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Robert Dixon and G. C. Lim

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Robert Dixon[†] and G. C. Lim[‡]
[†] Department of Economics, The University of Melbourne
[‡] Melbourne Institute of Applied Economic and Social Research,
The University of Melbourne

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Melbourne Institute of Applied Economic and Social Research
The University of Melbourne
Victoria 3010 Australia
***Telephone* (03) 8344 2100**
***Fax* (03) 8344 2111**
***Email* melb-inst@unimelb.edu.au**
***WWW Address* http://www.melbourneinstitute.com**

Abstract

In this paper, we set out a model of labour productivity which distinguishes between shocks which change productivity permanently and shocks which have transient affects on productivity. We show that this model is a type of unobserved components model –a random walk with drift plus noise model. The advantage of this approach is that it provides a coherent framework to identify the deterministic trend growth component and also the productivity-enhancing (or technology-related) stochastic components. The model is applied to aggregate labour productivity in Australia and the time series of technology shocks extracted is used to shed some light on the contributions of policy reforms to productivity.

1. Introduction

Macroeconomists take a great deal of interest in the level and growth rate of labour productivity (amongst other variables), for obvious reasons. As a case in point, an understanding of its trend growth is vital to an assessment of wage growth and inflationary pressures. Great attention is paid to perceived breaks in trend growth, and whether growth in productivity is slowing or accelerating but the analysis can be quite ad hoc. The aim of this paper is to propose the adoption of a particular univariate time-series framework to decompose, in a coherent manner, the deterministic and stochastic trend components of aggregate productivity. The former component captures the systemic growth in productivity while the latter component picks up the (un-systemic) contributions of technology and policy to growth in productivity.

In a classic paper, Nelson and Plosser (1982) argued that many macroeconomic time series behave more like a random walk (stochastic trend) than a deterministic trend and it has become common to model many macro variables, including productivity (and technical change), as a random walk with or without drift. However, the adoption of a random walk data generating process (DGP) is not innocuous because it assumes that every 'shock', every 'disturbance', has permanent effects. However, this may not be sensible for all macro data sets – especially if we recognize the presence of such things as measurement errors and data revisions which, by definition, cannot reflect some underlying economics process and hence can only have temporary effects on the data.

In this paper, we show that the DGP for labour productivity is best characterized as a random walk with drift plus noise (or stochastic trend plus noise model) which allows for both permanent and temporary shocks; in other words, the model separates out those shocks that change productivity permanently and those shocks that have only transient affects on productivity. More importantly, our estimated model is based on well accepted propositions from neoclassical growth theory and does not rely on the presence of measurement errors to justify the distinction between permanent and temporary effects.[1]

[1] Although the presence of measurement errors and frequent revisions in national accounts data provides a *prima facie* case for considering models which allows for both types of shocks.

The paper is organised as follows. In section 2 we set out the key propositions relating to labour productivity in the context of a Solow-Swan growth model when technical progress is a random walk. We then show that this implies that observed labour productivity will behave as a random walk with drift plus noise model (which is a specific example of an unobserved components model). In section 3, we set out the key statistical properties of the random walk with drift plus noise model. In so doing, we note a statistical property that may be used to check when it is appropriate to adopt a stochastic trend model and when it is more appropriately modelled as a stochastic trend plus noise model. In section 4, we discuss the behaviour of Australian labour productivity for two measures of aggregate labour productivity and estimate the random walk with drift plus noise model in an unobserved components setting. We then compare the underlying trend generated by this model with that generated by a purely deterministic trend. The final section contains concluding remarks.

2. Labour productivity and growth

Let the production function be a Cobb-Douglas function with constant returns to scale and let the technological change be labour saving:

$$Y_t = K_t^{\alpha}\left(Z_t L_t\right)^{1-\alpha} \tag{1}$$

where Y is aggregate production, K is an index of capital input, L is an index of labour input, Z is the level of technology, α is the elasticity of output with respect to the capital input and t is a time index. Equation (1) can be expressed in log terms

$$y_t = \alpha k_t + \left(1-\alpha\right) z_t \tag{2}$$

where y_t is the logarithm of labour productivity (Y/L), k is the logarithm of the capital-labour ratio (K/L) and z is the logarithm of Z.

In a Solow-Swan growth model, equilibrium or steady state growth entails aggregate output and capital growing at the same, constant, rate.[2] Defining this (constant) equilibrium capital-output ratio as:

[2] For further discussion on the constancy of the capital-output ratio in conditions of equilibrium growth see Dixon (2003) and (2006).

$$\left(\frac{K_t^*}{Y_t^*}\right) = e^{\phi}$$

where an asterisk indicates that the variable is growing at the steady-state rate. It follows from the above that:[3]

$$k_t^* = y_t^* + \phi \qquad (3)$$

Substituting (3) into (2) we obtain the steady-state (log) output per worker (y^*) as:[4]

$$y_t^* = \frac{\alpha}{(1-\alpha)}\phi + z_t \qquad (4)$$

Suppose that observed y_t is made up of y_t^* plus a transitory component ε_t:

$$y_t = \frac{\alpha}{(1-\alpha)}\phi + z_t + \varepsilon_t \qquad \varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right)$$

Let z, be a random walk with drift such that $z_t = z_{t-1} + \gamma + \eta_t$, where γ is the rate of (Harrod-neutral) technological progress and $\eta_t \sim NID\left(0, \sigma_\eta^2\right)$. We can then write the above as:

$$y_t = \frac{\alpha}{(1-\alpha)}\phi + z_{t-1} + \gamma + \eta_t + \varepsilon_t \qquad (5)$$

showing that the $\eta's$ can be interpreted as 'technology shocks' and the $\varepsilon's$ as 'non-technology shocks'.[5] As in Gali (1999), in our model a technology shock "has a permanent, one-for-one effect on productivity" and "only technology shocks can have a permanent effect on the level of labour productivity" (Gali, 1999, p 253 and p. 256.)

Equation (5) may also be written as a random walk with drift and a composite error term:

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[3] See also Shapiro & Watson (1988, p 114).

[4] In the context of the Solow-Swan growth model, the equilibrium level of output per worker is derived as: $\left(Y_t/L_t\right) = Z_t\left(s/\left(n+g+d\right)\right)^{\alpha/(1-\alpha)}$, where s is the savings propensity, n is the rate of population growth (assumed exogenous and equal to the rate of growth in labour supply and employment), g (= γ) is the rate of technological progress and d is the depreciation rate). When s, n, g and d are assumed to be constant over time, then taking logs we find that ϕ is equivalent to log(s/(n + g + d)).

[5] Non-technology shocks will include measurement errors.

$$y_t = y_{t-1} + \gamma + \eta_t + \varepsilon_t - \varepsilon_{t-1} \qquad (6)$$

In the next section of the paper we show that (6) is a particular example of an unobserved components model.

3. Random Walk with Drift plus Noise Model

A random walk with drift plus noise model for a variable (y) consists of two additive components. One is the 'permanent' or 'underlying' component (μ, which is a random walk - with drift in our case) and the other is the 'transitory' or 'pure noise' component (ε).[6]

$$y_t = \mu_t + \varepsilon_t \;; \qquad \varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right) \qquad (7a)$$

$$\mu_t = \mu_{t-1} + \gamma + \eta_t \,, \qquad \eta_t \sim NID\left(0, \sigma_\eta^2\right) \qquad (7b)$$

where μ_t is the mean growth rate of the series and, and η_t and ε_t are assumed to be independent of each other. This model can be collapsed into:

$$y_t = y_{t-1} + \gamma + \eta_t + \varepsilon_t - \varepsilon_{t-1} \qquad (8)$$

which is identical to (6) above. Given the initial condition (y_0) y_t can also be written as:

$$y_t = y_0 + \gamma t + \sum_{i=1}^{t} \eta_i + \varepsilon_t \qquad (9)$$

showing that the η shocks have permanent effects on y while the ε shocks have only temporary effects.

The advantage of the model is that it offers a coherent framework to discuss changes in productivity. The component γt shows the deterministic trend growth which is expected to increase by γ per period.[7] The component $\sum_{i=1}^{t} \eta_i$ shows the

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[6] Harvey (1989) has a good discussion of the time series characteristics of the model. Enders (2004, p 164) refers to this model as a "trend plus noise model".

[7] If (9) is the true model and an equation for y_t were to be estimated without including the second last term on the RHS of (9) the estimate of γ would be biased. Further, if (9) is the true model then any forecast should take into account both the deterministic **and** the stochastic trend components and should not rely only on the former.

cumulative effects of both productivity-enhancing ($\eta > 0$) and productivity-reducing technological shocks ($\eta < 0$); in other words, a series of positive shocks would be indicative of accelerations in productivity over and above that driven by the deterministic trend. Finally the component ε_t captures transient "noise" in productivity.

A correlogram for the time series can be used in a relatively straightforward way to assess whether or not the model we have presented is a suitable model. It follows from (9) that $\text{var}(y_t) = t\sigma_\eta^2 + \sigma_\varepsilon^2$, $\text{var}(y_{t-s}) = (t-s)\sigma_\eta^2 + \sigma_\varepsilon^2$ while $\text{covar}(y_t, y_{t-s}) = (t-s)\sigma_\eta^2$ and so the autocorrelation coefficients for y will be $\rho_s = \dfrac{(t-s)\sigma_\eta^2}{\sqrt{(t\sigma_\eta^2 + \sigma_\varepsilon^2)((t-s)\sigma_\eta^2 + \sigma_\varepsilon^2)}}$ indicating that regardless of the length of the lag the autocorrelation coefficients for y will be positive and that as we increase the length of the lag the autocorrelation coefficients will become smaller and tend towards zero.

Turning to the first-differences in y (ie the growth rate of labour productivity), it follows from (9) that the change in y $(\Delta y_t = y_t - y_{t-1})$ in any period will be:

$$\Delta y_t = \gamma + \eta_t + \Delta\varepsilon_t = \gamma + \eta_t + \varepsilon_t - \varepsilon_{t-1} \tag{11}$$

It can be shown that $\text{var}(\Delta y_t) = \text{var}(\Delta y_{t-s}) = \sigma_\eta^2 + 2\sigma_\varepsilon^2$ and that $\text{covar}(\Delta y_t, \Delta y_{t-1}) = -\sigma_\varepsilon^2$ while $\text{covar}(\Delta y_t, \Delta y_{t-s}) = 0$ for all $s \geq 2$. It follows that the first order autocorrelation coefficient for Δy will equal $\rho_1 = \dfrac{-\sigma_\varepsilon^2}{\sigma_\eta^2 + 2\sigma_\varepsilon^2}$ which will lie between −0.5 (if σ_ε^2 is very large relative to σ_η^2) and 0 (if σ_η^2 is very large relative to σ_ε^2). However, the autocorrelation coefficient will be 0 for any lags longer than 1, that is $\rho_s = 0$, for all $s \geq 2$. All of which is to say that, for this model the first differences will be negatively autocorrelated at lag one but there will be no autocorrelation at longer lags. Hence, this statistic serves as a convenient way to decide *apriori*, whether to estimate the econometrically more difficult unobserved components model or to estimate the simpler random walk model with/without drift.

4. Australian labour productivity

Much of the discussion of productivity growth in Australia (see Parham, 2004 for example) presumes that the time series can be regarded as a deterministic log-linear trend (possibly with an occasional break in the growth rate). For the reasons given in the previous section we think it worth exploring the hypothesis that productivity growth in Australia contains a stochastic trend component and both permanent and temporary shocks.

The Australian Bureau of Statistics (ABS) provides quarterly data for two measures of aggregate labour productivity. One is for the 'whole economy' and is measured as GDP divided by total hours worked in all sectors of the economy. Due to the well known problems with computing value-added in many sectors (and especially the public sector) a second measure is reported which refers to the 'market sector' alone.[8] We will examine both series and will compare and contrast the time series properties of the series for the market sector alone (real GDP per hour worked in the market sector) with that for the economy as a whole (real GDP per hour worked) for Australia over the period 1978:3 – 2007:4. The productivity series examined are the reported seasonally-adjusted series.

Figure 1 presents a time series for the logarithms of these two series.[9] Not surprisingly, for both series Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests fail to reject the null of a unit root in the levels while a Kwaitowski, Phillips, Schmidt and Shin (KPSS) test rejects the null of stationarity in the levels. The same tests reject the null of a unit root in the first differences or fail to reject the null of stationarity in the first differences for both series. We therefore infer that labour productivity in Australia is best viewed as having a stochastic trend component.

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[8] The "market sector" is defined as comprising 12 of the 17 ANZSIC divisions. The divisions included in the market sector are: Agriculture, forestry & fishing, Mining, Manufacturing, Electricity, gas & water, Construction, Wholesale trade, Retail trade, Accommodation, cafes & restaurants, Transport & storage, Communication services, Finance & insurance, and Cultural & recreational services. Excluded are: Property & business services, Government administration & defence, Education, Health & community services and Personal & other services.

[9] The (seasonally adjusted) data has been downloaded from the ABS web site for 5206.0 Australian National Accounts: National Income, Expenditure and Product, Dec 2007 and is contained in the spreadsheet 5206001_key_aggregates.xls downloaded on 12 March 2007. The series ID's are A2304192L and A2304194T. A description of the way in which the data is compiled may be found in ABS (2005).

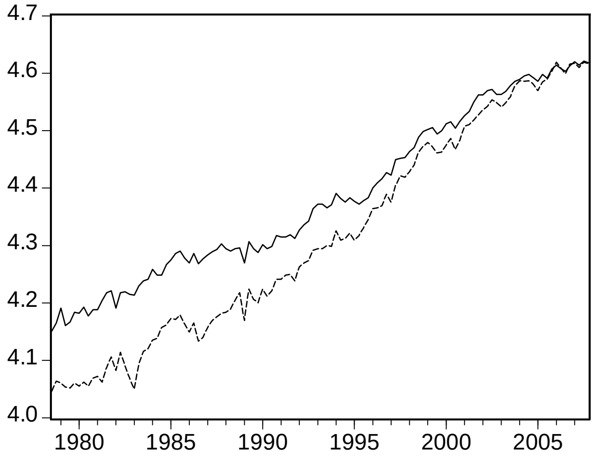

Fig. 1. Logarithms of GDP per hour worked in whole economy (solid line) and GDP per hour worked in the market sector alone (broken line): Australia, 1978:3 – 2007:4.

Correlograms for the logarithm of GDP per hour worked and GDP per hour worked in the market sector show that in both cases all of the autocorrelation coefficients are positive and tend towards zero as we increase the length of the lag, consistent with the model set out above. The first-order autocorrelations are 0.974 and 0.977 respectively. For the first differences in both series, the first order autocorrelations are negative and lie between −0.5 and 0, while the autocorrelation coefficients for lags greater than one are not significantly different from zero. The significant and negative first-order autocorrelation coefficients (−0.263 and −0.264 respectively) for the differenced series suggest that it is appropriate to model both series as "random walks with drift plus noise".

The "random walk with drift plus noise" model was estimated by maximum likelihood using a Kalman filter.

For real GDP per hour worked in the whole economy we find:

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t & \varepsilon_t &\sim N(0, 0.0000388) \\ \mu_t &= \underset{(0.000656)}{0.003971} + \mu_{t-1} + \eta_t, & \eta_t &\sim N(0, 0.0000456) \end{aligned}$$

The estimated average rate of technological progress (the drift parameter) for real GDP per hour worked in the whole economy is 0.0040 per quarter or 1.61% per annum (over the period 1978-2007). The 'signal-noise ratio' $\left(\sigma_\eta^2 / \sigma_\varepsilon^2\right)$ informs us about the relative size of technology shocks as against non-technology shocks (i.e., the relative size of shocks with permanent effects as against shocks with transitory effects). Given the estimates of σ_η^2 and σ_ε^2 reported above, the 'signal-noise' ratio for real GDP per hour worked in the whole economy is 1.1769, indicating that over the whole period technology shocks were (slightly) larger on average than non-technology shocks.

For real GDP per hour worked in the market sector we find

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t & \varepsilon_t &\sim N(0, 0.0000615) \\ \mu_t &= \underset{(0.000984)}{0.004865} + \mu_{t-1} + \eta_t, & \eta_t &\sim N(0, 0.0000899) \end{aligned}$$

Here, the estimated average rate of technological progress is 0.0049 per quarter or 1.97% per annum (over the period 1978-2007). This finding that technical progress is faster for the market sector taken alone than it is for the economy as a whole makes sense, given that labour productivity growth for the whole economy is a weighted sum of productivity growth in the market sector and productivity growth in the non-market sector and this latter includes a number of (government) sectors where by construction productivity growth is zero.[10] The 'signal-noise' ratio for this series is 1.4621, indicating for the market sector, technology shocks were almost one and one-half times as large on average than non-technology shocks.

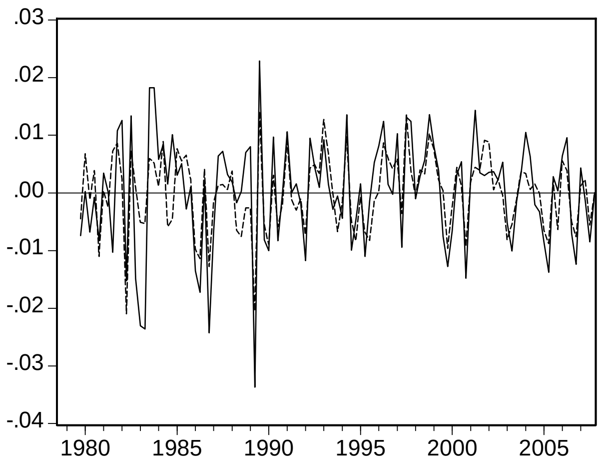

Fig. 2. Technology (ie permanent) shocks recovered from the model for the market sector alone (solid line) and Technology (ie permanent) shocks recovered from the model for the whole economy (broken line): Australia, 1979:4 – 2007:4.

[10] That is, where production is valued at labour cost.

Figure 2 presents the time series for the technology (ie permanent) shocks recovered from the model for the market sector (solid line) and from the model for the whole economy (broken line). There are three aspects of the permanent shocks which are noteworthy. First, the two series (although estimated separately) are positively correlated ($r = 0.76$) as we would expect. Second, there appears to be a decline in the volatility of the shocks in the second half of the period. Third, the series for the technology (ie permanent) shocks in the market sector may throw some light on a debate which has been on-going in Australia about the impact of the microeconomic reform program which began in the early 1980s. Many commentators argue that the reforms resulted in higher productivity growth in the 90s (see Parham (2004) for example), some are sceptical about the impact of the reforms (see Quiggin (2001) for example) while others wonder whether the reforms have permanent effects (see Gruen (2004) for example). It is common in this debate to compare indices for the market sector for the period 1984/5 – 1988/89 with those for the period 1993/4 – 1998/99. Our results show that average technology shocks was about −0.0006 in the period 1979Q1-1993Q and about +0.0008 in the period 1994Q1 to 2007Q4[11] and that these shocks have had a permanent effect on labour productivity – indeed, all of the difference in the average growth rate of labour productivity between the two periods can be attributed to the change in the average size of technology shocks. Taken together these findings are consistent with the impact of productivity enhancing reforms[12] and that these reforms have had a permanent effect on labour productivity.

Finally, our approach provides an alternative assessment of over and below trend growth. Figures 3 and 4 show the difference between a simple deterministic (log) linear trend approach and our broader framework to extract the deterministic trend component. We have estimated both models over the whole sample period 1978Q3-2007Q4, but for illustrative purposes, we display in Figures 3 and 4 the predicted and actual series for only the second half of our sample period. The main point to note here is that: according to the simple (log) linear trend approach actual productivity near the end of our sample period was "below-trend", whereas according to our methodology

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[11] And this is the case even if we use the start and end periods proposed in Quiggin (2001).
[12] But our evidence does not rule out the possibility that they do reflect instead increases in the 'intensity of work' as Quiggin (2000) has proposed.

which separates out the stochastic trend component, actual productivity was "above-trend" near the end of our sample period.

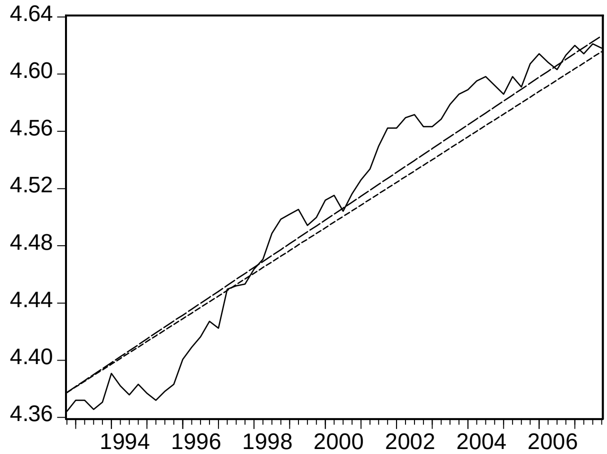

Fig. 3. Logarithms of GDP per hour worked in whole economy: actual vale (——), values predicted by a simple log-linear deterministic trend (– – – –) and values predicted by the deterministic trend component of the random walk plus drift plus noise model (- - - - -).

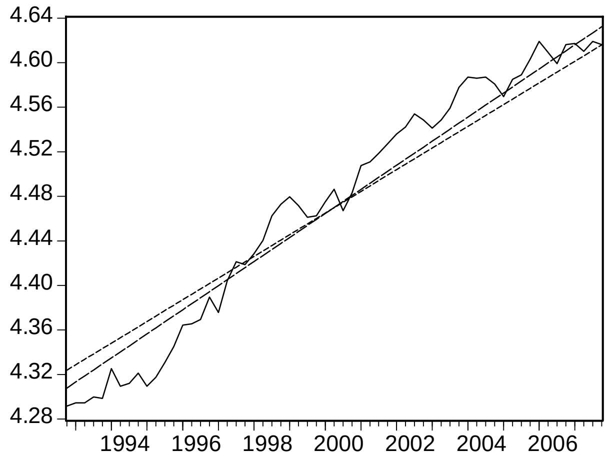

Fig. 4. Logarithms of GDP per hour worked in the market sector: actual vale (——), values predicted by a simple log-linear deterministic trend (– – – –) and values predicted by the deterministic trend component of the random walk plus drift plus noise model (- - - - -).

5. Concluding remarks

In this paper, we set out a model of labour productivity which distinguished between shocks which change productivity permanently and shocks which have transient affects on productivity. We showed that this model is equivalent to a particular time series model – the random walk with drift plus noise model. The advantage of the analysis is that it provided a coherent framework to identify both the deterministic trend growth component and the stochastic trend component which is driven by productivity-enhancing (or productivity-slowing) shocks. The empirical analysis showed that the "random walk with drift plus noise" model, which is based on the steady state in the Solow-Swan model when technical progress is a random walk, appeared to capture well the behavior of Australian aggregate labour productivity. Also, we saw that estimates for the average rate of technological progress and also for both the absolute and the relative size of technology shocks c.f. non-technology shocks were all higher for the market sector taken alone than they were for the whole economy. If the results from applying our model to Australian data have implications for the equivalent data sets for other countries it would suggest that studies for (say) the USA which have looked at data for the whole economy may be under-stating both the rate of technological progress and the importance of technology shocks.

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