The Definition and Measurement of Productivity*

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Abstract

This paper introduces the various methods that have been used to analyse productivity. Productivity is defined as the ratio of output to input for a specific production situation. Productivity changes can be caused by either movements in the 'best practice' production technology, or a changes in the level of efficiency. The paper discusses the various problems encountered in measuring productivity when there are multiple outputs and inputs. Also, the problems concerning the measurement of inputs and outputs are discussed. Methods that analyse the level of inefficiency within a sample of firms are reviewed. These include data envelopment analysis, stochastic production functions and panel data methods. Lastly, a few Australian productivity studies are reviewed to illustrate the empirical use of the various definitions and techniques.

Keywords: productivity, data envelopment analysis, stochastic production frontiers, panel data.

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1 Introduction

The issues surrounding the definition and measurement of productivity have been the topic of research for a variety of disciplines, including accountancy, economics, engineering and operations research. At a basic level, the concept of 'productivity' is relatively easy to define. It is the ratio of output to input for a specific production situation. Rising productivity implies either more output is produced with the same amount of inputs, or that less inputs are required to produce the same level of output. In either case, it is not difficult to understand the importance of productivity changes for general welfare, including environmental concerns. The concept of productivity is linked closely with the issue of efficiency. If a firm is efficient it is said to be operating on the production frontier (i.e. it is achieving 'best practice'), where the production frontier is defined at some point in time with reference to a particular set of firms (i.e Australian firms in a certain industry). Rising efficiency would therefore imply rising productivity. Equally, the shift outwards of a production frontier also implies productivity growth. In this paper, therefore, productivity growth encompasses both changes in efficiency and changes in best practice. The issue of productivity and efficiency is elaborated on in section 4.

Although the basic concept of productivity is straightforward, difficulties are soon encountered when one confronts various measurement problems, the presence of multiple inputs and outputs, and uncertainty over how to model the production process. In addition, productivity measures can be made at the process, plant, firm, industry or economy level; each of which involves some specific issues and concepts. This paper aims to introduce the various issues and techniques involved in productivity analysis. The paper is written at the start of a project which will analyse firm performance, hence the paper concentrates on issues surrounding firm-level productivity. Issues specific to macroeconomic productivity analysis are not stressed although, of course, macroeconomic data originates from measurement of firm-level data.

The structure of the paper is as follows. Section 2 outlines some basic theory of productivity measurement for a single firm (or workplace). The central aspect of such work is the production function from which various productivity measures can be constructed on the basis of certain assumptions about firm behaviour and market conditions. Section 2 also discusses the role of index number theory to construct productivity measures. Section 3 considers some the problems involved with measuring outputs and inputs. At a theoretical level these are easy to define but in practice data are often not available to allow the theoretically correct measurement of outputs and inputs. Section 4 returns to the issue of efficiency and productivity and clarifies the points made in the opening paragraph above. Section 4 also provides the theoretical framework for section 5 which considers quantitative techniques to identify relative levels of efficiency when a data set of many firms is available. The essential concept in such work is the construction of a theoretical production frontier from a sample of firms. This can be done using mathematical programming (a technique from operations research) or econometric methods. These techniques have been developed not only to assess efficiency at a point in time but also to assess how efficiency changes over time, including the shift in the production frontier. Section 6 considers a few Australian studies into productivity. This is done to provide some examples of how the methods and issues discussed in previous sections have been used. Section 7 concludes.

2 Theory of productivity measurement

2.1 Defining productivity for a single firm

The most basic case to consider is that of a single firm that produces one output (y) using a single input (x), the ratio of these two (y/x) yields a measure of the level of productivity. Both output and input are measured in real units (i.e. their level reflects the real quantity of either output or input). Diewert (1992), in fact, considers this ratio an output-input coefficient and only attributes the word 'productivity' to the measurement of changes in such coefficients over time (t), i.e.

[1] $$prod^{t-1,t} \quad {y^t}/{y^{t-1}} \Big| \Big/ \; {x^t}/{x^{t-1}} \Big| \; .$$

In such a simple single output, single input case there are various equivalent ways of assessing productivity change between two periods. Diewert (1992) lists five methods that yield equivalent measures of productivity (basically these utilise data on costs and prices to express [1] in different ways). Diewert (1992) also refers to the Jorgenson and Griliches (1967) method of using only input price (*w*) and output price (*p*) data to yield the ratio

[2] $$\phi^{t-1,t} \quad \left(w^t \big/ w^{t-1} \right) \Big/ \left(p^t \big/ p^{t-1} \right) \ .$$

If a firm's revenues equals its costs then [2] is an equivalent productivity measure to [1] (which can be shown by using the identity py=wx). This latter productivity measure is an indication of how various assumptions, in this case that of perfect competition (revenue equal to costs), may be used to allow productivity measurement when only certain data is available. These various ways of calculating a well-defined productivity measure for the single input, single output case are primarily illustrative, since in practice almost all interesting cases involve multiple outputs or inputs or both. When we move to consider these cases the analysis becomes much more complex. In Diewert's words "we are faced with a bewildering array of alternative conceptual measures of productivity change" (Diewert, 1992, p.169).

To take a simple example, suppose that a firm has one output (y) and three inputs: labour (*l*), capital (*k*) and materials (*m*) and that the relationship between them (at time *t*) is given by

[3] $$y \ = f^{\,t}(l,k,m) \ .$$

Even in such a situation, it is still possible to specify partial productivity measures, such as *y/l* or *y/k* (and consider either the level or the growth rates of such ratios). Indeed, there may be specific reasons for calculating such partial productivity measures (e.g. wage negotiators may wish to study the level/path of *y/l*). However, the use of partial productivity measures can be misleading since different inputs can *substitute* for each other (e.g. if *k* is increased while all other inputs are held constant, this raises *y* and also the partial productivity measure *y/l*, even though there may have been no increase in the productivity of labour *per se*). This type of situation has led people to adopt multifactor measures of productivity. A basic adjustment is to consider value added ($y_a = y - m$) rather than gross output as the left hand side variable in [3] (see Morrison, 1993, p.29). This is one method of removing the influence of materials input so as to consider y_a/l or y_a/k.

To decompose the movements of y we can differentiate [3] with respect to time, yielding,

$$[4] \qquad \dot{y} \quad \frac{f}{l}\dot{l} + \frac{f}{k}\dot{k} + \frac{f}{m}\dot{m} + \frac{df}{dt} \quad ,$$

where the dot notation is used for a time derivative. Dividing equation [4] by y yields

$$[5] \qquad \frac{\dot{y}}{y} \quad \frac{f}{l}\frac{l}{y}\frac{\dot{l}}{l} + \frac{f}{k}\frac{k}{y}\frac{\dot{k}}{k} + \frac{f}{k}\frac{m}{y}\frac{\dot{m}}{m} + \frac{df}{dt}\frac{1}{y} \ .$$

In words, equation [5] states that the growth of output is a weighted sum of the growth rates of the inputs, and an additional term which includes the shift in the production function (f(.)) over time. The latter term is referred to as the rate of multifactor productivity (MFP) growth or, sometimes, the rate of technical advance. In either case it represents the output change not accounted for by changes in inputs or changes in input mix.

For empirical work, equation [5] is not directly useable as it stands. The problem is – assuming appropriate (real) measures of y, k, l and m are available – that the marginal product terms (e.g.$\partial f/\partial l =$ MP) are not directly observable. Past researchers have, therefore, resorted to theoretical assumptions to yield values for the marginal products which, in turn, allows the calculation of MFP. To illustrate, assume that the firm is profit maximising and subject to constant returns to scale (these assumptions imply the firm operates in a perfectly competitive industry). In such a situation, the firm's maximum profits are achieved when the value of the marginal product of an input equals the input price, ie. p.MP = w , where p is the output unit price and w is the input price. This equation allows us to substitute out the MP terms in [5] and write the more familiar (see Morrison, 1993, p.45 for a more detailed derivation)

$$[6] \qquad \frac{\dot{y}}{y} \quad S_l \frac{\dot{l}}{l} + S_k \frac{\dot{k}}{k} + S_m \frac{\dot{m}}{m} + mfp \, ,$$

where S_i represents the relative input share of value added output. This expression is the method used by the ABS to provide multifactor productivity estimates at the aggregate level (see Aspen, 1990).

The derivation of multifactor productivity can also be achieved by using the cost structure of a firm. Mathematically, the derivation above is based on the production function or 'primal' measure; an equivalent method of expressing the constraints faced by a firm is to use the cost function or 'dual' measure. Using the cost function, *C(w, Y, t)*, and again the assumptions of constant returns and cost minimisation, multifactor productivity can be shown to equal

$$[7] \qquad mfp_c \quad \frac{\dot{c}}{c} - \sum_j \frac{w_j x_j}{C} \frac{\dot{w}_j}{w_j} \quad ,$$

where C equals total costs, c (= C/Y) is unit costs, w is input prices and x is input quantities (for j inputs). Under the assumptions of constant returns to scale and cost minimisation it can be shown that the measure for multifactor productivity in [7] is identical to that in [6] (see Morrison, 1993, p.48 for a full derivation of [7] and proof of this statement). Note that the dual method would require detailed data on the prices of inputs.

Whether the primal or dual methodologies are used in the measurement of productivity is dependent on the assumptions made. As stated, the methods assume that firms experience constant returns to scale and undertake profit maximisation (cost minimisation). If, in reality, these assumptions are not true, the multifactor productivity measure may be biased. Moreover, the production and cost function have an implicit assumption that inputs can be adjusted instantaneously (i.e. no adjustment costs). In the real world the level of various inputs may be fixed over the short or medium term (capital being the most obvious example but even labour and other inputs may exhibit some adjustment costs). The problem of 'input fixity' again mean that the equations for multifactor productivity are likely to be biased.

There are various methods that can be used to adjust for non-constant returns to scale and input fixity. These involve a more general treatment of equation [5] so as not to impose these assumptions. Morrison (1993, Chapters 4, 6 and 7) details some of these methods. Hall (1986) provides the initial paper in a literature that seeks to adjust MFP measures for the degree of market power, as proxied by the mark-up ratio (which is defined as the ratio of output price to marginal costs). Again, Morrison (1993, p.111 and Chapter 9) contains a full discussion.

2.2 Index numbers

An alternative way to consider the problem of assessing multifactor productivity comes from index number theory. In short, index number theory considers how to obtain a single index of, say, the prices of goods over time, when there are many goods. Various indices are commonly used including Laspeyres, Paasche, Fisher Ideal and the Tornqvist index. The Tornqvist index can be applied to the problem of creating an index for output from a number of inputs to yield,

$$\ln \frac{Y_t}{Y_{t-1}} \qquad \sum_i S_i \ln \frac{x_{i,t}}{x_{i,t-1}}$$

[8]

$$\text{where} \quad S_i \quad \frac{1}{2}\; w_{i,t-1}x_{i,t-1} \Big/ \sum_i w_{i,t-1}x_{i,t-1} \Big| \quad \frac{1}{2}\; w_{i,t}x_{i,t} \Big/ \sum_i w_{i,t}x_{i,t} \Big|$$

This, somewhat intimidating equation, simply states that output growth is a weighted sum of the input growth rates[1], where the weights are the average input shares in total

[1] Note that $\ln(x_t/x_{t-1})$ is a discrete approximation of $dx/dt/x$.

costs. This, therefore, is almost identical to equation [5], which is stated in continuous time. Thus, when a data series for output and inputs are available the Tornqvist index can be used to yield MFP, by subtracting the left hand side of [8] from the right hand side. This is the normal method used by the ABS (see Aspen, 1990).

It is worthwhile elaborating a little more of the Tornqvist index number approach to productivity. In particular, it should be noted that the index is a discrete approximation to the continuous time equation [5]. This raise the question of how good an approximation it is? To answer this an exact functional form for the production function *f( )* needs to be considered. When the production function is translog, Diewert (1976) has shown that the Tornqvist index is exact (ie. the index [8] is the appropriate choice for empirical work). Moreover, since the translog function is 'flexible' (i.e. provides a second order approximation to an aggregator function), Diewert calls the Tornqvist index superlative.

3 Measuring inputs and outputs

The various methods discussed above have not mentioned the difficulties that are encountered when practical measures of productivity are sought. This section outlines some of the basic measurement issues that face researchers. More extensive discussions are contained in Morrison (1993) and Baily and Gordon (1988).

3.1 Output

The discussion above has referred to measures of real output, either gross or value added. In a time series study this implies the need for price deflators which may or may not be available at the, preferred, firm or industry level. A more difficult issue is that output quality may increase over time, even though unit prices are static or declining (a prime example is personal computers). There is a literature on methods to adjust for quality issues (see Gandal, 1994, for a computer software example), and indeed some statistical agencies adjust the output of certain sectors (Griliches, 1994, notes that the US government introduced a quality adjusted (hedonic) price series for computers in 1986). Calculation of real output in service industries also presents problems. Lowe (1995) discusses the problems of measuring the output of the retail sector when quality changes, such as an increase in opening hours, have occurred. Ritzman (1995), in a study of Australia banking, assumes that real output is proportional to the level of deposits and loans (i.e. assuming that an output *flow* is proportional to a *stock*). There are, therefore, various difficult measurement issues and adjusting for these will depend on the specific data available and various assumptions.

Output should be defined as the real output produced in a set time period. The sales or revenue figure normally reported in accounts will not coincide with this if inventory levels have risen or fallen over the period. Hence, adjustments for the level of inventories should be made and also, if possible, the impact of any output given away for promotions, etc.[2]

3.2 Labour

Labour quantity is normally measured in terms of the number of employees. Adjustments should be made for the extent of part time work, hence the idea of 'full time equivalent' employees. Even if these adjustments are possible, there is also the issue of adjusting for hours worked by, for example, the inclusion of overtime. Adjustments for the quality of labour is also an issue to be considered. In theory, labour could be split into various separate inputs depending on skill, education or other classification, and each of these could be entered as a separate input in [5]. Such an approach would require the necessary data on the quantities of each and the wage bill of each category.

3.3 Capital

The measurement of capital is, perhaps, the most problematic of inputs to measure. Importantly, the productivity measures discussed above are concerned with the real value of capital services in a set time period. This is not the same as the stock of capital held by a firm or industry; instead we need to measure the *flow* of services from such a stock. Morrison (1993, p.139) reports that one method of tackling this

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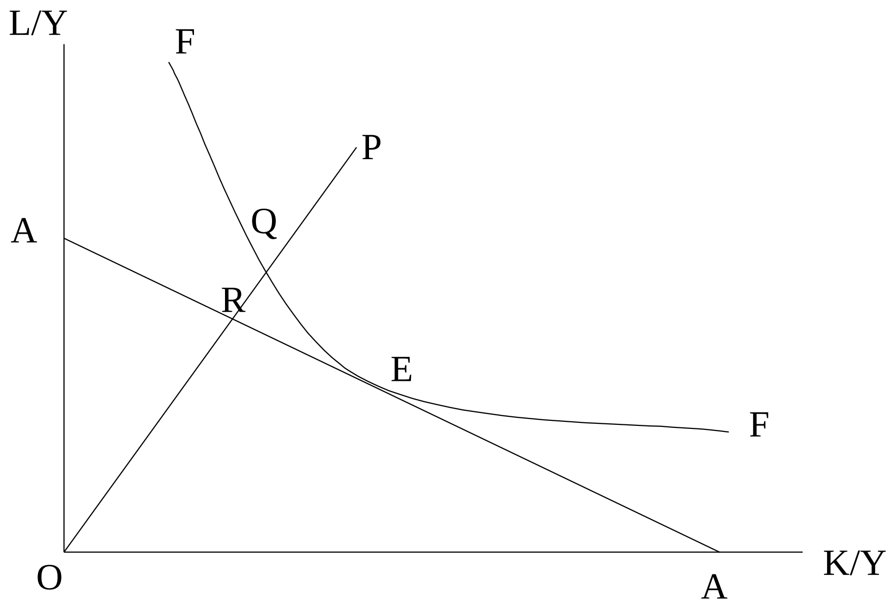
[2] See Diewert and Smith (1994) for a detailed case study of inventories and productivity in a distribution firm.

issue is to: first, create a capital stock series by adding up the investment in different assets over time, allowing for depreciation, maintenance, inflation in asset prices, etc; second, assume the flow of capital services is a constant fraction of this stock; third, calculate a price for each component of capital services by using the price of relevant capital goods. The vector of service flows and service prices can then be aggregated into a single capital flow and price (using an appropriate index). The data requirements for such a procedure, however, mean that such a method is often impossible in practice. Instead, some researchers have assumed that the *value flow* of capital for a firm equals sales less all variable costs (effectively a 'residual' measure of the value of capital). A capital stock is also calculated (using best available data). Then the researcher assumes that the flow value equals the price of capital services multiplied by the quantity (stock of capital). This, in turn, allows a price for capital services to be calculated. This method calculates the price of capital from the two other components. An alternative method is to assume a price of capital (say, to reference to bond prices) and then calculate the value of capital services.

4 Efficiency and productivity

The above discussion focussed only on productivity measurement with no mention of the concept of 'efficiency'. Efficiency is normally defined as comprising of two components: technical and allocative efficiency. Technical inefficiency occurs if a firm is not obtaining maximal output from a set of inputs. Allocative inefficiency occurs when a firm fails to choose the optimal balance of inputs given input prices (even though it may be obtaining maximal output from the inputs actually used). To illustrate, Figure 1 is reproduced from Coelli (1995, p.222),

Figure 1 **Technical and allocative efficiency**



The figure has axes for the input of labour and capital (per unit of output). The (unknown) best practice *unit* isoquant is given by FF (note the axes are in input per unit of output). The input combination to produce a single unit of output for a single firm is shown by the point P (at the end of the line OP). Thus, the firm is producing outside the efficient unit isoquant and can be termed inefficient. Technical inefficiency is measured by the ratio OQ/OP (i.e. the hypothetical extent to which inputs can be proportionally reduced without reducing output). Allocative efficiency can only be determined in relation to the prices of inputs. The isocost line AA illustrates the input price ratio. Given this ratio, production at point Q is not optimal and the firm should locate at point E. A measure of the inefficiency caused by this non-optimal point is OR/OQ and is termed the extent of allocative inefficiency. Thus, overall economic inefficiency can be shown by the ratio OR/OP. Overall inefficiency can be thought of as the combination of technical and allocative inefficiency (represented by the ratio OR/OQ).

The measurement of productivity discussed in section 3 assumes that all firms were fully efficient (i.e. all operate on the production frontier and select optimal quantities of each input). This assumption was implicit in the profit maximising/cost minimising assumptions made. In reality, we would not expect all firms to be fully efficient. If this is the case, then the measure of TFP arising from, say, a Tornqvist index, may be partly due to a shift in the production frontier and partly due to firms moving closer to the frontier over time (i.e. becoming more efficient). This means that TFP may be measuring both 'efficiency' improvements and 'shifts in the production frontier' and should not, therefore, be defined as 'technical change' (unless various adjustments for efficiency changes have been made). This distinction can cause confusion as some researchers consider 'productivity growth' as only referring to the shift in the frontier. Here we follow Grosskopf (1993, p.160) who states, "I define productivity growth as the net change in output due to change in efficiency and technical change, where the former is understood to be the change in how far an observation is from the frontier of technology and the latter is understood to be shifts in the production frontier".[3]

In this paper, therefore, we consider the analysis of inefficiency as an aspect of productivity. Such analysis relies on having a data set of various firms (with information on input and output levels) and using this information to construct a theoretical production frontier.

5 Analysis of efficiency

There are three main categories of techniques that can be used: data envelopment analysis, stochastic production frontier, and panel data techniques. Each of these techniques has an extensive literature and our aim here is to provide an overview with associated references.

[3] There is a further possible confusion arising from calling shifts in the production frontier 'technical change' since this may create the impression that technological change only accounts for a fraction of overall output growth. Such an impression is only correct if technological change and input accumulation are independent (e.g. new technology does not give rise to investment). See Fagerberg (1994) for further discussion of these issues.

5.1 Data envelopment analysis

The data envelopment analysis (DEA) method is based on work by Farrell (1957) and Koopmans (1951), although its first full implementation was by Charnes, Cooper and Rhodes (1978). Useful recent summaries of data envelopment analysis methods are given by Ali and Seiford (1993) and Coelli (1995), both of which we draw on here.[4]

Data envelopment analysis (DEA) is a mathematical programming approach to assessing the inefficiency of a firm (plant) relative to a sample of firms (plants). In essence, the approach maps out a hypothetical production frontier on the basis of all the output and input information from the sample. Inefficiency is then assessed by the 'distance' a firm's specific output/input mix is from the frontier. Such a distance can be assessed in terms of the amount of inputs that are wasted in producing a given output, or the level of output that could be produced given the level of inputs.

The method used to identify the production function involves the solution to a mathematical programming problem for each of the firms in the sample. If there are *n* firms which produce a single (identical) output y with two inputs (*l*, *k*), the mathematical programming problem for firm *i* is

$$\max_{,\ l,\ k,\omega} \quad \mathrm{y_i} \ - \upsilon_{\mathrm{l}} l \quad \upsilon_{\mathrm{k}} k$$

$$[9] \qquad s.t. \qquad \mathrm{y_i} \ - \upsilon_{\mathrm{l}} l_i \quad \upsilon_{\mathrm{k}} k_i \qquad 0 \ \text{ for i } \ 1,\ldots,\mathrm{n}$$
$$\geq 1 \qquad \upsilon_{\mathrm{l}} \geq 1 \quad \upsilon_{\mathrm{k}} \geq 1$$

[4] See also Seiford and Thrall (1990) for a review.

where μ, $_i$, and are parameters. The solution to this problem defines a 'hyperplane' (in this case this is simply a plane in *y*,*l*,*k*-space) which can be intuitively thought of as the closest frontier to the output/input combinations of firm *j*. Note that this frontier is defined using the information on all the other firms in the sample (hence the *n* constraints in [9]). If firm *j* is located on this hyperplane it is defined as efficient. If the firm is not located on this hyperplane it is inefficient, and a measure of the level of inefficiency can be derived from the distance the actual output/input vector is from the frontier. A similar hyperplane can be defined for each of the firms in the sample and these can then be regarded as forming the various facets of the overall production frontier. Some firms may share the same 'hyperplane. In Coelli's (1995, p.231) words, the overall production frontier "can be visualised as a number of intersecting planes forming a tight fitting cover over a scatter plot of points in three-dimensional space". The solution to the *n* programming problems is normally pursued by using the dual representation of the above problems (see Ali and Seiford, 1993, p.124, or Coelli, 1995, p.231).

Equation [9] has effectively defined the production process as a linear combination of inputs/outputs. The presence of in the maximisation equation means that each hyperplane does not have to pass through the origin (i.e. there is no requirement that zero inputs produce zero outputs). Although this may seem odd, it allows a more flexible overall production frontier, which is normally called the 'variable returns to scale' case. If is set to equal zero, all hyperplanes are forced to pass throught the origin, which is known as the 'constant returns to scale' case. Note also that the hyperplanes need not be linear combinations of outputs and inputs *y*, *k* and *l*, since they could be defined in logarithms, yielding a Cobb-Douglas based frontier.

The DEA approach, according to Ali and Seiford (1993), has been used extensively in studies on efficiency (they reference a literature review in 1990 with 400 DEA references) and is, therefore, a well established method. Potential drawbacks of the approach include the fact that data points are taken as representing "true" values, with no allowance for measurement error and other noise. Also, the process of creating a production frontier from a set of intersecting hyperplanes can sometimes cause oddities in measures of efficiency (essentially inefficiency is measured as the distance to the nearest plane, even if this plane is parallel to an axis which implies one input could be reduced without loss of output[5]). Another potential drawback is discussed by Sengupta (1998) who shows that DEA analysis can be misleading if it does not correctly incorporate capital goods inputs.

5.2 Stochastic production frontier approach

The DEA method outlined above takes each data point as an accurate description of the production process. This may not be the case since errors can occur in the measurement of outputs or inputs. The likelihood of such errors has led to the stochastic production frontier (SPF) approach, which is a method that allows for such 'noise' in data points. Again, the SPF approach is concerned with the construction of a production frontier on the basis of data on a sample of firms. For example, suppose that firm *i*'s production can be represented by

$$[10] \qquad y_i = f(x_i;\beta) \quad v_i \quad u_i \qquad\qquad \mathrm{i} = 1, ..., \mathrm{n}$$

where *i* refers to one of *n* firms. The v_i and u_i terms are stochastic. For example, v_i could be defined as having a normal distribution, while u_i has a positive half normal distribution. The reasoning is that v_i captures possible measurement errors, etc while u_i represents the distance observation *i* is from the frontier, hence u_i enters as a negative in [10]. Equation [10] can be solved by either maximum likelihood or COLS (corrected OLS) to yield the coefficients . [6] Furthermore, using the available values for the residuals (v_i + u_i) it is also possible to calculate the mean or mode of the distribution of u_i (see Bauer, 1990, p.42-3).

An important issue in the SPF approach is the choice of distribution for u_i. Obviously, the distribution needs to be one-sided, but they are a range of possible forms including the exponential, truncated normal and two-parameter gamma (see Greene, 1990).

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[5] Coelli (1995, p.233) illustrates this point. He reports on some methods that try to avert this problem.

[6] Coelli (1995, p.225) states that the maximum likelihood method is to be preferred (following a Monte Carlo experiment on the difference between COLS and ML).

Following from this is the fact that the efficiency measures from the SPF may be sensitive to the distribution used. The SPF approach may be augmented in various familiar ways. First, the functional form used for the production function can be Cobb-Douglas, translog or other form. Second, the cost dual representation can also be used if there is data on input prices. There are also other, less familiar, modifications, for example, Kalirajan and Obwona (1994a, 1994b) modify [10] so that the coefficients are random. As they note, equation [10] assumes that all firms share the same coefficients, with movement in the production frontier being a ‘neutral shift’ from the current position. One method of relaxing this assumption is the use of a random coefficients model.

The SPF approach is able to deal with the fact that measures of outputs and inputs have an element of randomness, however, this comes at the cost of imposing a functional form for the production function and assuming a distribution for inefficiency. Harris (1992, p.203) notes that use of different functional forms in SPF analysis can lead to different efficiency measures. In contrast, the DEA method does not have this drawback, in Bauer’s words (1990, p.29), "The Chief advantage of the mathematical programming or DEA approach is that no explicit functional form is imposed on the data."

5.3 Panel data methods

The estimating equation shown by [10] can also be estimated if panel data is available. Given *T* periods of data the equation is now

$$[11] \quad y_{i,t} = f(x_{i,t};\beta) \quad v_{i,t} \quad u_{i,t} \qquad i = 1, ..., n \quad t = 1, .., T \ .$$

Estimation of [11] is only different from the cross-sectional estimation [10] in that it has greater degrees of freedom (although note the implicit assumption that the coefficients, , are constant over time – something that can be tested for (see Baltagi, 1995, Chapter 4).[7] Different estimation methods can be used if the efficiency term, u_{it}, is considered constant over time (i.e. u_i). This then allows the use of random or fixed effects models (see Greene, 1993, or Baltagi, 1995, for basic explanations of such estimators). The assumption that the inefficiency terms (u_i's) are constant overtime is a major assumption, which may be valid only for a small number of years.

Kumbhakar (1990) and Battese and Coelli (1992) suggest methods of allowing for systematic variation in the u_i's overtime, but this again might be construed as imposing major assumptions on efficiency adjustment. Baltagi, Griffin and Rich (1995) estimate a translog cost function which contains both an industry wide technical change term and individual firm efficiency terms. The former reflects the overall shift in the production function and is allowed to be a function not only of time, but also output and prices. Use of the industry technical change and the residual from the regression allows an index of firm specific technical change to be calculated. They compare this with a traditional TFP measure (calculated for the same data set of 24 airlines over 1971 to 1986) and, although the overall correlation coefficient between the percentage changes of each is 0.61, they find substantial differences between the measures.

6 Australian studies

This section discusses a number of Australian based studies to illustrate the use of the techniques mentioned above. Table 1 provides a summary of some recent firm-level or specific industry productivity studies (for a more complete review of productivity studies see Dawkins and Rogers, 1998).

[7] A panel data set may also allow the calculation of firm level inefficiency measures, even when using the SPF approach, see Battese and Coelli (1988) for a discussion and an example on three years of dairy farm data in Victoria and New South Wales.

Table 1 Examples of Australian firm-level productivity studies

Author, Year	*Title*	*Details*
Esho and Sharpe, 1996	X-Efficiency in Australian Permanent Building Societies, 1974-1990	SPF approach (with quadratic time trend for efficiency)
Ritzman, 1995	Productivity in Australian Banking: Two Alternative Approaches	Tornqvist indexes (TFP) and econometric analysis of cost functions
Harris, 1992	Technical Efficiency in Australia: Phase I	SPF approach using manufacturing census (1977) data at 4 digit level
Tasman Pacific	The Scope for Productivity Improvement in Australia's Open Cut Black Coal Industry, 1997	Multi-lateral productivity index (TFP) and partial productivity measures
Bureau of Industry Economics, 1994[8]	International Performance Indicators: Electricity Update	MFP and DEA analysis
Lawrence et al, 1991	The Comparative Efficiency of State Electricity Authorities	Multi-lateral productivity index (TFP)

Note: the Table summarises some firm-level studies that have used similar techniques to those discussed in this paper. Studies based on survey data responses are not included, although they are briefly discussed below.

Lawrence, Swan and Zeitsch (1991) look at the productivity of Australian state electricity suppliers.[9] They employ a growth accounting approach based on the *translog multilateral input* index (see Caves, Christensen and Diewert, 1982). This index is preferable to the Tornqvist when the data is cross-sectional or panel in nature.[10] This preference is motivated by the fact that the index is transitive (i.e. that the comparison of productivity between two units should be the same whether

[8] There are a number of BIE studies in the International Performance Indicators project, including gas, telecommunications and coastal shipping. These reports use various methods of analysing productivity but are not listed for reasons of space. There are also studies by the Industry Commission and related bodies which include firm-level productivity analysis (e.g. Industry Commission, 1995).

[9] In fact, the authors use the term efficiency, but according to our definition above, we can interpret their paper as being concerned with productivity.

[10] Caves et al (1982, p.84) suggest it may not be preferable for purely time series studies.

compared directly or through a third observation). The particular form of the index used by Lawrence et al (1991) compares the TFP between electricity providers *m* and *n* according to,

[12]
$$\log(TFP_m / TFP_n) \quad \frac{1}{2}\sum_i (R_{im} + R_i^*)\log\frac{Y_{im}}{Y_i^*} \quad \frac{1}{2}\sum_i (R_{in} + R_i^*)\log\frac{Y_{in}}{Y_i^*}$$
$$- \frac{1}{2}\sum_j (S_{jm} + S_j^*)\log\frac{X_{jm}}{X_j^*} + \frac{1}{2}\sum_j (S_{jn} + S_j^*)\log\frac{X_{jn}}{X_j^*}$$

where the * indicates the average over all observations, *R* is revenue share, *S* is cost share, *i* refers to outputs and *j* to inputs. In terms of measuring the capital input, Lawrence et al use a series for the user cost of capital service, which takes account of "investment streams, asset lives, interest during construction and capacity commissioned each year" (Lawrence et al, 1991, p.186). The study finds quite large differences in TFP across states, although the differences have narrowed overtime.

Esho and Sharpe (1996) use a panel data approach to investigate the efficiencies of Australian permanent building societies over the period 1974-1990. They use a variant of the fixed effects panel data model, which allows firm efficiencies to be a quadratic function of time. Using a translog cost function, Esho and Sharpe consider three output measures (housing loans, current assets and average depositors' balances) and two inputs prices (cost of funds and a wage index). They find an average level of firm inefficiency of 25% using the econometric approach. This they compare to an accounting based approach (using ratios of operating expense to total assets, and income to operating expenses), finding that if such ratios are averaged over seven or more years they are "remarkably similar" (Esho and Sharpe, 1996, p.254) to the results of the econometric analysis.

The Bureau of Industry Economics (1995) undertook a study on the telecommunications industry that utilised various productivity techniques. Various partial productivity measures were calculated (e.g. revenue per telephone line, revenue per employee), but the report also recognised the problems of measuring output (e.g. issues concerning the quality of service). An output index comprising of the total number of calls and the total number of lines was used, with the weights of 30:70 used respectively (the weights are based on the assessment that around 70% of employees were used in maintaining networks, and 30% were used in 'traffic' related functions). To calculate an annual user cost of capital (i.e. the value of capital services) the report used an estimate given by

$$[13] \quad VC \quad (g \quad d - \dot{P})K \ ,$$

where, *g* is opportunity cost of holding capital (taken as the 10 year bond rate), *d* is the declining balance depreciation rate (equal to 13% assuming asset life of 13 years), d*P*/dt is the annual rate of change in capital prices, and *K* is the (estimated) stock of capital.

There are also studies based on survey data which rely on the answers to questions concerning productivity levels and change. For example, the Australian Workplace and Industrial Relations Surveys (AWIRS) ask the general manager of a workplace how its productivity compares with two years ago and also how productivity compares with its major competitors (in both cases using a five point scale). Although such a subjective ranking of productivity may have its limitations, it should be noted that all the various quantitative methods discussed above also have limitations. Many of the studies based on the AWIRS data have concerned industrial relations and productivity (see Crockett et al, 1990, and Drago and Wooden, 1992), although some studies consider other aspects (see Blanchflower and Machin, 1996, on product market competition and productivity). These studies tend to use the responses to the productivity questions as dependent variables in ordered probit regressions with a host of explanatory variables describing workplace and market characteristics.

7 Conclusion

This paper has considered a variety of methods for analysing the level and growth of productivity. The initial sections discussed how productivity measures can be derived from a production function. Various assumptions have to be made to allow empirical application, including the assumptions of profit maximisation (cost minimisation) and constant returns to scale. Section 2.2 also showed how the theoretical methods are similar to methods based on index number theory. In particular, the Tornqvist or translog multilateral index are the most common empirical methods of creating an index of MFP. Section 3 discussed the problems of measuring various outputs and inputs. These problems should not be underestimated, as in any particular study measurement problems are likely to raise the possibility of substantial bias in productivity estimates. As explained in section 4, this paper views efficiency changes as being one element of productivity changes, hence the paper also reviews the literature on efficiency analysis. This analysis includes the techniques of data envelopment analysis (DEA), stochastic production frontier (SPF) and panel data methods. Each of these methods has been the topic of extensive research and has certain benefits and drawbacks. The choice of technique may also be determined by the nature of the data set. A few studies compare some of the results of using more than one technique on the same data set. These show that the results from different techniques can be significantly different. This again suggests that productivity studies are complex, with the possibility that various measurement problems and technical assumptions will influence the results.

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