the basis of the new iterative procedure introduced in the text [Section 3], as well as expressions for the associated covariance matrices. The starting point of the derivations below is the stochastic model described in Section 3, where the notation is set.

A1. Derivation of Maximum Likelihood Estimators

Noting that $\mathbf{V}_{1}$, $\mathbf{V}_{2}$, …, $\mathbf{V}_{H}$ in the system of equations defined in (20) are independent, the log-likelihhood for all parameters, given data on all household types, can be written as

$$\begin{aligned}\log \mathbf{L} = {} & \frac{nM}{2}\log(2\pi) - \frac{1}{2}\sum_{h=1}^{H} M_h \log\left|\Omega_h\right| \\ & - \frac{1}{2}\sum_{h=1}^{H}\left(\mathbf{V}_h - \mathbf{Z}_h\Theta_h - \mathbf{X}_h\eta\right)'\left(\Omega_h^{-1}\otimes I_{M_h}\right)\left(\mathbf{V}_h - \mathbf{Z}_h\Theta_h - \mathbf{X}_h\eta\right)\end{aligned} \tag{24}$$

where $M = \sum_{h=1}^{H} M_h$. To maximise this function, the possibility of concentrating out the Θ_h is first investigated. Working in this direction, the last term can be written (without the summation) as

$$\begin{aligned}\mathbf{Q}_h &= \left(\mathbf{V}_h - \mathbf{Z}_h\Theta_h - \mathbf{X}_h\eta\right)'\left(\Omega_h^{-1}\otimes I_{M_h}\right)\left(\mathbf{V}_h - \mathbf{Z}_h\Theta_h - \mathbf{X}_h\eta\right) \\ &= \left(\mathbf{V}_h - \mathbf{X}_h\eta\right)'\left(\Omega_h^{-1}\otimes I_{M_h}\right)\left(\mathbf{V}_h - \mathbf{X}_h\eta\right) + \Theta_h'\mathbf{Z}_h'\left(\Omega_h^{-1}\otimes I_{M_h}\right)\mathbf{Z}_h\Theta_h \\ &\quad - 2\Theta_h'\mathbf{Z}_h'\left(\Omega_h^{-1}\otimes I_{M_h}\right)\left(\mathbf{V}_h - \mathbf{X}_h\eta\right)\end{aligned} \tag{25}$$

Now,

$$\frac{\partial \log \mathbf{L}}{\partial \Theta_h} = \frac{1}{2}\frac{\partial \mathbf{Q}_h}{\partial \Theta_h} \tag{26}$$

$$= -\frac{1}{2}\left[2\mathbf{Z}_h'\left(\boldsymbol{\Omega}_h^{-1} \otimes I_{M_h}\right)\mathbf{Z}_h\Theta_h - 2\mathbf{Z}_h'\left(\boldsymbol{\Omega}_h^{-1} \otimes I_{M_h}\right)\left(\mathbf{V}_h - \mathbf{X}_h\eta\right)\right]$$

Setting this derivative to zero and solving for the maximising value $\hat{\Theta}_h$ gives

$$\hat{\Theta}_h = \left[\mathbf{Z}_h'\left(\boldsymbol{\Omega}_h^{-1} \otimes I_{M_h}\right)\mathbf{Z}_h\right]^{-1}\mathbf{Z}_h'\left(\boldsymbol{\Omega}_h^{-1} \otimes I_{M_h}\right)\left(\mathbf{V}_h - \mathbf{X}_h\eta\right) \tag{27}$$

Now,

$$\begin{aligned}\mathbf{Z}_h'\left(\boldsymbol{\Omega}_h^{-1} \otimes I_{M_h}\right)\mathbf{Z}_h &= \left(I_n \otimes z_h'\right)\left(\boldsymbol{\Omega}_h^{-1} \otimes I_{M_h}\right)\left(I_n \otimes z_h\right) \\ &= \boldsymbol{\Omega}_h^{-1} \otimes z_h' z_h \\ &= \boldsymbol{\Omega}_h^{-1} \otimes M_h \\ &= M_h \boldsymbol{\Omega}_h^{-1}\end{aligned} \tag{28}$$

Also,

$$\mathbf{Z}_h'\left(\boldsymbol{\Omega}_h^{-1} \otimes I_{M_h}\right) = \boldsymbol{\Omega}_h^{-1} \otimes z_h' \tag{29}$$

Using (28) and (29) in (27) gives

$$\begin{aligned}\hat{\Theta}_h &= \left[\boldsymbol{\Omega}_h^{-1} \otimes M_h\right]^{-1}\left(\boldsymbol{\Omega}_h^{-1} \otimes z_h'\right)\left(\mathbf{V}_h - \mathbf{X}_h\eta\right) \\ &= \left(\boldsymbol{\Omega}_h \otimes M_h^{-1}\right)\left(\boldsymbol{\Omega}_h^{-1} \otimes z_h'\right)\left(\mathbf{V}_h - \mathbf{X}_h\eta\right) \\ &= \left(I_n \otimes \frac{1}{M_h} z_h'\right)\left(\mathbf{V}_h - \mathbf{X}_h\eta\right)\end{aligned} \tag{30}$$

Considering the i^{th} row in equation (30)

$$\begin{aligned}\hat{\theta}_{ih} &= \frac{1}{M_h} z_h'(v_{ih} - x_h\eta_i) \\ &= \bar{v}_{ih} - \bar{x}_h\eta_i\end{aligned} \tag{31}$$

where $\bar{v}_{ih} = \frac{1}{M_h} z_h' v_{ih}$ is the average expenditure on commodity i for all households of type h and $\bar{x}_h = \frac{1}{M_h} z_h' x_h$ is the average income of all h-type households. The result in (31) is an important one. It means that $\hat{\theta}_{ih}$'s do not depend on Ω_h and can be computed at the end of the maximum likelihood algorithm, after we have estimated Ω_h and η_i.

Let $\overline{\mathbf{V}}_h' = (\bar{v}_{1h}, \bar{v}_{2h}, \ldots, \bar{v}_{nh})$ and $\overline{X}_h = I_n \otimes \bar{x}_h$. Then,

$$\hat{\Theta}_h = \overline{\mathbf{V}}_h - \overline{\mathbf{X}}_h \eta \qquad (32)$$

Substituting (32) into (25) yields

$$\begin{aligned}\mathbf{Q}_h &= \left[\left(\mathbf{V}_h - \mathbf{Z}_h\overline{\mathbf{V}}_h\right) - \left(\mathbf{X}_h - \mathbf{Z}_h\overline{\mathbf{X}}_h\right)\eta\right]' \left(\Omega_h^{-1} \otimes I_{M_h}\right)\left[\left(\mathbf{V}_h - \mathbf{Z}_h\overline{\mathbf{V}}_h\right) - \left(\mathbf{X}_h - \mathbf{Z}_h\overline{\mathbf{X}}_h\right)\eta\right] \\ &= \left(\mathbf{V}_h^* - \mathbf{X}_h^*\eta\right)' \left(\Omega_h^{-1} \otimes I_{M_h}\right)\left(\mathbf{V}_h^* - \mathbf{X}_h^*\eta\right)\end{aligned} \qquad (33)$$

where $\mathbf{V}_h^* = \mathbf{V}_h - \mathbf{Z}_h\overline{\mathbf{V}}_h$ is a vector of expenditures expressed in terms of deviations from the mean expenditures for each commodity and household type, and $\mathbf{X}_h^* = \mathbf{X}_h - \mathbf{Z}_h\overline{\mathbf{X}}_h$ is a vector of incomes expressed in terms of deviations from the mean incomes for each household type. The concentrated log-likelihood function can now be written as

$$\begin{aligned}\log \mathbf{L}^* &= -\frac{nM}{2}\log(2\pi) - \frac{1}{2}\sum_{h=1}^{H} M_h \log\left|\Omega_h\right| \\ &\quad - \frac{1}{2}\sum_{h=1}^{H}\left(\mathbf{V}_h^* - \mathbf{X}_h^*\eta\right)'\left(\Omega_h^{-1} \otimes I_{M_h}\right)\left(\mathbf{V}_h^* - \mathbf{X}_h^*\eta\right)\end{aligned} \qquad (34)$$

$$= -\frac{nM}{2}\log(2\pi) + \frac{1}{2}\sum_{h=1}^{H} M_h \log\left|\Omega_h^{-1}\right| - \frac{1}{2}\sum_{h=1}^{H} tr\left[\mathbf{W}_h\Omega_h^{-1}\right] \qquad (35)$$

where $\mathbf{W}_h$ is an $(n \times n)$ matrix of $(i,j)^{th}$ element given by[6]

$$\left[\mathbf{W}_h\right]_{ij} = (v_{ih}^* - x_h^*\eta_i)'(v_{jh}^* - x_h^*\eta_j). \qquad (36)$$

Differentiation with respect to Ω_h^{-1} yields

[6] See Judge, et.al. (1988, p.553) for details of the two alternative specifications in (34) and (35).

$$\frac{\partial \log \mathbf{L}^{*}}{\partial \Omega_h^{-1}} = \frac{M_h}{2}\Omega_h - \frac{1}{2}\mathbf{W}_h \tag{37}$$

Setting this derivative equal to zero yields the maximum likelihood estimator for Ω_h given η_i as follows:

$$\hat{\Omega}_{\boldsymbol{h}} = \frac{1}{\boldsymbol{M_h}}\mathbf{W}_{\boldsymbol{h}} \tag{38}$$

To find an expression for the maximum likelihood estimator for η, given Ω_h, we return to the last term in (34) and rewrite it as

$$\begin{aligned}\sum_{h=1}^{H}\mathbf{Q}_h^{*} &= \sum_{h=1}^{H}\left(\mathbf{V}_h^{*}-\mathbf{X}_h^{*}\eta\right)'\left(\Omega_h^{-1}\otimes I_{M_h}\right)\left(\mathbf{V}_h^{*}-\mathbf{X}_h^{*}\eta\right)\\ &= \sum_{h=1}^{H}\left[\mathbf{V}_h^{*\prime}\left(\Omega_h^{-1}\otimes I_{M_h}\right)\mathbf{V}_h^{*}+\eta'\,\mathbf{X}_h^{*\prime}\left(\Omega_h^{-1}\otimes I_{M_h}\right)\mathbf{X}_h^{*}\eta-2\eta'\mathbf{X}_h^{*\prime}\left(\Omega_h^{-1}\otimes I_{M_h}\right)\mathbf{V}_h^{*}\right]\end{aligned} \tag{39}$$

Now,

$$\begin{aligned}\frac{\partial \log \mathbf{L}^{*}}{\partial \eta} &= -\frac{1}{2}\sum_{h=1}^{H}\frac{\partial \mathbf{Q}_h^{*}}{\partial \eta}\\ &= -\frac{1}{2}\sum_{h=1}^{H}\left[2\mathbf{X}_h^{*\prime}\left(\Omega_h^{-1}\otimes I_{M_h}\right)\mathbf{X}_h^{*}\eta-2\mathbf{X}_h^{*\prime}\left(\Omega_h^{-1}\otimes I_{M_h}\right)\mathbf{V}_h^{*}\right]\end{aligned} \tag{40}$$

Setting this quantity equal to zero yields

$$\left[\sum_{h=1}^{H}\mathbf{X}_h^{*\prime}\left(\Omega_h^{-1}\otimes I_{M_h}\right)\mathbf{X}_h^{*}\right]\hat{\eta} = \sum_{h=1}^{H}\mathbf{X}_h^{*\prime}\left(\Omega_h^{-1}\otimes I_{M_h}\right)\mathbf{V}_h^{*} \tag{41}$$

Now, using obvious notation

$$\begin{aligned}\mathbf{X}_h^{*\prime}\left(\Omega_h^{-1}\otimes I_{M_h}\right)\mathbf{X}_h^{*} &= \left(I_n\otimes x_h^{*\prime}\right)\left(\Omega_h^{-1}\otimes I_{M_h}\right)\left(I_n\otimes x_h^{*}\right)\\ &= \Omega_h^{-1}\otimes x_h^{*\prime}x_h^{*}\\ &= x_h^{*\prime}x_h^{*}\boldsymbol{\Omega}_h^{-1}\end{aligned} \tag{42}$$

Also,

$$\mathbf{X}_h^{*\prime}\left(\Omega_h^{-1}\otimes I_{M_h}\right)\mathbf{V}_h^{*}=\left(I_n\otimes x_h^{*\prime}\right)\left(\Omega_h^{-1}\otimes I_{M_h}\right)\mathbf{V}_h^{*} \tag{43}$$
$$=\left(\Omega_h^{-1}\otimes x_h^{*\prime}\right)\mathbf{V}_h^{*}$$

In light of the 2nd line in (42), equation (43) can be written as

$$\begin{aligned}\mathbf{X}_h^{*\prime}\left(\Omega_h^{-1}\otimes I_{M_h}\right)\mathbf{V}_h^{*} &=\left(\Omega_h^{-1}\otimes x_h^{*\prime}x_h^{*}\right)\left(\Omega_h^{-1}\otimes x_h^{*\prime}x_h^{*}\right)^{\mathbf{-1}}\left(\Omega_h^{-1}\otimes x_h^{*\prime}\right)\mathbf{V}_h^{*}\\ &=\left(x_h^{*\prime}x_h^{*}\Omega_h^{-1}\right)\left(\Omega_h\otimes\left(x_h^{*\prime}x_h^{*}\right)^{-1}\right)\left(\Omega_h^{-1}\otimes x_h^{*\prime}\right)\mathbf{V}_h^{*}+ \qquad (44)\\ &=x_h^{*\prime}x_h^{*}\Omega_h^{-1}\left[I_n\otimes\left(x_h^{*\prime}x_h^{*}\right)^{-1}x_h^{*\prime}\right]\mathbf{V}_h^{*}\\ &=x_h^{*\prime}x_h^{*}\Omega_h^{-1}\hat{\eta}^{h}\end{aligned}$$

where $\hat{\eta}^{h}=\left[I_n\otimes\left(x_h^{*\prime}x_h^{*}\right)^{-1}x_h^{*\prime}\right]\mathbf{V}_h^{*}$ is the OLS estimator for η from observations corresponding only to the h-type households. The i^{th} element in $\hat{\eta}$ is given by $\hat{\eta}_i^{h}=\left(x_h^{*\prime}x_h^{*}\right)^{-1}x_h^{*\prime}v_{ih}^{*}$. Substituting (42) and (44) into (41) yields

$$\left[\sum_{h=1}^{H}\left(x_h^{*\prime}x_h^{*}\right)\Omega_h^{-1}\right]\hat{\eta}=\sum_{h=1}^{H}\left(x_h^{*\prime}x_h^{*}\right)\Omega_h^{-1}\hat{\eta}^{h} \tag{45}$$

or

$$\hat{\eta}=\left[\sum_{h=1}^{H}\left(x_h^{*\prime}x_h^{*}\right)\Omega_h^{-1}\right]^{-1}\sum_{h=1}^{H}\left(x_h^{*\prime}x_h^{*}\right)\Omega_h^{-1}\hat{\eta}^{h} \tag{46}$$

Conditional on Ω_h and Θ_h, the maximum likelihood estimator for η is given by a matrix-weighted average of the h-type household OLS estimators $\hat{\eta}_h$ with weights given by $\left(x_h^{*\prime}x_h^{*}\right)\Omega_h^{-1}$. Maximum likelihood estimators for all the parameters in Θ_h, Ω_h and η are given by the simultaneous solution of equations (31), (38) and (46).

A2. Derivation of the Asymptotic Covariance Matrices

This section outlines the derivation of the asymptotic covariance matrices for the parameters Θ_h and η. A second section details the derivation of the asymptotic covariance matrices for b, a_h, b_i and a_{ih}. The last section derives asymptotic variances for the s_{ih} estimators.

A2.1 Variance Matrices for Θ_h and η

To specify the asymptotic covariance matrix for the maximum likelihood estimator, the second derivatives of the log-likelihood function specified in (24) are required. From (26), (28) and (29), these second derivatives are obtained as follows:

$$\begin{aligned}\frac{\partial^2 \log \mathbf{L}}{\partial\Theta_h \partial\Theta_h'} &= -\mathbf{Z}_h(\Omega_h^{-1} \otimes I_{M_h})\mathbf{Z}_h \\ &= -M_h \Omega_h^{-1}\end{aligned} \qquad (47)$$

$$\frac{\partial^2 \log \mathbf{L}}{\partial\Theta_h \partial\Theta_k'} = 0 \qquad (h \neq k) \qquad (48)$$

$$\begin{aligned}\frac{\partial^2 \log \mathbf{L}}{\partial\Theta_h \partial\eta'} &= -\mathbf{Z}_h'(\Omega_h^{-1} \otimes I_{M_h})\mathbf{X}_h \\ &= -M_h \bar{x}_h \Omega_h^{-1}\end{aligned} \qquad (49)$$

From (24) and (42), we obtain

$$\begin{aligned}\frac{\partial^2 \log \mathbf{L}}{\partial\eta \partial\eta'} &= -\sum_{h=1}^{H} \mathbf{X}_h'(\Omega_h^{-1} \otimes I_{M_h})\mathbf{X}_h \\ &= -\sum_{h=1}^{H} x_h' x_h \Omega_h^{-1}\end{aligned} \qquad (50)$$

It can be shown that the expectations of the cross partial derivatives with respect to θ_{ih} and η_i and the elements of Ω_h are zero. Thus the information matrix is block diagonal, and, providing the interest is not on the standard errors of the maximum likelihood estimator of Ω_h, concern may be confined to the derivatives (47) – (50).

Specifically, let $\Theta' = \left(\Theta_1', \Theta_2', \ldots, \Theta_H', \eta'\right)$, then

$$-\frac{\partial^2 \log \mathbf{L}}{\partial\Theta\partial\Theta'} = \begin{bmatrix} M_1\mathbf{\Omega}_1^{-1} & 0 & \cdots & 0 & M_1\bar{x}_1\mathbf{\Omega}_1^{-1} \\ 0 & M_2\mathbf{\Omega}_2^{-1} & \cdots & 0 & M_2\bar{x}_2\mathbf{\Omega}_2^{-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & M_H\mathbf{\Omega}_H^{-1} & M_H\bar{x}_H\mathbf{\Omega}_H^{-1} \\ M_1\bar{x}_1\mathbf{\Omega}_1^{-1} & M_2\bar{x}_2\mathbf{\Omega}_2^{-1} & \cdots & M_H\bar{x}_H\mathbf{\Omega}_H^{-1} & \sum_{h=1}^{H} x_h' x_h \mathbf{\Omega}_h^{-1} \end{bmatrix} \tag{51}$$

Since this matrix does not contain any stochastic elements, the information matrix obtained by taking expectations of (51) is the same as (51). Let $D = \sum_{h=1}^{H}\left[\left(x_h' x_h - M_h\bar{x}_h^2\right)\mathbf{\Omega}_h^{-1}\right]$, $\bar{x} = \left(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_H\right)'$ and

$$A = \begin{bmatrix} M_1^{-1}\mathbf{\Omega}_1 & & & \\ & M_2^{-1}\mathbf{\Omega}_2 & & \\ & & \ddots & \\ & & & M_H^{-1}\mathbf{\Omega}_H \end{bmatrix} \tag{52}$$

Using results on the partitioned inverse of a matrix and using $V(.)$ to denote the asymptotic covariance matrix, it can be shown that

$$\begin{aligned} V(\hat{\Theta}) &= \left[E - \frac{\partial^2 \log \mathbf{L}}{\partial\Theta\partial\Theta'}\right]^{-1} \\ &= \begin{bmatrix} A^{-1} & A^{-1}(\bar{x}\otimes I) \\ (\bar{x}'\otimes I)A^{-1} & \sum x_h x_h' \mathbf{\Omega}_h^{-1} \end{bmatrix}^{-1} \\ &= \begin{bmatrix} A + \bar{x}\bar{x}'\otimes D^{-1} & -\bar{x}\otimes D^{-1} \\ -\bar{x}\otimes D^{-1} & D^{-1} \end{bmatrix} \end{aligned} \tag{53}$$

The relevant variance components from (53) are $V(\hat{\Theta}_h) = M_h^{-1}\Omega_h + \bar{x}_h^2 D^{-1}$ and $V(\hat{\eta}) = D^{-1}$.

A2.2 Variance Matrices for b, a_h, b_i and a_{ih}

This section provides details of the derivation of the asymptotic covariance matrix of the estimators for the parameters b, a_h, b_i and a_{ih} defined in equations (9) – (12). From (9), $V(\hat{b}) = z'D^{-1}z$ where $z = (1,1,\ldots,1)'$. Let $\mathbf{B} = (b_1, b_2, \ldots, b_n)'$. Then

$$V(\hat{\mathbf{B}}) = \left(\frac{\partial \mathbf{B}}{\partial \eta'}\right) D^{-1} \left(\frac{\partial \mathbf{B}}{\partial \eta'}\right)' \tag{54}$$

Now,

$$\frac{\partial b_i}{\partial \eta_j} = \begin{cases} \dfrac{1}{b}\left(1 - \dfrac{\eta_i}{b}\right) & \textit{for} \quad i = j \\ \dfrac{1}{b}\left(-\dfrac{\eta_i}{b}\right) & \textit{for} \quad i \neq j \end{cases} \tag{55}$$

Let $C = I_n - \frac{1}{b}\eta z'$. From (54) and (55), it follows that $\dfrac{\partial \mathbf{B}}{\partial \eta} = \dfrac{1}{b}C$ and $V(\hat{\mathbf{B}}) = \dfrac{1}{b^2} C D^{-1} C'$.

Consider now the covariance matrix for the $\hat{a}_{ih}$. Let $\alpha_h = (a_{1h}, a_{2h}, \ldots, a_{nh})'$. By definition, we have

$$V(\hat{\alpha}_h) = \begin{bmatrix} \dfrac{\partial \alpha_h}{\partial \Theta_h'} & \dfrac{\partial \alpha_h}{\partial \eta'} \end{bmatrix} V\begin{pmatrix} \hat{\Theta}_h \\ \hat{\eta} \end{pmatrix} \begin{bmatrix} \left(\dfrac{\partial \alpha_h}{\partial \Theta_h'}\right)' \\ \left(\dfrac{\partial \alpha_h}{\partial \eta'}\right)' \end{bmatrix} \tag{56}$$

Now, $\dfrac{\partial \alpha_h}{\partial \Theta_h'} = C^*$ and $\dfrac{\partial \alpha_h}{\partial \eta'} = a_h C^*$ where $C^* = I_n + \dfrac{1}{1-b}\eta z'$. Thus

$$\begin{aligned} V(\hat{\alpha}_h) &= \begin{bmatrix} C^* & a_h C^* \end{bmatrix} \begin{bmatrix} M_h^{-1}\Omega_h + \bar{x}_h^2 D^{-1} & -\bar{x}_h D^{-1} \\ -\bar{x}_h D^{-1} & D^{-1} \end{bmatrix} \begin{bmatrix} C^{*\prime} \\ a_h C^{*\prime} \end{bmatrix} \\ &= C^*\left[M_h^{-1}\Omega_h + (\bar{x}_h - a_h)^2 D^{-1} \right] C^{*\prime} \end{aligned} \tag{57}$$

Noting that $\hat{a}_h = z'\hat{a}_{ih}$, we also have

$$V(\hat{a}_h) = z'C^*\left[M_h^{-1}\Omega_h + (\bar{x}_h - a_h)^2 D^{-1} \right] C^{*\prime} z \tag{58}$$

A2.3 Variance Expressions for the $\hat{s}_{ih}$'s.

By definition, the commodity specific scales are ratios of the subsistence expenditures of households *h* to the reference household *r*. Thus, $\hat{s}_{ih} = \dfrac{\hat{a}_{ih}}{\hat{a}_{ir}}$. In this regard, the following expression for the variance of the commodity-specific scales $V(\hat{s}_{ih})$ is obtained

$$\begin{aligned} V(\hat{s}_{ih}) &= \left(\frac{\partial s_{ih}}{\partial a_{ih}}\right)^2 V(\hat{a}_{ih}) + \left(\frac{\partial s_{ih}}{\partial a_{ir}}\right)^2 V(\hat{a}_{ir}) + 2\left(\frac{\partial s_{ih}}{\partial a_{ih}}\right)\left(\frac{\partial s_{ih}}{\partial a_{ir}}\right)\operatorname{cov}(\hat{a}_{ih},\hat{a}_{ir}) \\ &= \frac{1}{a_{ir}^2}V(\hat{a}_{ih}) + \frac{a_{ih}^2}{a_{ir}^4}V(\hat{a}_{ir}) - \frac{2a_{ih}}{a_{ir}^3}\operatorname{cov}(\hat{a}_{ih},\hat{a}_{ir}) \end{aligned} \tag{59}$$

The elements $V(\hat{a}_{ih})$ and $V(\hat{a}_{ir})$ are given by the appropriately selected diagonal elements in (56) and its counterpart for the reference household. The elements $\operatorname{cov}(\hat{a}_{ih},\hat{a}_{ir})$ are given by the diagonal elements of

$$\begin{aligned} \boldsymbol{cov}(\hat{\alpha}_h,\hat{\alpha}_r) &= \begin{bmatrix} \dfrac{\partial\alpha_r}{\partial\Theta_h'} & \dfrac{\partial\alpha_r}{\partial\Theta_r'} & \dfrac{\partial\alpha_r}{\partial\eta'} \end{bmatrix} V\begin{pmatrix} \hat{\Theta}_h \\ \hat{\Theta}_r \\ \hat{\eta} \end{pmatrix} \begin{bmatrix} \left(\dfrac{\partial\alpha_h}{\partial\Theta_h'}\right)' \\ \left(\dfrac{\partial\alpha_h}{\partial\Theta_r'}\right)' \\ \left(\dfrac{\partial\alpha_h}{\partial\eta'}\right)' \end{bmatrix} \\ &= \begin{bmatrix} 0 & C^* & a_r C^* \end{bmatrix} V\begin{pmatrix} \hat{\Theta}_h \\ \hat{\Theta}_r \\ \hat{\eta} \end{pmatrix} \begin{bmatrix} C^{*\prime} \\ 0 \\ a_h C^{*\prime} \end{bmatrix} \\ &= \begin{bmatrix} 0 & C^* & a_r C^* \end{bmatrix} \begin{bmatrix} M_h^{-1}\Omega_h + \bar{x}_h^2 D^{-1} & \bar{x}_h\bar{x}_r D^{-1} & -\bar{x}_h D^{-1} \\ \bar{x}_h\bar{x}_r D^{-1} & M_h^{-1}\Omega_r + \bar{x}_r^2 D^{-1} & -\bar{x}_r D^{-1} \\ -\bar{x}_h D^{-1} & -\bar{x}_r D^{-1} & D^{-1} \end{bmatrix} \begin{bmatrix} C^{*\prime} \\ 0 \\ a_h C^{*\prime} \end{bmatrix} \\ &= (\bar{x}_h - a_h)(\bar{x}_r - a_r) C^* D^{-1} C^{*\prime} \end{aligned} \tag{60}$$

Appendix 2: Detailed Description of 11 Broad Expenditure Categories

The HES is concerned with the expenditure patterns of private households. It is restricted to goods and services that are for private consumption. For this study, the following general expenditure categories were used:

1. **Housing** includes expenses incurred for the payment of rent, mortgage, property rates, house and contents insurance as well as housing repairs and maintenance.

2. **Fuel and power** includes all expenses towards electricity, gas and other fuels.

3. **Food** includes all expenses towards bakery products, flour and other cereals, meat and fish, dairy products, fruits and vegetables, miscellaneous food (jam, jellies, coffee, tea), non alcoholic beverages, meals out and take away food.

4. **Alcohol and tobacco** refers to all expenses towards the purchase of cigarettes and all types of alcoholic beverages.

5. **Clothing and footwear** includes all expenses towards the purchase of clothing and footwear for men, women and children, clothing accessories (e.g. ties, gloves, hankerchiefs) as well as clothing and footwear services (e.g. drycleaning and shoe repairs).

6. **Household furnishings and equipment** includes all expenses towards furniture and floor coverings, blankets and rugs, household linen and furnishings, household appliances, glassware, tableware, household utensils and cleaning agents. This category also includes expenditures incurred for the operation of the household such as gardening services, housekeeping, childcare and the repair and maintenance of household durables.

7. **Medical and health care** covers items such as accident and health insurance premiums, practitioner's fees, prescriptions, medicines, pharmaceutical products, hospital and other health charges.

8. **Transport** refers to all expenses made for the purchase of motor vehicles, petrol and fuels, vehicle registration and insurance, vehicle servicing and repairs, driver's licenses, driving lessons, subscriptions to motor organisations, vehicle hire, as well as public transport fees.

9. **Recreation and entertainment** includes expenses for the purchase of television and other audiovisual equipment, books, newspapers and other printed materials, recreational equipment (cameras, musical instrument, toys), gambling, entertainment and recreational services. Holiday expenses as well as those incurred for animal pets are also included in this category.

10. **Personal care** pertains to expenses towards toiletries, cosmetics, hair dressing and beauty services.

11. **Others** includes expenses for miscellaneous goods (watches, jewellery, stationery), interest payments on selected credit services, education fees, life insurance and other miscellaneous services.

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