AN INVESTIGATION INTO CAPTIVITY AND CORRELATION IN THE LABOR SUPPLY MODEL OF MITTS

June 2003

Tim R.L. Fry, Mark N. Harris and Hsein Kew

Melbourne Institute of Applied Economic and Social Research
The University of Melbourne

Final report prepared for the Department of Family and Community Services

Acknowledgements: We would like to thank the Department of Family and Community Services for funding this research. The views expressed in this paper are those of the authors and do not represent the views of the Minister for Family and Community Services, the Department of Family and Community Services or the Commonwealth Government.

1. Introduction

This research is concerned with investigating whether there is any evidence of captivity to or correlation between the discrete hours outcomes in the labor supply model embodied in the Melbourne Institute Tax and Transfer Simulator (MITTS). Our approach is to take the modeling framework used in MITTS and to investigate the use of alternative specifications of the "core" discrete choice model. Details of the modeling framework of MITTS are given in Kalb (2002). That report documents the results of specifying and estimating the discrete choice labor supply model that underlies the behavioral results in MITTS. The model is based upon the quadratic utility function and allows for the presence of fixed costs associated with working and heterogeneity in preferences for labor supply and income.

The "core" discrete choice model utilized is the conditional Logit model with eleven labor supply points (hours outcomes). Thus inherent in the labor supply estimations are the probabilistic expressions derived from the Logit model. That is, the individual is assumed to make a choice, from a finite set of choices, of how many hours to work. Estimations are subsequently based on the probability that an individual will "choose" a particular hours outcome implied by the Logit model. However, such a model may be inappropriate on two fronts.

Firstly, there is a natural ordering in the labor supply choices – with 35 hours of work being closer to 40 hours of work than 30 hours of work is. Therefore it seems plausible to expect a correlation between outcomes that are close neighbors, especially when particular numbers of hours are not available. Suppose for example, that the preferred labor supply of 17.5 hours per week is not on offer. Then one might expect a correlation between the available alternative choices of 15 and 20 hours per week. This potential correlation is ignored in the Logit model and this may lead to biased parameter estimates of the utility function, and hence to biased estimates of labor supply responses to policy shocks.

Secondly, there are invariably spikes in the distribution of observed hours, for example, due to institutional reasons such as "full-time" employment being defined as a 40-hour working week. This implies that certain individuals may be "captive" to particular hours

sets. Another example would be the involuntary unemployed being captive to zero hours of work. Again, the Logit model is not equipped to handle such captivity once more potentially causing erroneous inference.

In this report we use a new discrete choice model – the DOGEV model (Fry and Harris (2002)). The DOGEV model has the advantage over the Logit model in these two respects, as it accounts for both correlation between neighbouring outcomes and for captivity of individuals to particular outcomes. Use of the DOGEV specification will therefore reduce the chance of biased parameter estimates due to misspecification of the functional form of the discrete choice model.

The plan of the rest of this report is as follows. In section 2 we discuss the DOGEV model and a number of other specifications that it embodies in the context of the labor supply model. Section 3 presents the results of the estimation of the models. Section 4 discusses an illustrative policy simulation using traditional MITTS and alternative specifications. Finally section 5 presents some conclusions.

2. The Models

2.1 Random Utility and The Logit Model

In this section we describe the modeling framework used in the labor supply models. We start by summarizing the Logit approach to the problem. It is assumed that an individual, *i*, derives "utility", V_{ij}, from each hours outcome, $j = H^j$, over the range of discretized hours outcomes, $j = H^1, \ldots, H^J = 1, \ldots, J$. Further it is assumed that individuals choose the outcome that yields the maximum value of the set of utilities.

The utility that an individual derives from an hours outcome comprises two components: a deterministic component, U_{ij}, and a random component, ε_{ij}. This is given by:

$$V_{ij} = U_{ij} + \varepsilon_{ij}$$

In traditional MITTS, U_{ij} is given by:

$$U_{ij} = U(T - H^j, Y_{H^j}; X)$$

where the quadratic utility function is used:

$$U(Y,H) = \alpha_{YY} Y^2 + \alpha_{HH} H^2 + \alpha_{YH}(Y \times H) + \beta_Y Y + \beta_H H$$

$$\beta_Y = \beta_{y0} + \beta_y^{'} X$$
$$\beta_H = \beta_{h0} + \beta_h^{'} X$$

with the assumption that the random components in the model are independent and identically distributed as Type 1 Extreme Value the Logit form for the probability that individual *i* will choose outcome *j* is obtained as:

$$P_{ij} = \frac{\exp(U_{ij})}{\sum_{k=1}^{J} \exp(U_{ik})}.$$

We mentioned above two possible shortcomings with the use of the Logit formulation – captivity and correlation. It is possible to generalize the modeling approach to allow for these. We begin by considering the idea of captivity. An individual may be captive to a particular hours outcome, such as zero (involuntary unemployment), 20 hours (part-time employment), 40 hours (full-time employment) for a range of institutional or other reasons. In this situation the individual is not necessarily choosing the outcome that maximizes their utility.

2.2 Choice Set Generation and The DOGIT Model

Manski (1977) introduced a generalization of the random utility maximization model that allows for such a phenomenon – choice set generation. In this approach it is assumed that the set of choice outcomes faced by an individual is itself generated according to a probabilistic model. The individual then exercises their choice of outcome conditional upon the choice set that they face. In general we have:

$$P_{ij} = \sum_{C \subseteq B_i} (P_i(C) \times P_i(j \mid C))$$

That is the probability that probability that individual *i* will choose outcome *j* is given by the sum of over all choice sets available to the individual of the probability of the

individual facing that choice set, $P_i(C)$, times the probability that the individual chooses the outcome given the choice set, $P_i(j|C)$. The model is made operational by the choice of the choice set generating process and the conditional selection probabilities.

In our labor supply application there are eleven hours points: 0-2.5, 2.5-7.5, 7.5-12.5, 12.5-17.5, 17.5-22.5, 22.5-27.5, 27.5-32.5, 32.5-37.5, 37.5-42.5, 42.5-47.5, 47.5+ hours. Hereafter we denote these as: $H^1 = 0, H^2 = 5, \ldots, H^{10} = 45, H^{11} = 50$. In terms of the choice set generation process, we assume that an individual may be captive to any one of these hours outcomes or choose freely from the full set of eleven hours outcomes. Thus there are twelve possible choice sets that an individual may face – eleven "singleton" sets and the full choice set comprising of all eleven choice outcomes. We then define the probabilistic choice set generating process over the twelve choice sets faced by individuals: $C_1 = \{H^1\}, C_2 = \{H^2\}, \ldots, C_{11} = \{H^{11}\}, C_{12} = \{H^1, \ldots, H^{11}\}$. This process is given by:

$$P_i(C_s) = \frac{\theta_s}{1 + \sum_{k=1}^{11} \theta_k}, s = 1, \ldots, 11$$

$$P_i(C_{12}) = \frac{1}{1 + \sum_{k=1}^{11} \theta_k}$$

with $\theta_j \geq 0, j = 1, \ldots, 11$.

Now if an individual is captive to an outcome, say 40 hours $C_9 = \{H^9\}$, then conditional upon being faced with the singleton captive choice set the probability that they will choose the outcome is equal to one. However, if the individual is faced with the full choice set, $C_{12} = \{H^1, \ldots, H^{11}\}$, then we assume that the conditional probability of choosing an outcome is consistent with a random utility maximization model. Under these assumptions we have:

$$P_{ij} = \left(\frac{\theta_j}{1 + \sum_{k=1}^{11} \theta_k} \times 1 \right) + \left(\frac{1}{1 + \sum_{k=1}^{11} \theta_k} \times P_i(j \mid C_{12} = \{H^1, \mathrm{K}, H^{11}\}) \right)$$

All that remains is to specify $P_i(j \mid C_{12} = \{H^1, \mathrm{K}, H^{11}\})$ the conditional probability of an individual choosing a particular outcome. One such choice, consistent with a random utility maximization model, is the Logit formulation used in traditional MITTS. In this case:

$$P_i(j \mid C_{12} = \{H^1, \mathrm{K}, H^J\}) = \frac{\exp(U_{ij})}{\sum_{k=1}^{11} \exp(U_{ik})}$$

which, after simplification, yields:

$$P_{ij} = \frac{\exp(U_{ij}) + \theta_j \sum_{k=1}^{11} \exp(U_{ik})}{(1 + \sum_{k=1}^{11} \theta_k) \sum_{k=1}^{11} \exp(U_{ik})}$$

The resultant model here is identical to the DOGIT model of Gaudry and Dagenais (1979).

2.3 The OGEV Model

The choice set generation process has enabled us to add into our model the idea of captivity for institutional or other reasons. What it has not enabled us to do is capture the idea that adjacent hours outcomes might be related – or correlated. To deal with this possibility we will use the Ordered Generalized Extreme Value (OGEV) model of Small (1987). The underlying motivation for the OGEV model is to provide a suitable model for outcomes that are ordered and potentially related whilst retaining consistency with random utility maximization and providing the computational flexibility of the Logit model. Thus, unlike the Logit or DOGIT probabilities, the OGEV ones embody a correlation between outcomes in close proximity.

In this research we restrict our attention to what Small (1987) terms the standard OGEV model. The standard OGEV model implies a correlation between outcomes that are near neighbors. Analogously to a moving average process, this correlation decreases the further

away two outcomes j and s are and is zero when $|j-s|>2$. Although they cannot be written explicitly in closed form the correlations are inversely related to the parameter ρ. The standard OGEV probabilities are given by:

$$P_{ij} = \left(\frac{\exp\left(\rho^{-1}U_{ij}\right)}{\sum_{k=r}^{J+1}\left[\exp\left(\rho^{-1}U_{ir-1}\right)+\exp\left(\rho^{-1}U_{ir}\right)\right]^{\rho}} \right) \times$$
$$\left(\left[\exp\left(\rho^{-1}U_{ij-1}\right)+\exp\left(\rho^{-1}U_{ij}\right)\right]^{\rho-1} + \left[\exp\left(\rho^{-1}U_{ij}\right)+\exp\left(\rho^{-1}U_{ij+1}\right)\right]^{\rho-1} \right)$$

with the convention that $\exp(\rho^{-1}U_{i0}) = \exp(\rho^{-1}U_{iJ+1}) = 0$ and $0 < \rho \leq 1$. As $\rho \to 1$ the OGEV probabilities converge to the Logit ones. This gives a simple parameter restriction test (ρ = *1*) of the OGEV versus the Logit model. Such a test is also a test of zero correlation.

2.4 The DOGEV Model

Unfortunately, the OGEV model does not allow for the phenomenon of captivity and the DOGIT model does not allow either for the ordering of outcomes or for the (potential) correlation of proximate outcomes. Thus we now combine the ideas underlying both the DOGIT and OGEV models to produce the Dogit Ordered Generalized Extreme Value (DOGEV) model (see Fry and Harris (2002) for an extensive discussion of this model). The idea is to combine the two-part choice generating process of Manski (1977) that led to the DOGIT model with the proximate correlation and ordering of the OGEV model.

Specifically, we assume that the choice set generation process leading to captivity is that that led to the DOGIT. However when free choice is exercised from the full choice set the (conditional) selection probabilities are given by the OGEV formulation:

$$P_i(j \mid C^{12} = \{H^1, \mathrm{K}, H^{11}\}) = \left(\frac{\exp\left(\rho^{-1}U_{ij}\right)}{\sum_{k=r}^{J+1}\left[\exp\left(\rho^{-1}U_{ir-1}\right)+\exp\left(\rho^{-1}U_{ir}\right)\right]^{\rho}} \right) \times$$
$$\left(\left[\exp\left(\rho^{-1}U_{ij-1}\right)+\exp\left(\rho^{-1}U_{ij}\right)\right]^{\rho-1} + \left[\exp\left(\rho^{-1}U_{ij}\right)+\exp\left(\rho^{-1}U_{ij+1}\right)\right]^{\rho-1} \right)$$

Combined with the choice set generation process the DOGEV specification is given by:

$$P_{ij} = \left(\frac{\theta_j}{1 + \sum_{k=1}^{11} \theta_k} \times 1 \right) + \left(\frac{1}{1 + \sum_{k=1}^{11} \theta_k} \times P_i(j \mid C_{12} = \{H^1, \mathrm{K}, H^{11}\}) \right).$$

The DOGEV model is attractive in that it allows both for captivity to particular outcomes and correlation amongst adjacent outcomes. It also embodies, or nests, a number of (sub)models – DOGIT (where captivity but not correlation is present), OGEV (where correlation but not captivity is present) and Logit (where neither captivity nor correlation are present). The parameters of the DOGEV model and its sub-models can be consistently estimated using the maximum likelihood criterion. Moreover, the DOGEV model allows for simple hypothesis tests of appropriate parameter restrictions to guide model selection. The parameter values (restrictions) leading to each of the models are given in Table 1:

Table 1: DOGEV and Parameter Values

DOGEV:	At least one $\theta_j > 0,\ j = 1, \mathrm{K}, 11;\ 0 < \rho < 1$
DOGIT:	At least one $\theta_j > 0,\ j = 1, \mathrm{K}, 11; \rho = 1$
OGEV:	$\theta_1 = \mathrm{K} = \theta_{11} = 0;\ 0 < \rho < 1$
LOGIT:	$\theta_1 = \mathrm{K} = \theta_{11} = 0;\ \rho = 1$

3. Estimation Results

3.1 Model Selection

We now turn to the estimation of the DOGEV model and its sub-models for the labor supply decision for three demographic groups in Australia – Single Men, Single Women and Sole Parents[1]. In common with other estimations for traditional MITTS we use the Survey of Income and Housing Costs 1994-95, 1995-96, 1996-97 and 1997-98 as released by the Australian Bureau of Statistics. Consistent with Kalb (2002) we utilize the quadratic form of the utility function in all our probability specifications (DOGEV, DOGIT, OGEV and LOGIT). Thus, for the Logit specification the results will be identical to those in traditional MITTS as documented in Kalb (2002).

[1] The DOGEV, DOGIT and OGEV models are not applicable to two adult households as in traditional MITTS there are 66 (=6 × 11) paired hours outcomes that do not exhibit the ordering required for (D)OGEV and that would require an additional 66 θ parameters to capture potential captivity. Thus this demographic group is not considered in this research.

Our initial emphasis is on the hypothesis based model selection based upon the parameter restrictions contained in Table 1. The restrictions can be tested using likelihood ratio tests. The results of these tests are presented in Table 2 below:

Table 2: Likelihood Ratio Test Results

	Single Men		Single Women		Sole Parents	
DOGEV v. OGEV	2391.2[b]	*5 d.f*	*		268.9	*11 d.f.*
DOGEV v. DOGIT	0[a]		*		0[a]	
DOGEV v. LOGIT	2391.2[a]	*6 d.f*	*		4820.7[a]	*12 d.f.*
DOGIT v. LOGIT	2391.2	*5 d.f*	1264.5	*11 d.f.*	4820.7[a]	*11 d.f.*
OGEV v. LOGIT	0[a]		0[a]		4551.9	*1 d.f.*

[a] In the (D)OGEV estimations the parameter ρ went to the boundary value of one yielding identical maximized log-likelihood values for the (D)OGEV model and the associated sub-model.

[b] Only 5 θ parameters could be estimated for single men as several hours outcomes had very few observations.

* The DOGEV model estimations suffered large numerical problems. Five θ values could be estimated for DOGEV but 11 for DOGIT making the models non-nested and testing inappropriate.

The clear evidence from this hypothesis testing is that for all three demographic groups that the DOGIT model is the preferred specification. Thus, there is clear evidence of captivity in labor supply but not of correlation[2]. Moreover, the statistical evidence is that the DOGIT model is more consistent with the data than the traditional MITTS Logit specification.

3.2 Estimation Results

We now turn to a discussion of the DOGIT estimation results as contained in Table 3. For comparative purposes the traditional MITTS Logit estimation results are given in Table 4.

[2] Whilst this may appear surprising it is possible that this result may reflect the choice of the number of labor supply hours outcomes used in the discretization of the models.

Table 3: DOGIT Estimation Results

	Single Men		***Single Women***		***Sole Parents***	
	Estimated coefficient	t-ratio	Estimated coefficient	t-ratio	Estimated coefficient	t-ratio
Quadratic terms						
Income × 100,000	-0.0304	-0.64	0.1718	1.41	-10.9878	-6.19
Labor supply × 100	0.0083	0.25	-0.5379	-3.36	-0.3089	-4.10
Cross product						
Inc. & lab.sup. × 10,000	-0.2997	-2.07	-1.9619	-4.30	1.9258	1.43
Linear terms						
Income × 100						
Constant	-0.0841	-0.84	1.0011	4.16	14.8402	3.64
Children 0-2 yrs old					0.5600	0.62
Children 3-4 yrs old					0.1841	0.22
Children 5-9 yrs old					1.0685	1.16
Number of children					1.5617	4.67
Age/10	0.2997	3.71	0.2193	2.02	-1.5329	-0.97
Age squared/100	-0.0332	-3.32	-0.0210	-1.49	0.0782	0.43
Vocational education	0.0449	1.53	-0.0171	-0.35	-0.4776	-0.99
Diploma	-0.0518	-1.32	0.0468	0.67	1.1498	2.40
Degree	-0.0356	-1.00	0.1278	1.49		
Female					0.3417	0.43
Labor supply						
Constant	-0.1194	-5.94	0.0604	0.65	-0.3222	-2.27
Children 0-2 yrs old					-0.0774	-1.94
Children 3-4 yrs old					-0.0572	-1.52
Children 5-9 yrs old					-0.0670	-1.30
Number of children					-0.0329	-2.76
Age/10	0.0799	7.47	0.1520	9.10	0.1624	3.63
Age squared/100	-0.0109	-7.27	-0.0208	-9.04	-0.0186	-3.44
Vocational education	0.0183	4.58	0.0080	1.31	0.0229	1.89
Diploma	0.0180	3.27	0.0403	5.04	-0.0331	-1.78
Degree	0.0258	5.38	0.0725	7.32		
Female					-0.0969	-2.59
Fixed costs/100						
Constant	8.3764	5.11	6.7550	3.67	1.6955	7.24
Live in capital city	-0.1720	-0.76	-0.5114	-2.21	0.0036	0.13
Children 0-4 yrs old					-0.1745	-1.57
Children 5-9 yrs old					-0.1732	-1.25
Live in NSW	-0.1512	-0.57	0.0208	0.10	0.0816	2.27
Female					-0.3930	-2.32
Captivity Parameters						
Captivity at hours 0	-	-	0.0895	3.89	0.0141	1.78
Captivity at hours 5	-	-	0.0172	6.62	0.0384	6.74
Captivity at hours 10	-	-	0.0249	6.92	0.0370	4.93
Captivity at hours 15	-	-	0.0211	4.69	0.0157	2.49
Captivity at hours 20	-	-	0.0232	3.52	0.0127	2.89
Captivity at hours 25	-	-	0.0056	1.08	0.0094	2.47
Captivity at hours 30	0.0030	0.60	0.0000	0.00	0.0088	2.10
Captivity at hours 35	0.0727	6.38	0.0234	1.77	0.0139	2.01
Captivity at hours 40	0.6573	12.43	0.3487	13.01	0.0965	7.10

Captivity at hours 45	0.0288	1.54	0.0000	0.00	0.0095	2.50
Captivity at hours 50	0.0960	2.49	0.0273	2.60	0.0384	5.65
% of observations with						
U quasi-concave	18.4		100		92.3	
U increases in Y	100		100		92.3	
IC convex	18.4		100		92.3	

Table 4: Logit Estimation Results

	Single Men		***Single Women***		***Sole Parents***	
	Estimated coefficient	t-ratio	Estimated coefficient	t-ratio	Estimated coefficient	t-ratio
Quadratic terms						
Income × 100,000	-0.0132	-0.57	-0.1421	-2.03	-0.5858	-2.54
Labor supply × 100	-0.4313	-12.79	-0.2711	-11.82	-0.0199	-0.51
Cross product						
Inc. & lab.sup. × 10,000	-0.4969	-3.74	-1.5207	-8.65	-1.4297	-2.44
Linear terms						
Income × 100						
Constant	0.2214	2.85	0.7474	5.10	2.9512	2.18
Children 0-2 yrs old					0.4762	1.33
Children 3-4 yrs old					-0.1161	-0.40
Children 5-9 yrs old					0.8649	2.72
Number of children					0.1135	1.28
Age/10	0.1412	4.40	0.1013	1.42	-0.2572	-0.40
Age squared/100	-0.0154	-3.76	-0.0028	-0.31	0.0127	0.17
Vocational education	0.0209	1.82	-0.0166	-0.50	-0.0452	-0.30
Diploma	-0.0005	-0.03	0.0289	0.60	-0.0165	-0.11
Degree	0.0118	0.67	0.1162	2.40		
Female					0.0300	0.12
Labor supply						
Constant	0.1502	4.55	0.0110	0.63	-0.1473	-4.08
Children 0-2 yrs old					-0.0335	-2.05
Children 3-4 yrs old					-0.0214	-1.57
Children 5-9 yrs old					-0.0534	-3.41
Number of children					-0.0020	-0.63
Age/10	0.0794	8.32	0.0914	12.88	0.0909	6.12
Age squared/100	-0.0103	-7.87	-0.0127	-13.63	-0.0111	-5.85
Vocational education	0.017	4.67	0.0043	1.45	0.0169	3.89
Diploma	0.0141	2.28	0.0207	5.33	0.0242	4.11
Degree	0.0234	4.37	0.0308	8.06		
Female					-0.0486	-4.10
Fixed costs/100						
Constant	17.3972	6.74	5.2641	9.44	2.3595	5.95
Live in capital city	-0.3486	-1.65	-0.1248	-0.90	0.0563	1.10
Children 0-4 yrs old					-0.2301	-0.91
Children 5-9 yrs old					-0.6367	-2.62
Live in NSW	-0.3047	-1.24	0.0476	0.35	0.2290	3.38
Female					-0.4903	-1.88

We first consider the parameters of the utility function. In the squared terms of hours and income we see quite large changes between the two specifications. In moving to the DOGIT the quadratic term in income for Single Women and for Labor Supply for Single men both change sign and are no longer statistically significant. For Sole Parents the quadratic terms for income and for labor supply both dramatically increase both in size and significance. The interaction term remains largely unchanged for Single Men and Single Women. However, for Sole Parents the interaction term both changes sign and is no longer statistically significant.

The control variables chosen to account for variation in tastes for work these were dummies for the age of the youngest child (0-2, 3-4 and 5-9); total number of children; age and age squared; and dummies for highest education attainment (vocational and diploma or higher). Those to account for variations in fixed costs were: a dummy representing residence in a capital city; number of pre-school aged children; number of school-aged children; and a dummy variable for residence in New South Wales[3]. Looking at the results in both specifications none of them appear to adequately capture the linear preference for income term particularly well, with the exception of the total number of children for Sole Parents. However, greater significance levels are achieved for the linear preference for the hours term. Moreover, with the exception of the constant terms, these coefficients are similar with regard to magnitudes, signs and significance levels across the Logit and DOGIT models. Finally, the same can ostensibly be said for the variables in the fixed costs equation.

We now consider the additional captivity parameters in the DOGIT specification, the θ's[4]. With the exception of those for 30 and 45 hours for Single Women none of the captivity parameters have gone to the lower boundary of zero. Overall, these parameters are all typically statistically significant – recalling that these are one-sided tests. The largest effect appears, not surprisingly, to be afforded by full-time hours of work. The next largest effects are those corresponding to zero hours and to primarily part-time casual employment hours of (in order) 10 and 5. Those for both very high hours of work (50) and

[3] A dummy variable for gender is also used in the case of Sole Parents.

[4] In the case of Single Men there were too few observations at hours outcomes 0, 5, 10, 15,20, and25 to reliably estimate the captivity parameters for these outcomes. Hence, *a priori*, they were constrained to zero.

the more usual part-time hours levels (of 15 and 20) also appear to be significant, but of a smaller magnitude.

We also note that the DOGIT model estimates for Single Women and for Sole Parents do not unduly affect the underlying preference structure as evidenced by the percentage of observations exhibiting quasi-concavity, increasing utility with income and convex indifference curves. Unfortunately, this is not the case for Single Men. However, for Single Men the estimated probability of "free choice" according to the utility model is just 0.538[5]. Thus, about half of the sample for this demographic group is estimated to be captive to a particular outcome and, as a result, not choosing according to a utility specification.

The quite large changes in the quadratic terms of the utility functions and the lack of quasi-concavity suggest that the addition of the captivity component is having some impact on the underlying preference structure. We, therefore, re-estimated the DOGIT model restricting the utility parameters to the values obtained in the Logit estimations. These restrictions were strongly rejected on statistical grounds. Thus, we conclude that the DOGIT model is consistent with the data and should be used in simulations.

3.3 Goodness of Fit

Due to the complexity of the above models, it is not clear how well they describe the data. In Tables 5a – 5c the sample proportions are presented along with the percentage predicted probabilities for each hours outcome – derived from so-called hit-miss tables. For both models two sets are presented; one from the usual hit-miss table (Traditional) and the other where the stochastic elements of the model are explicitly taken into account (over 1,000 random draws; Simulated). As the number of random draws increases both sets of probabilities tend towards the sample proportions.

[5] For Single Women it is 0.367 and for Sole Parents it is 0.227.

Table 5a: Single Men

		Traditional		Simulated	
Outcome	Actual	Logit	Dogit	Logit	Dogit
0 – 2.5	0.2021	0.7233	0.0503	0.2019	0.2022
2.5 – 7.5	0.0048	0.0000	0.0000	0.0003	0.0059
7.5 – 12.5	0.0086	0.0000	0.0000	0.0012	0.0073
12.5 – 17.5	0.0085	0.0000	0.0000	0.0044	0.0089
17.5 – 22.5	0.0130	0.0000	0.0000	0.0135	0.0116
22.5 – 27.5	0.0148	0.0000	0.0000	0.0349	0.0162
27.5 – 32.5	0.0235	0.0000	0.0000	0.0751	0.0250
32.5 – 37.5	0.0746	0.0000	0.0000	0.1293	0.0731
37.5 – 42.5	0.4031	0.0000	0.9497	0.1775	0.4035
42.5 – 47.5	0.0876	0.0000	0.0000	0.1938	0.0882
> 47.5	0.1594	0.2767	0.0000	0.1681	0.1580

Table 5b: Single Women

		Traditional		Simulated	
Outcome	Actual	Logit	Dogit	Logit	Dogit
0 – 2.5	0.2581	0.5044	0.2530	0.2509	0.2406
2.5 – 7.5	0.0111	0.0000	0.0000	0.0037	0.0110
7.5 – 12.5	0.0170	0.0000	0.0000	0.0062	0.0167
12.5 – 17.5	0.0172	0.0000	0.0000	0.0209	0.0170
17.5 – 22.5	0.0272	0.0000	0.0000	0.0357	0.0260
22.5 – 27.5	0.0305	0.0000	0.0000	0.0552	0.0328
27.5 – 32.5	0.0446	0.0000	0.0000	0.1075	0.0599
32.5 – 37.5	0.1136	0.0516	0.0000	0.1333	0.1075
37.5 – 42.5	0.3375	0.2966	0.7470	0.1480	0.3266
42.5 – 47.5	0.0670	0.1460	0.0000	0.1351	0.0890
> 47.5	0.0764	0.0015	0.0000	0.1031	0.0728

Table 5c: Sole Parents

		Traditional		Simulated	
Outcome	Actual	Logit	Dogit	Logit	Dogit
0 – 2.5	0.5329	0.7640	0.7640	0.5330	0.4925
2.5 – 7.5	0.0313	0.0011	0.0011	0.208	0.0315
7.5 – 12.5	0.0368	0.0011	0.0011	0.0332	0.0360
12.5 – 17.5	0.0329	0.0022	0.0022	0.0405	0.0316
17.5 – 22.5	0.0384	0.0071	0.0066	0.0155	0.0425
22.5 – 27.5	0.0296	0.0368	0.0302	0.0477	0.0496
27.5 – 32.5	0.0335	0.0401	0.0357	0.0487	0.0502
32.5 – 37.5	0.0538	0.0269	0.0176	0.0511	0.0507
37.5 – 42.5	0.1306	0.0379	0.0757	0.0553	0.1127
42.5 – 47.5	0.0296	0.0423	0.0148	0.0601	0.0429
> 47.5	0.0505	0.0406	0.0510	0.0640	0.0601

In both models in the Traditional hit-miss setting, both models heavily over-predict the most heavily observed outcomes (either forty or zero hours of work) at the expense of all other hours outcomes. Because of this, focus is on the simulated approach. Here, predicted probabilities closely replicate sample proportions, although now the DOGIT specification actually under predicts the zero hours outcome. Where the DOGIT model clearly surpasses the Logit specification, is in predicting the traditional full-time working week state of ($37.5<H\leq 42.5$) – presumably as a result of the strong captivity parameters for all demographic groups corresponding to this hours outcome. Moreover, there are strong a priori (institutional) reasons as to why individuals are captive to this hours level, in addition to preferences for this hours outcome as determined by personal characteristics. That is, there are presumably relatively few employment opportunities at the margin of the usual full-time hours ones.

4. Simulations

4.1 Introduction

We have seen that the DOGIT specification appears to be a better representation of the data generating process than the Logit specification utilized in traditional MITTS. Thus, there is evidence of captivity in the labor supply decision. We now turn to an investigation of whether using the DOGIT specification of the labor supply decision will yield different results in a behavioral microsimulation than those obtained from traditional MITTS. The policy change used in this evaluation is that of reducing all taper rates from 50 and 70 per cent to 30 per cent. The data that we use are from the Income Distribution Survey 1997/98 to ensure consistency with the base tax system of the observations.

4.2 The Simulation Process

The estimated models of preferences can be used to simulate the potential impact of tax and/or welfare policy reform on participation and hours choices. The estimated preference function, varying by observed characteristics, is brought together with the level of net incomes to be enjoyed at each possible hours outcome. This yields an optimal employment choice under some benchmark tax system[6]. Next, a new tax system following

[6] There is nothing that ensures that the hours state corresponding to the maximum predicted utility corresponds to the observed hours state.

some policy reform is instigated. This alters the budget constraint and therefore, potentially, also the optimal choice of hours post-policy reform. By comparing the simulated employment optima for a large and representative sample of individuals it is possible to build a pattern of employment transitions that indicate both the direction and degree of behavioral response to the tax reform. Microsimulation methods of this sort are necessarily supply-side in nature, as including potential demand-side and price impacts of labor supply responses, would be computationally infeasible.

We begin by describing how this is carried out in traditional MITTS. The approach followed uses the probabilistic form of the discrete model – the Logit specification with quadratic utility function. Specifically, the estimated model is used to predict the probability of choosing each hours outcome under the base tax system[7], and under a range of alternative policy scenarios. Re-sampling methods are then used to generate estimates of the probability of transition from one labor market outcome to another following the policy reform.

Replicating, or repeating, for all individuals the following process builds up the transition probabilities over a large number of replications[8]:

1. Under the base system we calculate the value of $U_{ij} = U(T - H^j, Y_{H^j}; X)$ using the net-incomes for each hours outcome generated by MITTS and the estimated utility parameters from the Logit model. To these we add a set of J=11 random draws from independent, identically distributed Type 1 Extreme Value distributions. This yields a set of eleven V_{ij} 's. We then find the maximum value of $V_{i1}, \ldots, V_{i11}$ and this yields our predicted hours outcome. We check whether the predicted outcome is that observed.
    a) If so, we store the vector of 11 draws and proceed.
    b) If not we draw a new set of J=11 random draws from independent, identically distributed Type 1 Extreme Value distributions and repeat the process. If we match the predicted and observed hours outcome we store

[7] The March 1998 tax system.

[8] In MITTS 100 replications are typically used.

the vector of 11 draws and proceed. If not we continue drawing until either a match is found or 1,000 draws have occurred. Should 1,000 draws occur with no match then we "fix" the individual for this replication at their observed hours outcome.

2. In stage 1 all individuals are either (a) matched to their observed hours outcome with an associated set of random draws or (b) fixed to their observed outcome .

   a) We now implement the reform. This will change the net-incomes for each of the eleven hours outcomes that are generated by MITTS. We re-calculate $U_{ij} = U(T - H^j, Y_{H^j}; X)$ using these <u>new</u> net-incomes for each hours outcome generated by MITTS and the estimated utility parameters from the Logit model. For outcomes matched in the base we add the associated set of random draws from the base regime to the new U_{ij}'s to form a new set of V_{ij}'s. We then predict the hours outcome that maximizes the new set of $V_{i1}, \ldots, V_{i11}$.

   b) For those fixed to their observed hours outcome in the base system we assume that they do not change their hours outcome under the new system. Thus there is no transition. We label these pseudo-captives.

After the process above has been replicated a large number of times we average the re-sampled transition frequencies to arrive at our simulated transitions probabilities the sample. By aggregating individual transitions probabilities, it is possible to simulate the overall labor supply response to a policy reform.

The idea underlying the simulations in the Dogit model is the same. However, the exact process is more involved as in each replication we need to consider both the choice set generation (or captivity) process and the utility based free choice process – the latter being analogous to traditional MITTS but using the utility parameters estimated in the DOGIT specification. Thus the DOGIT based simulations begin by dealing with the captive versus free choice categorization. This is achieved in the following manner[9]:

[9] It is possible to utilize a random utility type model for this stage as done in Fry and Harris (1996) but our approach is equivalent to theirs and faster to implement.

1. Using the estimated θ parameters we compute the captivity probabilities for individual i $P_i(C_s) = \dfrac{\theta_s}{1+\sum_{k=1}^{11}\theta_k}, s = 1,\text{K},11$. These are then used to determine the number of captive replications $n_{ij} = \dfrac{\theta_j}{1+\sum_{k=1}^{11}\theta_k} \times 100, j = 1,\text{K},11$.

   a) For $100 - n_{ij}$ replications we conduct a traditional MITTS style utility-based simulation (but using the utility parameters estimated in the DOGIT specification).

   b) Additionally, there are n_{ij} replications for each outcome that are categorized as captive and do not change their hours outcome under the new system. Thus there is no transition.

4.3 Simulation Results

4.3.1 Captivity

An important component of the DOGIT formulation is that of captivity. However, we saw above that the use of traditional MITTS in policy simulations produces what we termed pseudo-captives. We begin our discussion of the simulation results by considering the average number of captives or pseudo-captives in a typical replication. These are given in Tables 6a and 6b:

Table 6a: Captivity in DOGIT

Outcome	Single Men	Single Women	Sole Parents
0	126.33	57.1	0
5	8.88	10.82	0.21
10	17.31	11.02	0.94
15	12.56	8.65	0
20	11.92	3.36	0
25	0	0	0
30	0	0	0
35	16.6	17.52	0
40	407.32	221.02	2.16
45	0	0	0.14
50	102.55	3.8	0.94

We see that the DOGIT does have reasonable levels of captivity – especially to the full-time hours outcome of 40. What is surprising is that traditional MITTS also produces a number of pseudo-captives at low hours.

Table 6a: Pseudo-Captivity in Traditional MITTS

Outcome	Single Men	Single Women	Sole Parents
0	0	0	0
5	5.95	2.22	0
10	3.20	0.56	0
15	0	0	0
20	0	0	0
25	0	0	0
30	0	0	0
35	0	0	0
40	0	0	0
45	0	0	0
50	0	0	0

4.3.2 Policy Results

We now consider the results of the simulation of the policy change of reducing all taper rates from 50 and 70 per cent to 30 per cent. The summary of the simulation results from the two specifications is given in Tables 7a and 7b:

Table 7a: Dogit Simulations

Behavioral Response	Single Men	Single Women	Sole Parents
Workers(% base)	59.48	44.81	42.27
Workers(% reform)	59.75	45.42	49.56
Non-work --> work (%)	0.30	0.66	7.45
Work --> non-work (%)	0.03	0.05	0.17
Workers working more	0.00	0.04	1.50
Workers working less	0.53	1.52	2.12
Average hours change	0.01	-0.06	2.25

Table 7b: Traditional MITTS (Logit) Simulations

Behavioral Response	Single Men	Single Women	Sole Parents
Workers(% base)	59.48	44.81	42.27
Workers(% reform)	59.97	45.58	49.51
Non-work --> work (%)	0.54	0.86	7.45
Work --> non-work (%)	0.05	0.09	0.22
Workers working more	0.02	0.07	1.72
Workers working less	1.21	2.07	2.66
Average hours change	-0.01	-0.19	2.20

It is clear that there are clear differences between the two specifications. To better understand these changes we present the detailed results of the simulations in Tables 8a – 8f. These clearly show the role that captivity plays in the DOGIT simulations. Typically the diagonal entries in the DOGIT results are higher than in the Logit indicating that less moves away from base hours occur. Moreover, the captivity component in DOGIT tends to act as a "gravitational" pull towards certain hours outcomes in the transitions.

Table 8a: Single Men's Labor Supply Transitions – DOGIT

Labour Supply Transitions (row percentages)												
From pre to post reform: rows to columns												
	0	5	10	15	20	25	30	35	40	45	50	Pre reform
0	99.3	0	0	0	0	0.1	0.2	0.2	0.1	0.1	0	40.5
5	-	100	-	-	-	-	-	-	-	-	-	0.7
10	-	-	100	-	-	-	-	-	-	-	-	1.8
15	-	-	-	100	-	-	-	-	-	-	-	1.1
20	0	-	-	-	99.9	-	0.1	-	-	-	-	1.2
25	-	-	-	-	0.1	99.9	-	-	-	-	-	1.2
30	-	-	0	-	0.2	0	99.7	0	-	-	-	1.8
35	0.1	-	-	0	0.1	0.2	0.3	99.3	-	-	-	5.5
40	0	-	0	0	0.1	0.2	0.1	0.1	99.4	0	-	27.4
45	0.1	-	0.1	0.1	0.3	0.5	0.8	0.7	0.7	96.7	-	6.3
50	0.1	-	0	0	0.1	0.2	0.2	0.3	0.2	0.1	98.9	12.4
Post reform	40.2	0.7	1.8	1.1	1.3	1.4	2	5.7	27.4	6.2	12.3	100

Table 8b: Single Men's Labor Supply Transitions – Traditional MITTS

Labour Supply Transitions (row percentages)												
From pre to post reform: rows to columns												
	0	5	10	15	20	25	30	35	40	45	50	Pre reform
0	98.7	0.0	0.0	0.0	0.1	0.2	0.3	0.3	0.2	0.1	0.0	40.5
5	-	100.0	-	-	-	-	-	-	-	-	-	0.7
10	-	-	100.0	-	-	-	-	-	-	-	-	1.8
15	-	-	-	98.4	-	1.6	-	-	-	-	-	1.1
20	0.1	-	-	-	99.9	-	0.1	-	-	-	-	1.2
25	-	-	-	-	0.1	99.9	-	-	-	-	-	1.2
30	-	-	0.0	-	0.2	0.0	99.7	0.0	-	-	-	1.8
35	0.1	-	-	0.1	0.1	0.2	0.4	99.2	-	-	-	5.5
40	0.1	-	0.0	0.1	0.3	0.8	0.8	0.6	97.3	0.0	0.0	27.4
45	0.1	-	0.1	0.1	0.3	0.5	0.8	0.7	0.7	96.7	-	6.3
50	0.1	-	0.0	0.0	0.2	0.4	0.4	0.6	0.3	0.1	97.9	12.4
Post reform	40.0	0.7	1.8	1.1	1.4	1.6	2.2	5.9	26.9	6.2	12.2	100.0

Table 8c: Single Women's Labor Supply Transitions – DOGIT

Labour Supply Transitions (row percentages)												
From pre to post reform: rows to columns												
	0	5	10	15	20	25	30	35	40	45	50	Pre reform
0	98.8	0.0	0.0	0.1	0.2	0.3	0.2	0.2	0.1	0.1	0.0	55.2
5	-	97.1	-	-	0.5	-	-	2.4	-	-	-	1.1
10	-	-	99.9	-	-	-	0.1	-	-	-	-	1.2
15	-	-	-	99.9	0.1	-	0.1	-	-	-	-	2.0
20	-	-	0.2	-	99.8	0.0	-	-	-	-	-	1.2
25	-	-	-	0.0	0.2	99.8	-	-	-	-	-	1.3
30	0.4	-	0.0	0.1	0.4	0.8	98.3	-	-	-	-	2.6
35	0.4	0.0	0.1	0.4	0.9	1.3	0.7	96.2	-	-	-	6.7
40	0.1	0.0	0.1	0.3	0.6	1.2	0.6	0.3	97.0	-	-	19.4
45	0.1	0.0	0.1	0.5	1.6	3.2	1.9	1.5	1.2	90.0	-	3.9
50	0.1	-	0.1	0.2	0.9	1.2	1.2	1.1	0.5	0.2	94.4	5.4
Post reform	54.6	1.1	1.3	2.1	1.6	2.0	3.0	6.8	19.0	3.5	5.1	100.0

Table 8d: Single Women's Labor Supply Transitions – Traditional MITTS

Labour Supply Transitions (row percentages)												
From pre to post reform: rows to columns												
	0	5	10	15	20	25	30	35	40	45	50	Pre reform
0	98.4	0.0	0.0	0.1	0.2	0.4	0.3	0.2	0.2	0.1	0.0	55.2
5	-	94.6	-	-	0.6	-	-	3.7	1.1	-	-	1.1
10	-	-	99.8	-	-	0.1	0.1	-	-	-	-	1.2
15	0.0	-	-	99.7	0.1	-	0.2	-	-	-	-	2.0
20	-	-	0.2	-	99.8	0.0	-	-	-	-	-	1.2
25	-	-	-	0.0	0.2	99.8	-	-	-	-	-	1.3
30	0.4	-	0.0	0.1	0.4	0.8	98.3	-	-	-	-	2.6
35	0.5	0.0	0.2	0.4	0.9	1.6	0.8	95.6	-	-	-	6.7
40	0.2	0.0	0.2	0.7	1.6	3.4	1.9	1.0	91.0	-	-	19.4
45	0.1	0.0	0.1	0.5	1.6	3.2	1.9	1.5	1.2	90.0	-	3.9
50	0.1	-	0.1	0.2	0.9	1.3	1.3	1.2	0.5	0.2	94.2	5.4
Post reform	54.4	1.1	1.3	2.2	1.8	2.5	3.3	6.9	17.8	3.5	5.1	100.0

Table 8e: Sole Parent's Labor Supply Transitions – DOGIT

Labour Supply Transitions (row percentages)												
From pre to post reform: rows to columns												
	0	5	10	15	20	25	30	35	40	45	50	Pre reform
0	87.1	0.0	0.2	0.9	1.5	2.5	1.9	1.7	1.6	1.5	1.1	57.7
5	-	90.0	0.2	0.2	0.9	2.0	2.4	0.7	0.8	1.0	1.7	3.3
10	-	-	83.5	1.2	0.5	2.0	3.6	3.2	2.7	2.1	1.2	3.3
15	-	-	-	93.1	0.5	0.3	1.1	1.1	2.1	0.4	1.4	2.8
20	-	-	0.2	0.1	93.9	0.7	1.4	0.9	1.2	1.2	0.5	3.9
25	0.0	-	-	-	-	99.6	0.1	-	0.2	0.1	0.0	3.7
30	1.7	-	0.1	0.7	0.7	0.7	93.3	0.6	1.0	0.8	0.3	3.4
35	-	0.1	0.6	0.3	0.9	2.1	0.5	93.9	0.7	0.8	0.3	3.2
40	0.5	0.1	0.5	1.2	1.7	2.1	1.5	0.3	91.7	0.3	0.2	12.3
45	1.4	-	0.8	0.8	2.2	4.0	6.6	0.5	1.2	82.2	0.3	1.0
50	0.6	0.1	0.8	0.7	1.6	2.7	3.4	2.4	2.3	1.1	84.4	5.5
Post reform	50.4	3.0	3.0	3.4	4.9	5.8	4.9	4.4	12.6	2.0	5.5	100.0

Table 8f: Sole Parent's Labor Supply Transitions – Traditional MITTS

Labour Supply Transitions (row percentages)												
From pre to post reform: rows to columns												
	0	5	10	15	20	25	30	35	40	45	50	Pre reform
0	87.1	0.0	0.2	0.9	1.5	2.5	1.9	1.7	1.6	1.5	1.1	57.7
5	-	85.3	0.2	0.4	1.0	2.5	3.8	1.1	2.1	1.6	2.1	3.3
10	-	-	82.7	1.4	0.6	2.2	3.6	3.5	2.7	2.1	1.3	3.3
15	-	-	-	93.1	0.5	0.3	1.1	1.1	2.1	0.4	1.4	2.8
20	-	-	0.2	0.1	93.9	0.7	1.4	0.9	1.2	1.2	0.5	3.9
25	0.0	-	-	-	-	99.6	0.1	-	0.2	0.1	0.0	3.7
30	1.7	-	0.1	0.7	0.7	0.7	93.3	0.6	1.0	0.8	0.3	3.4
35	-	0.1	0.6	0.3	0.9	2.1	0.5	93.9	0.7	0.8	0.3	3.2
40	0.9	0.2	0.6	1.7	2.7	3.1	2.7	0.6	86.8	0.5	0.2	12.3
45	1.4	-	0.8	0.8	2.2	4.0	6.7	0.5	1.3	82.0	0.3	1.0
50	0.7	0.1	0.8	0.7	1.6	2.8	3.4	2.4	2.3	1.2	84.1	5.5
Post reform	50.5	2.9	3.0	3.5	5.0	5.9	5.1	4.5	12.0	2.0	5.5	100.0

5. Conclusions

This research is concerned with investigating whether there is any evidence of captivity to or correlation between the discrete hours outcomes in the labor supply model embodied in the Melbourne Institute Tax and Transfer Simulator (MITTS). To this end we use a new discrete choice model – the DOGEV model Unlike the Logit model embedded in MITTS the DOGEV model is attractive in that it allows both for captivity to particular outcomes and correlation

amongst adjacent outcomes. It also embodies, or nests, a number of (sub)models – DOGIT (where captivity but not correlation is present), OGEV (where correlation but not captivity is present) and Logit (where neither captivity nor correlation are present).

Estimating the DOGEV model using Australian data yields strong evidence that for all three demographic groups considered (Single Men, Single Women and Sole Parents) the DOGIT model is the preferred specification and a better representation of the data generating process than the Logit model used in MITTS. Thus, there is clear evidence of captivity in labor supply but not of correlation. We then compare the results of an illustrative policy simulation using the DOGIT specification with that in MITTS based upon the Logit specification. The policy change used in this evaluation is that of reducing all taper rates from 50 and 70 per cent to 30 per cent. In this simulation exercise we find that there are marked differences between the results from the two specifications – especially for Sole Parents. The results clearly show the role that captivity plays in the DOGIT simulations with the captivity component in DOGIT tending to act as a "gravitational" pull towards certain hours outcomes in the transitions.

References

Fry T.R.L. and M.N. Harris (1996) "A Monte Carlo Study of Tests for the Independence of Irrelevant Alternatives Property", *Transportation Research, Series B*, **30B**, 19-30.

Fry T.R.L. and M.N. Harris (2002) "The DOGEV Model", Department of Econometrics and Business Statistics Working Paper 7/2002, Monash University.

Gaudry, M. and M. Dagenais (1979) "The DOGIT Model", *Transportation Research, Series B*, **13B**, 105-112.

Kalb, G. (2002) "Estimation of Labour Supply Models for Four Separate Groups in the Australian Population", Report to The Department of Family and Community Services, March 2002.

Manski, C. (1977) "The Structure of Random Utility Models", *Theory and Decision*, **8**, 229-254.

Small, K. (1987) "A Discrete Choice Model for Ordered Alternatives", *Econometrica*, **55**, 409-424.