Performance of the Operational Wansbeek-Bekker Estimator for Dynamic Panel Data Models

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Melbourne Institute Working Paper No. 8/98

ISSN 1328-4991

ISBN 0 7325 0971 8

April 1998

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Abstract

Wansbeek and Bekker (1996) considered a new estimator for simple dynamic panel data models (where there are no exogenous variables) which involved a complex weighting matrix. In this paper we propose an operational variant of this estimator which is applicable to the more realistic case where there are exogenous variables. We also propose an easy-to-compute approximation to the weighting matrix. The performance of this (these) new estimator(s) is examined, revealing very desirable small sample properties in a wide range of situations that the applied researcher is likely to encounter, especially in moderate time series length panels.

Key words: Panel data, dynamic models, IV/GMM estimation, Monte Carlo simulation, mean squared error (MSE).

JEL Classification: C13, C15 and C23.

1. Introduction

The estimation of dynamic autoregressive panel data models has been in the focus of econometrics research since the early seventies (Balestra and Nerlove [1966] and Sevestre and Trognon [1985]). The problem has been to find semi-asymptotically consistent estimation procedures (consistency in the cross-section component of the panel, holding the time-series component fixed) which combine small finite sample bias with semi-asymptotic efficiency. Although the number of estimators available to estimate such dynamic models has increase steadily, there still appears to be room for improvement as far as finite sample bias and efficiency is concerned (see, for example, Harris and Mátyás [1996]).

Wansbeek and Bekker (1996) proposed a new IV/GMM type estimator which seems to be more efficient than existing ones as it uses many more instruments (and hence orthogonality conditions). The new estimator in its proposed form, however, is not very attractive for practitioners. Firstly, it is only presented for the simple autoregressive model without exogenous variables, and secondly, because the calculation of the optimal weighting (covariance) matrix is quite complex. Moreover, nothing is known about the finite sample performance of this estimator *vis-a-vis* the existing ones.

In this paper we derive the Wansbeek-Bekker estimator for the auto-regressive error components panel data model with exogenous variables, then propose an easy-to-use approximation for the weighting matrix and finally analyse the relative performance of this estimator, compared to the more popular existing ones.

2. The model and the proposed estimators

The analysed model is:

$$y_{it} \quad y_{i,t-1} + x_{it} \quad + \square_i + u_{it}, \qquad i = 1,...,N;\ t = 1,...,T. \qquad (1)$$

or, in matrix notation,

$$y = \tilde{X}\delta + u.$$

The usual way to estimate this model is to apply the IV/GMM approach (see, for example, Sevestre and Trognon [1996]). The original Wansbeek-Bekker estimator (which assumes that $\Upsilon = 0$) extends the set of instruments proposed by Anderson and Hsiao (1982), for example, by including lags and leads of the dependent variable (and linear combinations of these). That is, by defining the variable y (from $t = 1$ to $t = T$) this estimator considers linear functions of y_+ as instruments, where y_+ is the stacked vector of observations defined from $t = 0$ to $t = T$ for each observation. The linear functions are defined by the $(T-1) \times T$ matrix A_i, which yields $A'y_+$ as the full set of instruments (where $A = I_N \otimes A_i$). Restrictions are then imposed A such that the individual effects are eliminated and consistency of the estimator ensured (Wansbeek and Bekker [1996]).

With exogenous variables in the model, two types of such IV estimators can be derived depending on whether we extend the instrument set to similarly include transformations of the exogenous variables, such that

$$Z = (A'y_+, X) \qquad \text{or} \qquad Z = (A'y_+, A'X_+, X).$$

To estimate model (1) using instrumental variables the variance-covariance matrix of the vector $Z'u$ or $Z_+'u$ is required (Bowden and Turkington [1984]). However, this matrix is quite complex due to the fact that $E(y_+u') \neq 0$. What we propose is to approximate this true covariance matrix by $\sigma_u^2(Z'Z)$ or $\sigma_u^2(Z_+'Z_+)$, which effectively means that the cross correlation components of this (these) matrix (matrices) are not taken into account. Accordingly, the approximation of these estimators' semi-asymptotic covariance matrices are respectively, $\sigma_u^2[\tilde{X}'Z(Z'Z)^{-1}Z'\tilde{X}]^{-1}$ and $\sigma_u^2[\tilde{X}'Z(Z'Z)^{-1}Z'\tilde{X}]^{-1}$, both of which are functions of *A*. Thus the optimal choice of *A* is that which minimises the trace of this (these) covariance matrix (matrices).[1]

Once *A* has been found using any constrained optimisation routine, *Z* and Z_+ are known and the estimators become simple applications of the IV technique (Bowden and Turkington [1984]). The question then is how these modified Wansbeek-Bekker estimators perform in practice for finite data sets.

3. The performance of the new estimators

To evaluate the performance of these two new estimators we carried out a Monte Carlo experiment. We generated artificial data using model (1) with two exogenous variables and

$$y_{i0} = \mu_i + \underline{x}_{i0}'\underline{\beta} + u_{i0},$$

$$\text{where: } u_{it} \sim iid\, N(0,1),\ \mu_i \sim iid\, N(0,1)\ \ x_{it}^{(k)} = x_{i,t-1}^{(k)} + \text{uniform}(-0.5,0.5),\ k = 1,2$$

$$\text{and: } \gamma = \beta_1 = \beta_2 = 0.5,$$

over two samples of small *N* (25) and small and moderate *T* (4 and 10, respectively), over 100 Monte Carlo repetitions. In order to compare the *relative* performance of the proposed estimators, we also calculated: the inconsistent OLS, OLS on the differenced model (OLS D), *Within* and FGLS estimators. We also considered: Arellano and Bond's (1991) IV estimator (widely used empirically - AB); GMM1 a minimum distance estimator using all possible orthogonality conditions (Harris and Mátyás [1996], Ahn and Schmidt [1996] and Crépon *et al* [1998]) and an identity waiting matrix and; GMM2 which uses the numerically maximum number of such conditions and an optimal weighting matrix (Hansen [1982]).

To ascertain the "robustness" of all the estimators to misspecification in the assumed data generating process, we also considered their performance to likely violations of such, letting: the residuals be autocorrelated; $u_{it} = \rho u_{i,t-1} + \varepsilon_{it}$, $\varepsilon_{it} \sim iid\, N(0,1)$; the explanatory variables

[1] It is also possible to minimise the determinant although but the resulting estimator is virtually numerically identical.

and the individual effects be correlated; $x_{it} \quad x^*_{it}$ H f□$_i$ and; the individual effect and the error term be correlated; $u_{it} \quad u^*_{it}$ H$/$□$_i$. The correlation parameters ±, , and , where set equal to 0 to 0.99 in the small T sample and 0, 0.1, 0.5 and 0.9 in the moderate T sample. The estimators' empirical MSE functions are depicted in Figures 1 and 2 in the Appendix.

In the small T samples (Figure 1), the GMM estimators clearly dominate when there is no misspecification in the assumed data generating process. (This can be seen in all figures where the correlation parameter equals zero). However, the performance of the proposed modified *WB* estimator is also quite encouraging, with both variants (especially the WB^+ version) having a smaller MSE than the *AB* estimator. Indeed, the *WB* estimators continue to perform well at all levels of correlation between □$_i$ and u_{it}. In the other misspecification scenarios, the GMM estimators dominate at low levels of serial correlation in the disturbances, being bettered only at medium to strong levels by the *Within*, and by most of the remaining estimators (excluding OLS) at strong levels. Correlating □$_i$ with X in this sample severely affects the *AB* and *WB* estimators, although the WB^+ estimator again has stable and reasonable performance. The GMM estimator continues to behave well, especially at low levels of correlation, although the dominant estimator for this form of misspecification appears to be the simple OLS one.

When T is increased (Figure 2) and the assumed data generating process is true, the rankings of the GMM and *WB* estimators are significantly reversed. Now both variants of the latter outperform those of the former, with the best performance of all estimators being achieved by WB^+. At low levels of serial correlation in the disturbances, WB^+ continues to be the dominant estimator, and has good performance at medium and strong levels, most notably uniformly dominating the GMM estimators (although such misspecification appears to favour the *AB* estimator[2]). Although correlating □$_i$ and X adversely affects the *WB* estimator, the WB^+ continues to perform well, being bettered only marginally at moderate (OLS) and strong (OLS and *Within*) levels of correlation. Finally, with moderate T and □$_i$ and u_{it} correlated, the WB^+

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[2] Being based on the first differenced model, as ± tends to unity the covariance matrix of the (first differenced) disturbances does indeed tend to the assumed moving average structure (Arellano and Bond [1991]).

estimator has excellent performance at all levels of correlation and clearly dominates all the other estimators.

We performed several simulations for different parameter values and sample sizes and it turned out that the ranking of the estimators (as expected) was invariant to the parameterisation of the model. The key issue was the time series length. The GMM type estimators tended to dominate in very small T samples, but were outperformed by the new proposed *WB* estimators (in particular WB^+) in samples of $T = 7$ or 8 or above.

4. Conclusion

In this paper we proposed an "operational" *WB* estimator, based upon Wansbeek and Bekker's (1996) simple auto-regressive estimator. Although requiring numerical optimisation, this estimator is no more computationally burdensome than GMM estimation, for example. The small sample performance of this new estimator was evaluated along with several others (most notably GMM and *AB*), and in its expanded instrument set variant WB^+, generally outperformed all other estimators when T was moderate *in all of the situations that an applied researcher might encounter*. Thus although in small T samples, the GMM estimator appears appropriate, we can recommend the WB^+ estimator in a wide range of situations when T is moderate or large.

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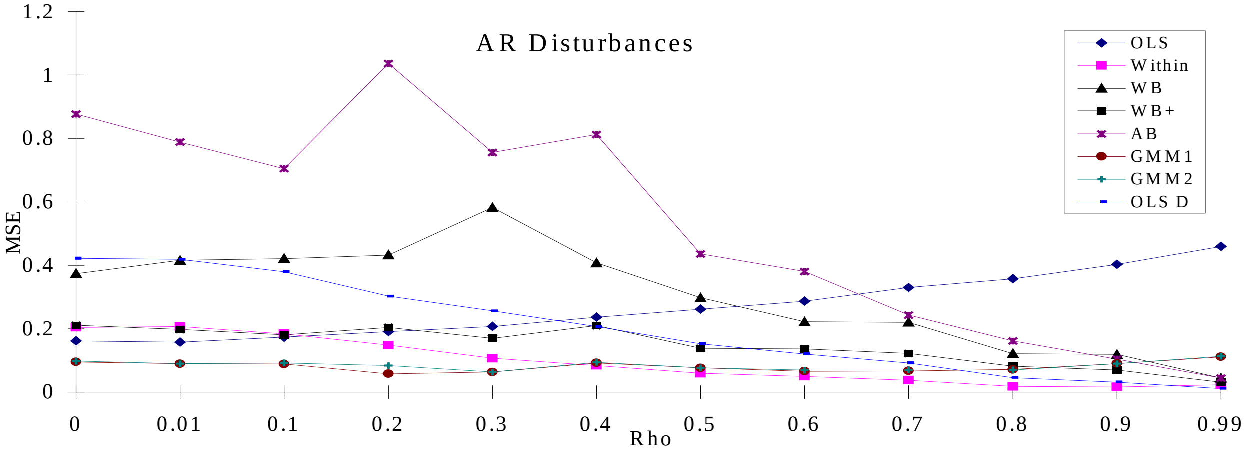
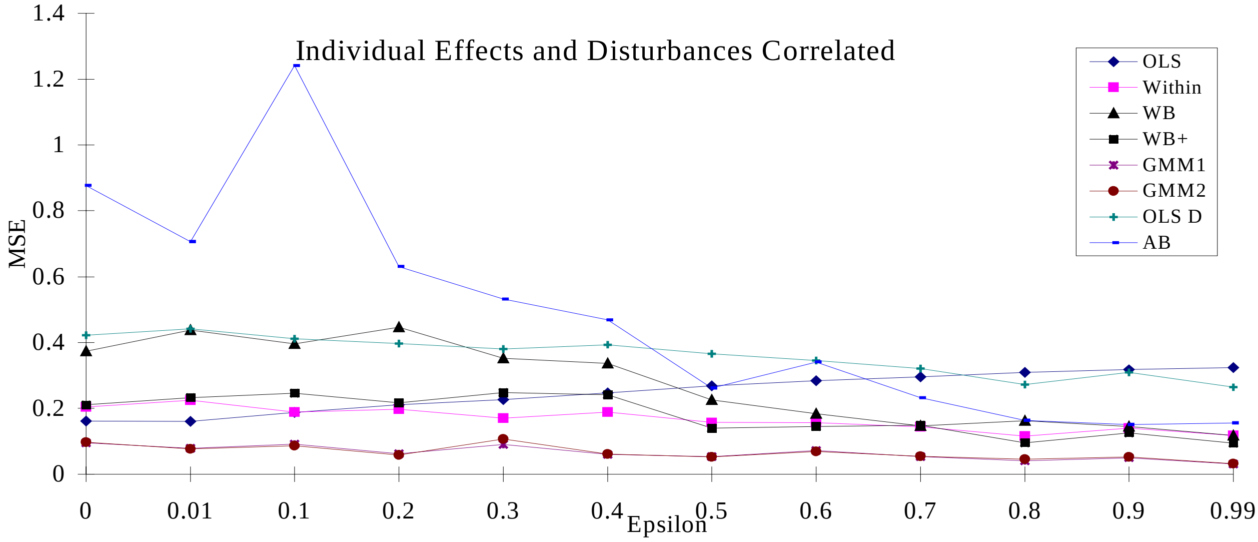
Appendix

Figure 1: Mean Squared Errors for T = 4; N = 25





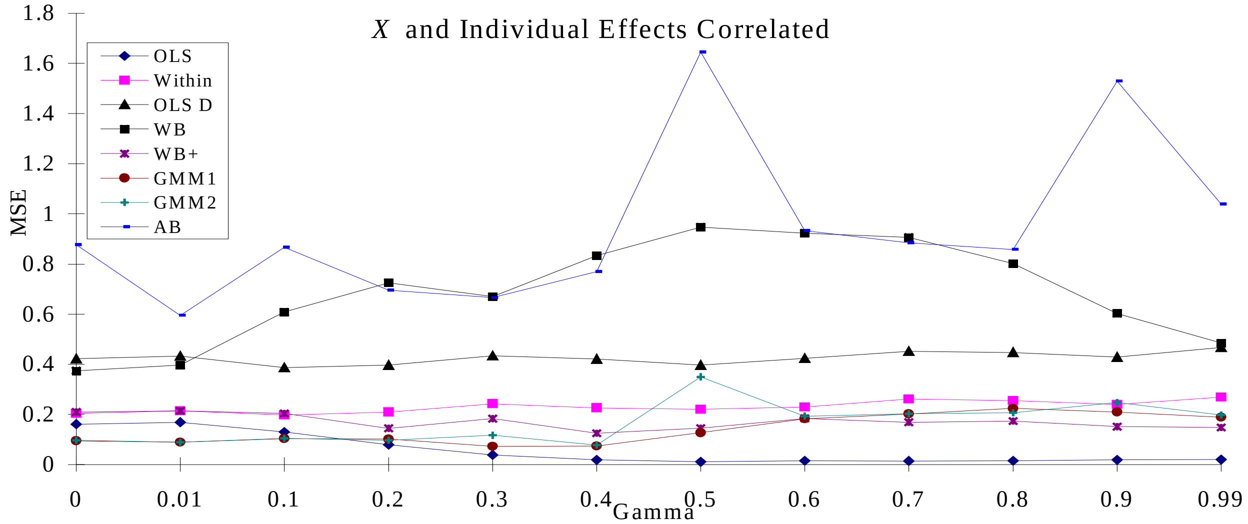


Figure 2: Mean Squared Errors for T = 10; N = 25