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Australian and Canadian Comparisons

Peter M. Summers*

Melbourne Institute of Applied Economic and Social Research

The University of Melbourne

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Melbourne Institute of Applied Economic and Social Research

The University of Melbourne

Parkville, Victoria 3052 Australia

***Telephone* (03) 9344 5288**

***Fax* (03) 9344 5630**

***Email* r.qin@iaesr.unimelb.edu.au**

***WWW Address* http://www.ecom.unimelb.edu.au/iaesrwww/home.html**

TRADE AND GROWTH IN SETTLER ECONOMIES: Australian and Canadian Comparisons

Peter M. Summers*
Melbourne Institute
University of Melbourne
Parkville, VIC 3052
ph./fax: (03) 9344-5313/5630
email: p.summers@iaesr.unimelb.edu.au

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Abstract

This paper analyses the historical relationship between exports, imports, and national income growth in Australia and Canada, using over 100 years of data for each country. I test for the presence of ìexport-led growthî and ìimport-led growthî in both countries over several time intervals, interpreting these concepts to mean a unidirectional causal ordering. I find no evidence to support the export-led growth hypothesis for Australia, but strong evidence for this hypothesis in Canadian data in the period 1915-38. The results here suggest a much stronger role for imports than has been found in previous analysis of Canadian data: imports as well as exports appear to lead growth over the 1915-38 period. In addition, import growth tends to be causally prior to export growth in both countries, but at different times. The strength of the relationship between trade and growth is generally comparable across countries. Finally, although there is little evidence of unidirectional causality in either country, there is substantial evidence of bi-directional causality.

Keywords: Trade and growth; causality; economic development
JEL Codes: F43, F14, C32

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Introduction

The relationship between a countryís overall economic performance and its trade performance is one which has received considerable attention from both theoretical and empirical economists. In particular, much effort has been devoted to explaining the role (if any) of the export sector in influencing the growth of aggregate income. The export-led growth hypothesis, that faster growth in exports results, *ceteris paribus,* in faster growth in national income (measured by GDP, for example), is one of the most enduring in international economics.

Beginning with Jung and Marshallís seminal 1985 study, numerous authors have employed time series econometric techniques, and in particular Granger-type causality tests, to assess the state of the empirical evidence regarding the export-led growth hypothesis.[1] From this perspective, the export-led growth hypothesis is interpreted as implying a unidirectional causal ordering from export growth to income growth; past export growth helps to predict current income growth, but not vice versa. Much of this evidence (again, beginning with Jung and Marshall, 1985) has been negative: the data do not seem to support such a causal ordering. One exception to this trend is Serletis (1992), who analyzed historical data for Canada, and concluded that ì(t)his finding [of Granger causality from export growth to income growth]Ösupports the export-led growth strategy in that expansion in exports promotes the growth of national incomeî (p. 144).

By including the growth of imports in his analysis, Serletis also provides an exception to the tendency for researchers in this area to restrict their attention to bivariate systems of exports and income.[2] Riezman, Whiteman and Summers (1996; RWS hereafter) demonstrate that inference in such bivariate systems is prone to a type of omitted variable bias; including imports can sometimes have dramatic effects on causal orderings as determined by Granger-type tests. Moreover, RWS point out that traditional causality tests are unable to answer the question of how strong a relationship exists between exports and income, say. They argue that the use of the conditional linear feedback measures developed by Geweke (1984) are more appropriate for such analysis, for three reasons. First, they allow the researcher to

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[1] See Riezman, Whiteman and Summers (1996) for a survey of this literature.

[2] For the sake of exposition, I will hereafter use the terms ìexports,î ìimports,î and ìincomeî synonymously with ìexport growth,î ìimport growth,î and ìincome growth,î except where I explicitly refer to the level of a variable. Also, ìgrowthî in phrases such as ìexport-led growthî means income growth.

control for the influence of other variables (e.g. imports) on the export-income relationship. Second, a simple transformation of the measures provides a measure of strength of such a relationship, analogous to a regression R^2. Third, the measures can be decomposed by frequency, allowing examination of the export-income relationship at, say, business cycle or long-run frequencies.

Applying Gewekeís measures to the data set of Summers and Heston (1991), RWS conclude that the evidence for export-led growth remains weak, if one concentrates on unidirectional causal orderings. However, linear feedback from exports to income (conditional on imports) tends to be stronger than in the opposite direction.

This paper has several objectives. First, I test for causal relationships between trade and growth using more than a century of data for both Australia and Canada. These two countries share many similarities; their geographical and population size, abundant natural resources and the predominance of primary commodities in their export base, and historical relationship with Great Britain. McLean (1994) finds that the savings and foreign investment patterns of the two countries are also similar. There are also important differences however, such as Australiaís geographic isolation and the severe depression which it experienced in the 1890ís. It is of interest to examine how (or whether) these similarities and differences impact on the trade-growth nexus.

Second, I use the frequency decomposition property of the conditional linear feedback measures to examine the strength of the trade-growth relationship at various horizons. Thus, I can answer questions such as whether a causal relationship is stronger at business cycle frequencies than long run frequencies. The clf measures also allow direct comparison of the trade-growth relationship across countries.

Finally, I extend Serletisís (1992) work on Canada. In addition to the use of an additional nine years of data, I identify historical episodes in a formal way, rather than arbitrarily splitting the sample in 1944.

Methods

In the empirical work reported here, I use a three-variable vector autoregressive (VAR) model consisting of the growth rates of real GDP (GNP for Canada), exports and imports. The data are annual, covering the period 1861-1993 for

Australia, and 1870-1993 for Canada.[3] I determined the order of the VAR by minimising the Schwartz Criterion (SC) over the entire data range, which resulted in a lag length of one.

Before conducting tests for causal orderings, I performed a preliminary analysis designed to detect possible structural breaks in the data series for the two countries. For each country, I tested for break points corresponding to the depression of the 1890ís, each of the two World Wars, and to the 1973 OPEC oil embargo. In identifying possible breaks, I used a variant of the Chow-type test as described in Geweke (1986, pp. 11-12). I first estimated a VAR over adjacent sub-periods (e.g., from the beginning of the data to 1914), obtaining an estimate S of the residual covariance matrix Σ. Then, for each of the possible break years listed in table 1, I estimated separate models for each part of the sample, obtaining estimates S_1 and S_2 of the corresponding covariance matrices. I then calculated the test statistic $T|S| - T_1|S_1| - T_2|S_2|$. Finally, using Normally-distributed errors, I simulated this procedure 200 times and recorded the fraction in which the simulated statistic exceeded the computed value. This fraction appears in the third column of table 1. I identified the break point as the first year for which the empirical marginal significance level (msl) was 10% or less. Table 1 provides a summary of the initial periods and possible break points, as well as the results of the search. This procedure identified the same sample periods for each country, which suggests that both countries have been subjected to similar external influences since at least the 1870's. The sub-periods identified are: from the beginning of the data (1863 for Australia, 1872 for Canada) until 1887, 1888-1914, 1915-38, 1939-71, and 1972-93.[4]

Table 1. Identification of sub-samples.

Initial Period[a]	Possible Break Years	Break Year (msl)[b]
Australia		
1863-1914	1887-91	1887 (0.055)
1888-1939	1914-20	1914 (0.000)
1915-73	1938-46	1938 (0.005)
1939-93	1971-75	1971 (0.025)
Canada		
1872-1914	1887-91	1887 (0.020)
1888-1939	1914-20	1914 (0.100)
1915-73	1938-46	1938 (0.015)
1939-93	1971-75	1971 (0.000)

[a]Initial periods and possible break years were selected *a priori*.
[b]msl is the (empirical) marginal significance level for the test of no structural change at the given year. Break years are the first of the years in column 2 for which msl≤0.10.

[3] Sources are listed in the Data Appendix.

[4]Data actually begin in 1861 for Australia and 1870 for Canada; two observations are lost by computing growth rates and allowing for lags.

Inference regarding causal orderings is made by comparing the percentiles of the posterior distributions of the conditional linear feedback (clf) measures, which I computed for each of the six possible causal directions.[5] To fix ideas, denote the clf measure from exports (x) to income (y) conditional on imports (m), as $F_{x\rightarrow y|m}$. For ease of interpretation, I use the transformation $f_{x\rightarrow y|m} = 1 - \exp\left(-F_{x\rightarrow y|m}\right)$ to express the clf measures in a manner analogous to a regression R^2. That is, $f_{x\rightarrow y|m}$ = 0.5 means that 50% of the variance in income growth is explained by innovations in export growth, after controlling for import growth.

Following RWS, I classify causal orderings in two ways: if the 10th percentile of the distribution of $f_{x\rightarrow y|m}$ exceeds the 90th percentile of the distribution of $f_{y\rightarrow x|m}$, and write x→y to describe this ordering. Alternatively, if the 50th percentile of the distribution of $f_{x\rightarrow y|m}$ exceeds the 90th percentile of the distribution of $f_{y\rightarrow x|m}$, *and* the 10th percentile of the distribution of $f_{x\rightarrow y|m}$ exceeds the 50th percentile of the distribution of $f_{y\rightarrow x|m}$, I take this as weaker evidence of a causal ordering, and denote it (x→y). Note that x→y ⇒ (x→y). Other orderings are defined analogously.

Results

Results of the conditional linear feedback tests for Australia and Canada are presented in tables 2 to 7. Each table gives results of the tests for two variables, conditional on the third. Entries in the table correspond to the ìR^{2}î version of the clf measures, as described above. For example, table 2 shows that, at long run frequencies, export growth explains between 3.1 and 56.1 percent of the conditional variance in income growth in Australia, over the 1861-87 period, with a median of just under 20 percent.

Examining the Australian results first (tables 2-4), it is apparent that export growth has not been an engine of income growth during any of the various periods, or over any of the frequency bands considered. In fact, there is relatively little evidence of any causal orderings between the three variables in the system, and none at all in the 1888-1914 or 1972-1993 periods. None of the orderings satisfy the strong criterion.

In the latter part of the 19th Century, it appears that GDP growth was the driving force behind growth in Australian trade, particularly import growth. There is

[5] Construction of these measures is detailed in Geweke (1984), and is described briefly in Appendix B.

moderate evidence of income leading imports, both in the long run and overall. Income growth accounts for 58% of the conditional variance of imports (based on the median), at business cycle frequencies, whereas imports account for only about 23% of income variance. Exports also appear to lead imports at business cycle frequencies.

The period between 1915 and 1938 is best characterized by ìimport-led exports.î This causal pattern appears (in its weaker form) at all three frequency bands. The only causality apparent between exports and income is in the 1939-71 period, with weak evidence of growth-led exports at long run frequencies. Imports are also causally prior to exports in this cell. Thus it seems that exports have historically been the ìhandmaiden of growth,î and particularly of import growth, in Australia.

In contrast, the Canadian data provide a good deal of support for a causal link from trade growth to income growth in the years between the World Wars. Export growth leads income growth at all frequency bands; imports also lead growth in this period, although causality from income to imports prevents these from being "strong" orderings. The strength of feedback from exports is particularly strong; two of the three 80% ìconfidence intervalsî in table 5 are disjoint, and the third is nearly so.

Figures 1 through 4 present the posterior distributions of the clf measures between exports and income, and imports and income, by country and frequency. These figures facilitate a comparison of the trade-growth relationship between the two countries. Note that, except for the first and third sample periods, the strength of the relationship between trade growth and income growth is of comparable magnitude in Australia and Canada. Median estimates of the (conditional) predictive power of Australian exports and income tend to lie between about 15% and 35% at long run frequencies; the corresponding range for Canada (excluding the 1872-87 and 1915-38 periods) is 16% to 33%. As mentioned above, the Canadian data show evidence of pronounced ìtrade-led growthî in the 1915-38 period. Also noteworthy is the difference in the strength of feedback from the trade variables to income growth in the first sample period. Although no unidirectional causal ordering is present for either country, the predictive power of exports and imports for income growth is much higher in Canada than in Australia. Median estimates of $f_{x \rightarrow y|m}$ or $f_{m \rightarrow yx}$ in this period are never below 35.8% in Canada, and never above 22.5% for Australia.

Finally, table 8 presents the empirical results for Canada obtained using Serletisís samples. While these results confirm the export-led growth inference drawn by Serletis (i.e., export led growth appears in the pre-World War II sample and over the full sample, but not post-1945), I find strong evidence of import-led growth as well. Moreover, the results for the pre-1945 sample seem to be driven exclusively by the 1915-38 period, as shown in tables 5 and 6. The causal relationship between exports and imports seems to be particularly sensitive to choice of sample periods.

Conclusions

The findings in this paper, while consistent with many different models of international trade and long-run growth, do not suggest a strong role for export growth in helping to predict growth in Australian GDP over the last 135 years. In fact, the only evidence for export-led growth in either country is confined to a brief span in Canadaís economic history. Contrary to Serletis (1992), I also find a significant role for imports in leading growth in Canada during this time.

It is interesting to note that causal inferences in the Australian data are uniformly from income growth to growth in the trade variables, while the opposite is true for the inferences drawn from Canadian data. This result suggests that, for all their similarities, the underlying sources of growth in these two countries are fundamentally different.

It is worth pointing out that the results presented here have been predicated on the interpretation of ìexport-led growthî as ìa unidirectional causal ordering from exports to growth.î However, even though there is a general lack of such causal orderings in either data set, the results in this paper show that both countries show evidence of a fairly large degree of bi-directional causality between exports or imports and income. As Chow (1987, p. 60) notes, such a bi-directional relationship suggests that growth and trade ìÖare mutually interdependent in the development process.î He points out that export growth can allow expansion of a countryís domestic markets, contributing to the development of economies of scale. These scale economies, combined with the increased global integration brought about by increasing trade, enhances the countryís comparative advantage, further promoting export growth and national income growth. This sort of interpretation seems most plausible in the context of Australian and Canadian economic development.

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Appendix A: Data Sources

Australia
Export and Import data: Australian Bureau of statistics, National income expenditure and product, cat 5204.0, 1992/92, tables 9 & 91 (years 1949-1993), and F25 & F26 (years 1861-1948). Earlier years from N. G. Butlin (1962, pp. 410-11 and 413-14, tables 247 and 248) and M. W. Butlin (1977, pp. 78 and 108 tables IV.1 and IV.17, col. 4).

GDP data: Australian Bureau of Statistics, National income expenditure and product, cat 5204.0, 1992/93, tables 9 & 91 (years 1949-1993), and McLean and Pincus (1982, Appendix 1) (years 1861-1948)
GDP deflator (1966/67 = 100) from McLean and Pincus (1982); rebased to 1910/11=100.

Note: Australian data are in calendar years from 1861-1900, and financial years (ending June 30th), 1901-93. Thus data for the second half of 1900 is double-counted; once in 1900 and again in 1901; I multiply this data point by 2/3 to correct this.

Canada
GNP data: million \$1900, (years 1870-1926), from Urquhart (1986); \$1971 million, (years 1926-83) from Urquhart (1988); \$1987 million (years 1983-93), World Bank.

Export and Import data: Adjusted series from Urquhart (1986), table 2.4, thousands, (years 1870-1926); Statistics Canada, Canadian Statistical Review, Historical Summary 1970, (years 1926-71); Bank of Canada Review, winter 1992-3 (years 1971-73) and spring 95 (years 1974-94), seasonally adjusted.

Price deflators: 1900=100, (years 1870-1915); 1913=100, (years 1913-1926); 1948=100, (years 1926-1961); 1971=100, (years 1961-1975); from Urquhart (1988) table a4; 1987=100, (years 1975-1993) from World Bank

All series (for both countries) were spliced at the first year of overlap.

Appendix B: Conditional Linear Feedback Measures

This appendix briefly describes the procedure for computing the measures of conditional linear feedback described in the text. The exposition, and much of the notation, follows Geweke (1984), to which the reader is referred for more details.

Consider the vector autoregressive system (VAR) given by

(1) $\mathbf{b}(L)\mathbf{z_t} = e_t,$

where $z_t = \left(y_t', x_t', m_t'\right)'$ is a vector consisting of the growth rates of income, exports and imports, respectively, b(L) is a matrix polynomial lag operator of order p, and e_t is a vector of normal disturbances with mean zero and covariance matrix Σ. I assume that the roots of the equation $\det\left(b(L)\right) = 0$ all exceed 1 in modulus, so that the system is stationary. The methods presented below use a lag length of 2; use of alternative lag lengths is straightforward.

In computing the measures of linear feedback, it is more convenient to orthogonalise the system using the (lower triangular) Choleski decomposition,[6] $\mathbf{G^{-1}}$, of Σ, so that $\Sigma = \mathbf{G}^{-1}\left(\mathbf{G}^{-1}\right)'$. Let **W** be the diagonal matrix whose elements are the reciprocals of the diagonal elements of **G**. Pre-multiplying (1) by $\mathbf{WG}^{-1}$ results in a new system in which the lag 0 coefficient matrix is lower block triangular, with unit diagonal. This system serves as the basis for computing the clf measures. Thus we have

(2)
$$\mathbf{B}(L)\mathbf{z_t} = \eta_t,$$
$$E\eta_t\eta_{tt}' = \mathbf{WW}'$$

where **WW'** is diagonal, $\mathbf{B}(L) = \mathbf{WG}^{-1}b(L)$, and $\eta_t = \mathbf{WG}^{-1}\varepsilon_t$.

In order to proceed, we must first answer the following question: if the representation in (2) is the "true" three-variable model, and we estimate a two-variable system omitting exports (say), what would this system look like? In other words, we want to find matrices $\mathbf{d_1}$ and $\mathbf{d_2}$ such that

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[6]This orthogonalisation is not unique, since the Choleski decomposition is not the only matrix square root. Also, a different ordering of the variables in the VAR will in general change G^{-1}.

$$\begin{pmatrix} y_t \\ m_t \end{pmatrix} + \mathbf{d_1}\begin{pmatrix} y_{t-1} \\ m_{t-1} \end{pmatrix} + \mathbf{d_2}\begin{pmatrix} y_{t-2} \\ m_{t-2} \end{pmatrix} = \begin{pmatrix} \eta_{1t} \\ \eta_{2t} \end{pmatrix}, \text{ or}$$

(3) $$\mathbf{h}_t + \mathbf{d}_1\mathbf{h}_{t-1} + \mathbf{d}_2\mathbf{h}_{t-2} = \nu_t,$$

where $\mathbf{h}_t$ and ν_t have been defined in an obvious way.[7] The least squares orthogonality conditions are

(4) $$\begin{aligned} E\mathbf{h}_t\mathbf{h}'_{t-1} + \mathbf{d_1}E\mathbf{h}_{t-1}\mathbf{h}'_{t-1} + \mathbf{d_2}E\mathbf{h}_{t-2}\mathbf{h}'_{t-1} &= \mathbf{0}, \\ E\mathbf{h}_t\mathbf{h}'_{t-2} + \mathbf{d_1}E\mathbf{h}_{t-1}\mathbf{h}'_{t-2} + \mathbf{d_2}E\mathbf{h}_{t-2}\mathbf{h}'_{t-2} &= \mathbf{0}. \end{aligned}$$

If we define $\gamma_j = E\mathbf{h}_t\mathbf{h}'_{t-j}$, (4) becomes

(5) $$\begin{aligned} \gamma_1 + \mathbf{d_1}\gamma_0 + \mathbf{d_2}\gamma_{-1} &= 0 \\ \gamma_2 + \mathbf{d_1}\gamma_1 + \mathbf{d_2}\gamma_0 &= 0. \end{aligned}$$

Solving this equation for $\mathbf{d_1}$ and $\mathbf{d_2}$, we have:

(6) $$\begin{pmatrix}\mathbf{d}_1 & \mathbf{d}_2\end{pmatrix} = -\begin{pmatrix}\gamma_1 & \gamma_2\end{pmatrix}\begin{pmatrix} \gamma_0 & \gamma_1 \\ \gamma_{-1} & \gamma_0 \end{pmatrix}^{-1}.$$

If Γ_j is defined as $E\mathbf{z}_t\mathbf{z}'_{t-j}$ (the covariance matrices of the original VAR system), then the elements of γ_j are found from the appropriate partitioning of Γ_j. The matrices Γ_j can be recovered from the system (2) as follows. Write

(7) $$\begin{pmatrix} \mathbf{z_t} \\ \mathbf{z_{t-1}} \end{pmatrix} = \begin{pmatrix} \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{I} & \mathbf{0} \end{pmatrix}\begin{pmatrix} \mathbf{z_{t-1}} \\ \mathbf{z_{t-2}} \end{pmatrix} + \begin{pmatrix} \eta_\tau \\ \mathbf{0} \end{pmatrix}.$$

Let $\mathbf{B}$ denote the first matrix on the right-hand side of (7). Multiply (7) by its transpose and take expectations of both sides to get

(8) $$\tilde{\Gamma}_\mathbf{0} = \mathbf{B}\tilde{\Gamma}_\mathbf{0}\mathbf{B}' + \begin{pmatrix} \mathbf{WW}' & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}.$$

where

(9) $$\tilde{\Gamma}_\mathbf{0} = \begin{pmatrix} \Gamma_\mathbf{0} & \Gamma_\mathbf{1} \\ \Gamma_{-\mathbf{1}} & \Gamma_\mathbf{0} \end{pmatrix} = \begin{pmatrix} \Gamma_\mathbf{0} & \Gamma_\mathbf{1} \\ \Gamma'_\mathbf{1} & \Gamma_\mathbf{0} \end{pmatrix}.$$

If the eigenvalues of $\mathbf{B}$ are all less than one in modulus, the solution to equation (8) is given by

$$vec\left(\tilde{\Gamma}_0\right) = (I - B \otimes B)^{-1} vec(V),$$

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[7]In general, the two-variable VAR would be of infinite order if the three-variable VAR has two lags. We approximate the infinite-order VAR (actually, a finite-order vector ARMA) by a VAR(2) in the interest of conserving degrees of freedom.

where *vec* denotes the column stacking operator, ⊗ is the Kronecker product, and V is the matrix in the second term of (8).

. Then Γ_2 is the upper right 3x3 submatrix of $\tilde{\Gamma}_1 \; (= \mathbf{B}\Gamma_0)$.

Once the Γ_j (and hence the γ_j) are obtained, we can use (6) to solve for $\mathbf{d_1}$ and $\mathbf{d_2}$, and write the two-variable VAR implied by (2) as

(10) $$\begin{pmatrix} \mathbf{d_{11}}(L) & \mathbf{d_{12}}(L) \\ \mathbf{d_{21}}(L) & \mathbf{d_{22}}(L) \end{pmatrix} \begin{pmatrix} y_t \\ m_t \end{pmatrix} = \begin{pmatrix} y_t^* \\ m_t^* \end{pmatrix},$$

where $\mathbf{d_{11}}(0) = \mathbf{d_{22}}(0) = 1$, and $\mathbf{d_{12}}(0) = 0$. Equation (10) is the answer to the question posed above.

Equations (10) and (2) provide the basis for decomposing the clf measures by frequency. Let the moving average representation of (2) be

(11) $$\mathbf{z_t} = \mathbf{A}(L)\eta_t .$$

Also let $z_t^* = \left(y_t^*, x_t, m_t^*\right)'$. Then the moving average representation of z_t^* is

(12) $$\mathbf{z_t^*} = \begin{pmatrix} \mathbf{d_{11}}(L) & \mathbf{0} & \mathbf{d_{12}}(L) \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{d_{21}}(L) & \mathbf{0} & \mathbf{d_{22}}(L) \end{pmatrix} \mathbf{A}(L)\eta_t = \mathbf{F}(L)\eta_t .$$

Letting $\text{var}(\eta_{it}) = P_i$, the spectral density matrix of y^* is

(13) $$S_{y^*}(\lambda) = F_{11}(\lambda)P_1F_{11}(\lambda)' + F_{12}(\lambda)P_2F_{12}(\lambda)' + F_{13}(\lambda)P_3F_{13}(\lambda)',$$

where $F_{11}(\lambda)$ is the Fourier transform of $\mathbf{F_{11}}(L)$ at frequency λ, etc. Finally, the measure of conditional linear feedback from exports to income at frequency λ is

(14) $$f_{x \to y|m}(\lambda) = \ln\left(\frac{\left|S_{y^*}(\lambda)\right|}{F_{11}(\lambda)P_1F_{11}(\lambda)'} \right).$$

Overall measures of conditional linear feedback can be obtained by averaging the quantities in (14) over all frequencies λ. In addition, clf measures at particular horizons can be obtained in the same way. For example, linear feedback at business cycle frequencies can be represented as the average of (14) over frequencies corresponding to periods of between 8 and 32 quarters.

Posterior distributions were computed using a standard non-informative prior: flat on the coefficients of **B**(L) and inverted-gamma on the elements of Σ (see Zellner, 1971, or Doan, 1992). The computations were carried out via Monte Carlo integration with 1,000 replications using version 4.2 of RATS.