BIVARIATE INCOME DISTRIBUTIONS FOR ASSESSING INEQUALITY AND POVERTY UNDER DEPENDENT SAMPLES

ANDREA VINH
Department of Econometrics and Business Statistics
Monash University, AUSTRALIA

WILLIAM E. GRIFFITHS
Department of Economics
University of Melbourne, AUSTRALIA

DUANGKAMON CHOTIKAPANICH
Department of Econometrics and Business Statistics
Monash University, AUSTRALIA

ABSTRACT

As indicators of social welfare, the incidence of inequality and poverty is of ongoing concern to policy makers and researchers alike. Of particular interest are the changes in inequality and poverty over time, which are typically assessed through the estimation of income distributions. From this, income inequality and poverty measures, along with their differences and standard errors, can be derived and compared. With panel data becoming more frequently used to make such comparisons, traditional methods which treat income distributions from different years independently and estimate them on a univariate basis, fail to capture the dependence inherent in a sample taken from a panel study. Consequently, parameter estimates are likely to be less efficient, and the standard errors for between-year differences in various inequality and poverty measures will be incorrect. This paper addresses the issue of sample dependence by suggesting a number of bivariate distributions, with Singh-Maddala or Dagum marginals, for a partially dependent sample of household income for two years. Specifically, the distributions being considered are the bivariate Singh-Maddala distribution, proposed by Takahasi (1965), and bivariate distributions belonging to the copula class of multivariate distributions, which are an increasingly popular approach to modelling joint distributions. Each bivariate income distribution will be estimated via full information maximum likelihood using data from the Household, Income and Labour Dynamics in Australia (HILDA) Survey for 2001 and 2005. Parameter estimates for each bivariate income distribution will be used to obtain values for mean income and modal income, the Gini inequality coefficient and the headcount ratio poverty measure, along with their differences, enabling the assessment of changes in such measures over time. In addition, the standard errors of each summary measure and their differences, which are of particular interest in this analysis, will be calculated using the delta method.

1. INTRODUCTION

The incidence of inequality and poverty is of ongoing concern not only in developing regions but also amongst some of the world's economic leaders. As indicators of social welfare, much of the literature has been drawn towards the accurate specification and estimation of various inequality and poverty measures. In particular, policy makers and researchers alike are often interested in assessing the changes in inequality and poverty over time. That is, one would ideally expect to observe a reduction in inequality and poverty from one year to another in order to determine whether policies implemented for that purpose have been effective. Such assessments are typically performed through the estimation of income distributions, from which income inequality and poverty measures, along with their differences and standard errors, can be derived and compared.

Traditionally, comparisons of inequality and poverty over time have been made with income distributions for different years being treated as independent. Various univariate functional forms have been suggested in the literature and income distributions have been estimated accordingly using conventional inference techniques. Initially, the gamma, lognormal and Pareto distributions were commonly used, with the gamma distribution generally found to fit better than the lognormal distribution (Salem and Mount, 1974; McDonald and Ransom, 1979; McDonald, 1984). Other two-parameter distributions that have been considered include the beta, Fisk and Weibull distributions. In addition, a number of three-parameter distributions have been proposed in the literature, including the Singh-Maddala distribution, which contains the Pareto, Fisk and Weibull distributions as special cases, and the Dagum distribution, which, although not as widely applied as the Singh-Maddala distribution, has shown to provide a better fit (Kleiber, 1996; Kleiber and Kotz, 2003).

A major issue in taking the univariate approach, however, is that panel data is becoming more frequently used to make comparisons of inequality and poverty over time. Consequently, as some members of the panel will be common between years, recorded incomes are likely to be correlated, resulting in a dependent sample. Therefore, treating one income distribution for any given year independently of another does not take into account that those who earned a high income in one year are also likely to earn a high income in a subsequent year and vice versa. This is of concern particularly in regions which exhibit low income mobility, as it coincides with a high degree of correlation.

There are two main consequences of estimating separate univariate distributions for different years of a panel. The first is that the parameter estimates are likely to be less efficient than a bivariate or multivariate approach that recognises correlation in incomes from year to year. The second is that the standard errors for between-year differences in various inequality and poverty measures will be incorrect. This is due to the estimated differences being functions of the parameter estimates from both marginal distributions. These parameter estimates will be correlated, and separate estimation does not provide a covariance term for computing the standard error of the difference. These features provide motivation for the use of multivariate techniques for the distribution of income when using panel data, in order to account for possible correlation between years.

The presence of dependence in a sample of income taken from a panel study has been left largely unaddressed in the literature. In a paper by Kmietowicz (1984), a bivariate lognormal distribution is suggested for the joint distribution of household size and income, rather than income over time, which is then used to derive estimates of the Gini inequality measure. Sarabia et al. (2005) adapt this model by deriving extensions of the bivariate lognormal distribution and applying each to data from the European Community Household Panel. In both papers, the proposed models have marginal income distributions which follow the univariate lognormal distribution. However, it has been historically found at the univariate case that although the lognormal distribution performs well at lower income levels, it fits poorly at higher income levels (Singh and Maddala, 1976). In addition, the Singh-Maddala and Dagum distributions have been subsequently shown to provide a better fit than the lognormal distribution (Singh and Maddala, 1976; McDonald and Ransom, 1979; McDonald, 1984). Therefore, a multivariate distribution which has either Singh-Maddala or Dagum marginals would be better suited to approximating the joint distribution of income rather than one with lognormal marginal distributions.

Other studies which have recognised the issue of dependent samples often ignore the problem by selecting a subsample of the data to create either an independent sample or a completely dependent sample, with both Kmietowicz (1984) and Sarabia et al. (2005) guilty of the latter. This is of concern as the disregard of large proportions of available data creates the potential for the marginal distributions of income to be estimated inaccurately. Intuitively, a solution would be the use of a partially dependent sample, which contains both the dependent observations within a panel as well as the independent observations. However, this approach has yet to be performed in the literature.

This paper seeks to address the issue of sample dependence by applying various bivariate distributions, with Singh-Maddala or Dagum marginals, to a partially dependent sample of household income for two (non-consecutive) years, with a view to assess the changes in inequality and poverty over that period. For ease of analysis only the bivariate case is being considered in this paper. One of the distributions suggested is the bivariate Singh-Maddala distribution proposed by Takahasi (1965). The appeal of this distribution is that both the marginal and conditional distributions follow a univariate Singh-Maddala specification (Kleiber and Kotz, 2003). Other bivariate distributions being considered belong to the copula class of multivariate distributions. Using copulas to model multivariate distributions is extremely popular in the finance and actuarial context, particularly for capturing dependence amongst stocks. This approach is appealing as copulas are easily estimated using maximum likelihood techniques, and there are many alternatives available in the literature which capture a wide range of dependence structures beyond simply correlation. In addition, copulas are flexible in that they can be applied to any specification of the marginal distribution, including allowing for the marginal distributions to have different specifications. This provides an attractive method for capturing the dependence structure contained in the joint distribution of income under partially dependent samples.

Each of the above bivariate income distributions will be estimated via full information maximum likelihood using income data from the Household, Income and Labour Dynamics in Australia (HILDA) Survey for 2001 and 2005. Once the parameters for each bivariate income distribution have been estimated, values for various measures of inequality and poverty can be obtained for each marginal distribution along with their differences, enabling the assessment of changes in such measures over time. More specifically, the summary measures to be considered in this paper include mean income and modal income, the Gini inequality coefficient and the headcount ratio poverty measure. In addition, the standard errors of each of the differences, which are of particular interest in this analysis, will be calculated using the delta method. For comparative purposes, estimates of each measure will also be obtained for the Singh-Maddala and Dagum marginal distributions which have been estimated at the univariate case under independence.

The remainder of this paper is organised as follows. Section 2 discusses and defines the concept of a partially dependent sample. A definition for each of the bivariate distributions proposed for the joint distribution of income, along with the inequality and poverty measures considered in this analysis are provided in Section 3. Section 4 defines a likelihood function for partially dependent samples with a particular emphasis on the likelihood function for a copula. Section 5 summarises the characteristics of the HILDA panel data used in the analysis. Empirical results of the analysis as applied to the data, including parameter estimates, tests for independence, and estimates for the inequality and poverty measures are presented and discussed in Section 6; conclusions appear in Section 7.

2. PARTIALLY DEPENDENT SAMPLES

Given the nature of panel studies, it is difficult to maintain a completely dependent sample over an extensive period of time, as members will typically enter and exit the panel from year to year. Therefore, if a sample of income is taken from any two years of a panel, there will be members of the panel common to both years as well as members who have only participated in one year. Consequently, it is impossible to obtain a completely dependent sample without discarding valuable data, inhibiting the accurate approximation of the marginal income distributions for each year. In order to include all of the available data when estimating dependent income distributions, a partially dependent sample should be used, where those members of the panel which remain in both years are paired and treated as dependent, but those members who have only recorded data in one year are retained in the sample and treated as independent. The concept of a partially dependent sample is further defined as follows.

Let y_1 be a sample of income for one particular year of a panel data set, with a sample size denoted by n_1, and y_2 a sample of income from a subsequent year of the panel, with a sample size, n_2. Note that n_1 and n_2 need not be of the same size. Consider that there are k members of the panel which have recorded incomes in both years, where $k \leq min(n_1, n_2)$. These observations can be matched (and ordered for ease of notation), giving $\{(y_{1,1}, y_{2,1}), \dots, (y_{1,k}, y_{2,k})\}$ paired observations which follow some bivariate income distribution, $f(y_1, y_2)$. With respect to the remaining observations in the sample which are not matched with another observation from the other year, $(y_{1,k+1},\dots,y_{1,n_1})$ is independent of y_2, and $(y_{2,k+1},\dots,y_{2,n_2})$ is independent of y_1. In addition, it should be noted that $(y_{1,1},\dots,y_{1,k})$ and $(y_{1,k+1},\dots,y_{1,n_1})$ are observations from the same marginal income density, $f_1(y_1)$. Similarly, $(y_{2,1},\dots,y_{2,k})$ and $(y_{2,k+1},\dots,y_{2,n_2})$ come from the same marginal income density, given by $f_2(y_2)$. The main objective of this paper is to suggest alternative functional forms for the joint distribution of income, $f(y_1,y_2)$, whilst preserving the marginal income distributions $f_1(y_1)$ and $f_2(y_2)$ using all available data.

3. BIVARIATE INCOME DISTRIBUTIONS

3.1 Marginal Income Distributions

At the univariate level, the Singh-Maddala and Dagum distributions have been shown to provide a better fit than any of the other functional forms considered previously in the literature (Singh and Maddala, 1976; Dagum, 1977; Kleiber, 1996). This makes both distributions an attractive choice for the marginal densities of each income distribution. Both distributions belong to a class of distributions owed to Burr (1942), and are also special cases of the class of generalised beta of the second kind (GB2) distributions established by McDonald (1984), which has a probability density function given by,

$$f(y) = \frac{ay^{ap-1}}{b^{ap}B(p,q)[1+(y/b)^a]^{p+q}}, \qquad y > 0, \tag{1}$$

where b is a scale parameter, and a, p and q are shape parameters. $B(p,q)$ represents the beta function, which is defined as,

$$B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} = \int_0^1 t^{p-1}(1-t)^{q-1}dt. \tag{2}$$

The Singh-Maddala distribution, which is also known as the Burr XII distribution, is the special case of the GB2 distribution where $p = 1$. It has a probability density function given by,

$$f(y) = \frac{aqy^{a-1}}{b^a[1+(y/b)^a]^{1+q}}, \qquad y > 0, \tag{3}$$

and corresponding cumulative distribution function given by,

$$F(y) = 1 - [1 + (y/b)^a]^{-q}, \qquad y > 0, \tag{4}$$

where the parameters a, b and q are all strictly positive. The mean and mode of the Singh-Maddala distribution can be written in terms of the parameters a, b and q respectively as follows,

$$\mu = \frac{b\Gamma(1/a)\Gamma(q-1/a)}{a\Gamma(q)}, \tag{5}$$

$$m = b\left[\frac{a-1}{aq+1}\right]^{1/a}, \tag{6}$$

with $a > 1$ required for the mode to exist.

The Dagum distribution, which was later identified as the Burr III distribution, is the special case of the GB2 distribution where $q = 1$. It has a probability density function given by,

$$f(y) = \frac{apy^{ap-1}}{b^{ap}[1+(y/b)^a]^{p+1}}, \qquad y > 0, \tag{7}$$

which is associated with a cumulative distribution function defined as,

$$F(y) = [1 + (y/b)^{-a}]^{-p}, \qquad y > 0, \tag{8}$$

where the parameters a, b and p are all strictly positive. The mean and mode of the Dagum distribution can be written in terms of the parameters a, b and p respectively as follows,

$$\mu = \frac{b\Gamma(p+1/a)\Gamma(1-1/a)}{\Gamma(p)}, \tag{9}$$

$$m = b\left[\frac{ap-1}{a+1}\right]^{1/a}, \tag{10}$$

with $ap > 1$ required for the mode to exist. For a more detailed discussion of the Singh-Maddala and Dagum distributions and their properties, refer to Kleiber and Kotz (2003). The closed form expressions given in (5), (6), (9) and (10) can be used to obtain estimates for the mean income and modal income summary measures, using estimates of the distribution parameters. Expressions for the Gini inequality coefficient and headcount ratio are defined in subsequent sections.

3.2 The Bivariate Singh-Maddala Distribution

One of the distributions considered for the joint distribution of income is the bivariate case of the multivariate Singh-Maddala distribution proposed by Takahasi (1965). Developed as a compound Weibull distribution with a gamma distribution as the compounder, this distribution contains both marginal and conditional distributions which follow a univariate Singh-Maddala specification. It is often referred to as the multivariate Burr distribution due to its introduction prior to the paper by Singh and Maddala (Kleiber and Kotz, 2003).

Consider two samples of income from different years, y_1 and y_2, where both marginal densities, $f_1(y_1)$ and $f_2(y_2)$ respectively, follow the univariate Singh-Maddala distribution. Under the bivariate case of Takahasi's multivariate Singh-Maddala distribution, the joint density of income is given by,

$$f(y_1, y_2) = q(q+1)a_1 a_2 b_1^{-a_1} b_2^{-a_2} y_1^{a_1-1} y_2^{a_2-1}\left[1 + \left(\frac{y_1}{b_1}\right)^{a_1} + \left(\frac{y_2}{b_2}\right)^{a_2}\right]^{-(q+2)}, \quad (11)$$

with a cumulative distribution function of the form,

$$F(y_1, y_2) = 1 - \left[1 + \left(\frac{y_1}{b_1}\right)^{a_1}\right]^{-q} - \left[1 + \left(\frac{y_2}{b_2}\right)^{a_2}\right]^{-q} - \left[1 + \left(\frac{y_1}{b_1}\right)^{a_1} + \left(\frac{y_2}{b_2}\right)^{a_2}\right]^{-q}. \quad (12)$$

The parameters a_1 and b_1 correspond with the univariate Singh-Maddala distribution for the marginal density of y_1, and a_2 and b_2 correspond with the marginal density for y_2. Note however, that the parameter q is common in both marginal distributions

3.3 Bivariate Copula Distributions

The estimation of copulas is becoming an increasingly popular approach to modelling joint distributions. Often the issue of dependence within a sample is addressed through the concept of correlation. However, when dealing with nonlinear distributions, which is usually the case when analysing strictly positive income data, more complex, nonlinear dependence structures can arise when considering their joint distributions (Trivedi and Zimmer, 2005). The copula approach makes it possible for a wide range of dependence structures to be captured beyond simply correlation. Popularised by Sklar (1959), copulas allow for the derivation of a joint distribution in terms of the marginal distributions of each variate. Therefore, they can be fitted to any specification for the marginal distribution, including allowing for the marginal variates to follow different distributions.

A copula is a multivariate distribution which is defined on the $[0,1]^m$ hypercube, where each of the m marginal variates is uniformly distributed. That is, consider a set of m random variates, $y_1, \dots, y_m$, each of which have a cumulative distribution function given by $F_1(y_1), \dots, F_m(y_m)$ respectively. Then each can be transformed into marginal variates defined on the unit interval $[0,1]$ using $u_j \sim F_j(y_j)$ for $j = 1, \dots, m$. Each variate also has an inverse cumulative distribution function such that $y_j \sim F_j^{-1}(u_j)$ for $j = 1, \dots, m$. Under Sklar's theorem, if the joint distribution of $y_1, \dots, y_m$ is given by some function, $F(y_1, \dots, y_m)$, then there exists a copula function, $C(u_1, \dots, u_m)$, with margins given by $F_1(y_1), \dots, F_m(y_m)$ such that,

$$\begin{aligned} F(y_1, \dots, y_m) &= F\left(F_1^{-1}(u_1), \dots, F_m^{-1}(u_m)\right) \\ &= C\left(F_1(y_1), \dots, F_m(y_m)\right) = C(u_1, \dots, u_m) \end{aligned} \tag{13}$$

Thus modelling the dependence between the uniformly distributed margins is equivalent to modelling the dependence between the variates themselves. If the marginal distributions $F_1(y_1), \dots, F_m(y_m)$ are continuous, then the copula function $C(u_1, \dots, u_m)$ is unique. If some or all of the marginal distributions are discrete, then the copula function still exists, however is not unique. In addition, a copula must satisfy the following three properties:

(1) $C\left(1, \dots, 1, u_j, 1, .., 1\right) = u_j$ for any $j \leq m$;

(2) $C(u_1, \dots, u_m) = 0$ if $u_j = 0$ for any $j \leq m$;

(3) $C(u_1, \dots, u_m)$ is m-increasing.

For a more detailed discussion of the properties of a copula refer to Trivedi and Zimmer (2005).

The choice of copula depends on the dependence structure between the variates of interest. This paper considers three of the most commonly applied bivariate copulas; the Gaussian copula, Clayton copula and Gumbel copula. The Gaussian copula is an extension of the bivariate Normal distribution, and belongs to the class of elliptical copulas. It is defined as,

$$C(u_1, u_2; \theta) = \Phi_{\mathrm{G}}\big(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\big), \tag{14}$$

where Φ_{G} is the standard bivariate Normal distribution, Φ is the standard univariate Normal distribution, and θ is the dependence parameter. The Gaussian copula captures dependence in the form of correlation, and as a result, the dependence parameter is defined on the $(-1, 1)$ interval. That is, it captures both positive and negative dependence. A dependence parameter with the value of 0 corresponds with the independence case.

The Clayton (1978) copula is defined as,

$$C(u_1, u_2; \theta) = \left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-1/\theta}, \tag{15}$$

where the dependence parameter, θ, is defined only on the $(0, \infty)$ interval. Whilst the independence case is not achieved for any value of θ, as it approaches zero the Clayton copula approaches independence. In addition, the Clayton copula can only capture positive dependence. It favours data which exhibits strong left tail dependence and weak right tail dependence. Therefore, if a sample of bivariate data is expected to be highly correlated at lower values and relatively less correlated at higher values, then the Clayton copula would be an appropriate choice.

The Gumbel (1960) copula is defined as,

$$C(u_1, u_2; \theta) = exp\left\{-\left(\tilde{u}_1^{\theta} + \tilde{u}_2^{\theta}\right)^{1/\theta}\right\}, \tag{16}$$

where $\tilde{u}_j = -\ln(u_j)$. In this case, the dependence parameter, θ, is defined on the $[1, \infty)$ interval, where a value of 1 represents the independence case. Like the Clayton copula, the Gumbel copula only captures positive dependence. However, it favours data which exhibits strong right tail dependence and weak left tail dependence. Each of the three copula functions defined above will be applied to the case where both marginal densities follow the univariate Singh-Maddala distribution and to the case where both marginal densities follow the univariate Dagum distributions.

3.4 The Gini Inequality Coefficient

The Gini coefficient is the most commonly used measure of the degree of inequality in an income distribution. It is derived from a Lorenz curve, which is an alternative representation of the distribution of income for a population of interest. The Lorenz curve is a graph which represents the cumulative proportion of total income, η, received by the poorest proportion of the population of interest, π. That is, each income earning unit in the population, typically a household, is ordered from the lowest income earner to the highest income earner and their cumulative income shares recorded. The curve which corresponds to the 45 degree line, as shown in Figure 1, represents perfect equality, where each household receives the same income. Perfect inequality coincides with the curve which follows the bottom and right axes, representing the case when one household receives all income. Subsequently, measures of inequality can be derived from the area between the 45 degree line and the Lorenz curve, with a smaller area indicating that the population is closer to equality.

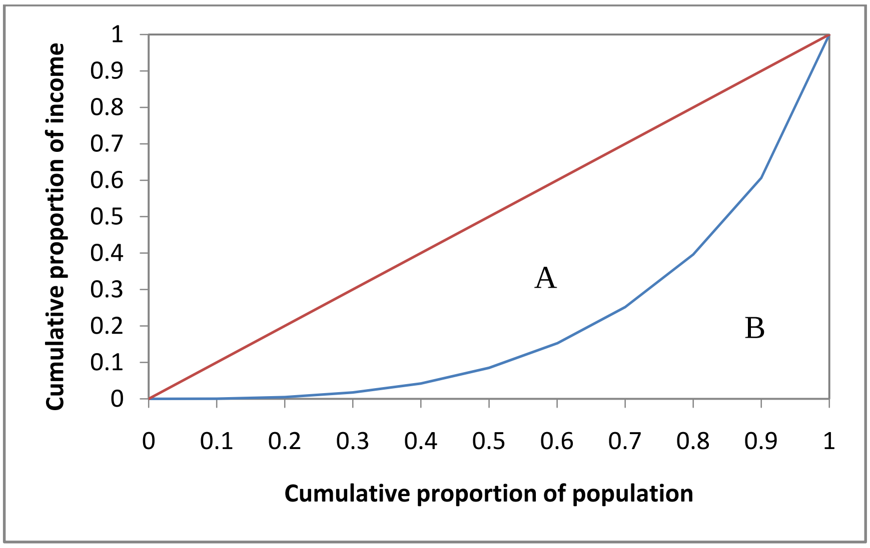

Figure 1 The Lorenz curve for income

The Gini coefficient is one such measure, which is simply the ratio of the area between the line of perfect equality and the Lorenz curve (A in Figure 1) and the area between the line of perfect equality and the line of perfect inequality (A + B). This enables the Gini coefficient to lie between 0 and 1, with a coefficient of 0 indicating perfect equality, and 1 representing perfect inequality. It is also useful to note that as both axes lie on a (0,1) interval, the Gini coefficient can also be defined as twice the area between the line of perfect equality and the Lorenz curve.

Closed form expressions for the Gini coefficient in terms of the parameters of the income distribution are available for both the univariate Singh-Maddala and Dagum distributions considered in this analysis. The Gini coefficient which corresponds to the Singh-Maddala distribution is defined as,

$$G = 1 - \frac{\Gamma(q)\Gamma(2q-1/a)}{\Gamma(q-1/a)\Gamma(2q)}, \quad (17)$$

and the Gini coefficient which corresponds to the Dagum distribution is defined as,

$$G = \frac{\Gamma(p)\Gamma(2p+1/a)}{\Gamma(p+1/a)\Gamma(2p)} - 1. \quad (18)$$

Estimates of the Gini coefficient can be found by using estimates of the income distribution parameters and substituting them into the above expressions.

3.5 The Headcount Ratio Poverty Measure

The poverty measure considered in this analysis is the headcount ratio. This is the most commonly used poverty measure in the literature, and measures the proportion of the population which are considered to be in poverty. If x represents the number of people or households which receive an income below a pre-specified poverty line, z, and N is the population size, then the headcount ratio, H_z, is given by,

$$H_z = \frac{x}{N}. \quad (19)$$

Alternatively, it can be calculated by evaluating the cumulative distribution function of the income distribution at the poverty line, z. That is,

$$H_z = F(z) = \int_0^z f(y)dy. \tag{20}$$

This function holds for any specification for the income distribution, $f(y)$, and therefore can be used to obtain estimates of the headcount ratio for the income distributions from each year considered in this paper. Although widely used, the headcount ratio has been heavily criticised for only indicating the level of poverty within a population and not the intensity of poverty or the extent of inequality amongst the poor. For example, consider two societies – society A and society B. If society A has a higher headcount ratio than society B, then the former may be said to have a higher incidence of poverty than the latter. However, it may be the case that society A has a large number of households which receive an income close to the poverty line, whilst society B has fewer households in poverty but households that receive an income much lower than the poverty line. Therefore, the headcount ratio may be misleading and as a result, many other measures of poverty have been proposed in the literature.

4. MAXIMUM LIKELIHOOD ESTIMATION

Maximum likelihood techniques have remained the most preferred estimation method when modelling joint distributions. When estimating copulas in particular, there are two alternative approaches to maximum likelihood estimation. The first uses two-stage maximum likelihood, which estimates each marginal distribution at the first stage, and then estimates the dependence parameter at the second stage, using the estimates obtained at the first stage. This method is attractive if there is a high number of marginal variates or if the marginal distributions contain a high number of parameters. The other approach, which is the one adopted by this paper, is the use of full information maximum likelihood (FIML) estimation, which estimates all the parameters simultaneously. Although this method can become cumbersome for highly parameterised marginal distributions or for high-dimensional copulas, this method gives more efficient estimates of the parameters (Trivedi and Zimmer, 2005). As only the bivariate case is being considered in this analysis, the number of parameters to be estimated is relatively low. Therefore, FIML will be used in favour of more efficient estimates.

4.1 The Log-Likelihood Function for a Partially Dependent Sample

Recall that when using a partially dependent sample in an analysis that some members of the panel will be common to both samples from each year, whilst some members will have been observed in only one year. Once again, let y_1 be a sample of income for one particular year of a panel data set, which follows a univariate income distribution given by $f_1(y_1)$, and y_2 a sample of income from a subsequent year of the panel, which follows a univariate income distribution given by $f_2(y_2)$. The observations which occur in both years follow some bivariate income distribution, $f(y_1, y_2)$, whilst the observations which occur in one year only are governed by their respective marginal distributions.

In addition, let d_1 be a dummy variable taking a value of 1 if the member of the panel only recorded an income in the first year, and d_2 be a dummy variable taking a value of 1 if the member of the panel only recorded an income in the subsequent year. Similarly, let $d_3 = (1 - d_1)(1 - d_2)$ be a dummy variable indicating whether a member of the panel recorded an income in both years. Then the likelihood function for a partially dependent sample can be defined as,

$$\mathcal{L}(\theta; y_1, y_2) = \prod_{i=1}^{n} d_{1i} f_1(y_{1i}) \prod_{i=1}^{n} d_{2i} f_2(y_{2i}) \prod_{i=1}^{n} d_{3i} f(y_{1i}, y_{2i}). \tag{21}$$

where $n = (n_1 - k) + (n_2 - k) + k = n_1 + n_2 - k$. That is, the dummy variable, d_1, is used to pick out the $(n_1 - k)$ observations of y_1 which follow the marginal income distribution, $f_1(y_1)$. Similarly, the dummy variable, d_2, picks out the $(n_2 - k)$ observations of y_2 which follow the marginal income distribution, $f_2(y_2)$. The dummy variable, d_3, is used to pick out the k observations of y_1 and y_2 which come from the joint income distribution, $f(y_1, y_2)$. The corresponding log-likelihood function for a partially dependent sample is then given by,

$$\ell(\theta; y_1, y_2) = \sum_{i=1}^{n} d_{1i} \ln f_1(y_{1i}) + \sum_{i=1}^{n} d_{2i} \ln f_2(y_{2i}) + \sum_{i=1}^{n} d_{3i} \ln f(y_{1i}, y_{2i}). \tag{22}$$

It is relatively straightforward to derive $\ln f(y_{1i}, y_{2i})$ from the probability density function for Takahasi's bivariate Singh-Maddala distribution. However, at this stage, it is useful to define the density function and likelihood function of a copula in order to derive $\ln f(y_{1i}, y_{2i})$ for each copula.

4.2 The Copula Log-Likelihood Function

Consider a bivariate copula denoted by $C(F_1(y_1), F_2(y_2); \theta)$. Its probability density function is then defined as,

$$\begin{aligned} c(u_1, u_2; \theta) &= \frac{\partial}{\partial y_1 \partial y_2} C(u_1, u_2; \theta) \\ &= C_{12}(u_1, u_2; \theta) f_1(y_1) f_2(y_2), \end{aligned} \tag{23}$$

where,

$$C_{12}(u_1, u_2; \theta) = \frac{\partial}{\partial u_1 \partial u_2} C(u_1, u_2; \theta). \tag{24}$$

It follows that the likelihood function of a copula is given by,

$$\mathcal{L}(\theta; y_1, y_2) = \prod_{i=1}^{n} C_{12}(u_{1i}, u_{2i}; \theta) f_1(y_{1i}) f_2(y_{2i}), \tag{25}$$

with the resulting log-likelihood function of a copula subsequently defined as,

$$\ell(\theta; y_1, y_2) = \sum_{i=1}^{n} \sum_{j=1}^{2} \ln f_j(y_{ji}) + \sum_{i=1}^{n} \ln C_{12}(F_1(y_{1i}), F_2(y_{2i}); \theta). \tag{26}$$

The expression given in (28) can be used to obtain the function $\ln f(y_{1i}, y_{2i})$ for the log-likelihood of the entire partially dependent sample as defined in (22).

The function, $C_{12}(F_1(y_{1i}), F_2(y_{2i}); \theta)$, represents the cross partial derivative of the copula distribution function with respect to its marginals. Closed form expressions for this are available for each of the three copulas considered in this paper as follows. For the Gaussian copula,

$$C_{12}(u_1, u_2; \theta) = (1-\theta^2)^{-1/2} exp\left\{-\frac{1}{2}(1-\theta^2)^{-1}(x_1^2 + x_2^2 - 2\theta x_1 x_2)\right\}$$
$$\times\, exp\left\{\frac{1}{2}(x_1^2 + x_2^2)\right\}, \qquad (27)$$

where $x_1 = \Phi^{-1}(u_1)$ and $x_2 = \Phi^{-1}(u_2)$.

For the Clayton copula,

$$C_{12}(u_1, u_2; \theta) = (1+\theta)(u_1 u_2)^{-\theta-1}\left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-2-(1/\theta)}, \qquad (28)$$

and for the Gumbel copula,

$$C_{12}(u_1, u_2; \theta) = C(u_1, u_2; \theta)(u_1 u_2)^{-1} \frac{(\tilde{u}_1 \tilde{u}_2)^{\theta-1}}{(\tilde{u}_1^{\theta} + \tilde{u}_2^{\theta})^{2-(1/\theta)}} \left[\left(\tilde{u}_1^{\theta} + \tilde{u}_2^{\theta}\right)^{1/\theta} + \theta - 1\right], \qquad (29)$$

where $\tilde{u}_j = -\ln(u_j)$.

5. DATA

This analysis was applied to unit-record data of Australian household disposable income obtained from the Household, Income and Labour Dynamics in Australia (HILDA) Survey for 2001 and 2005. The HILDA Survey is a longitudinal study of Australian households, which commenced in 2001 as an initiative of the Melbourne Institute of Applied Economic and Social Research, University of Melbourne. However, only private dwellings were considered in the sample of Australian households and as a result the homeless and those living in public housing were excluded from this analysis. Households which recorded a zero or negative disposable income were also removed from the data set.

The household disposable income data used in the analysis was derived by subtracting financial year taxes from gross income. Household gross income was obtained by summing all regular sources of income, from both private and public sources, including Family Tax Benefits and Child Care Benefits, however excludes irregular sources of income. In order to maintain the weighted mean of the distribution, households which recorded a disposable income greater than a pre-specified threshold of $275,000 were substituted with the top-coded value of their weighted average ($461,332 for 2001 and $418,490 in 2005).

In order to account for differences in the size of the households, the household disposable income data were adjusted using an equivalence scale. In particular, the OECD equivalence scale was used, which gives a weight of 1.0 to the first adult household member, 0.7 to each additional adult, and 0.5 to each child under the age of 15. The data were also adjusted to account for the effects of inflation using Consumer Price Index data obtained from the Australian Bureau of Statistics, which is based in 1948 dollars.

From this, a partially dependent sample was constructed by establishing whether a particular household had recorded an income in one or both years and its composition based on the number of adults and children. Households which only recorded an income in one year were treated as independent from the distribution of income in the other year. Households which recorded an income in both 2001 and 2005, and remained the same in composition, were paired and treated as dependent. However, households which recorded an income in both 2001 and 2005, and changed in composition, either by separating into more than one household or joining with another to form one household, were treated as independent. The motivation behind this is that it is difficult to identify the main income earner in a particular household, and the potential change in an individual's weight given by the equivalence scale. Therefore, whilst it is recognised that there is still some element of dependence among such households, it would not be as strong as the dependence inherent in households which remained the same in composition. It was found that 2750 households recorded an income in both years and did not change in composition and, as a result, were treated as dependent observations.

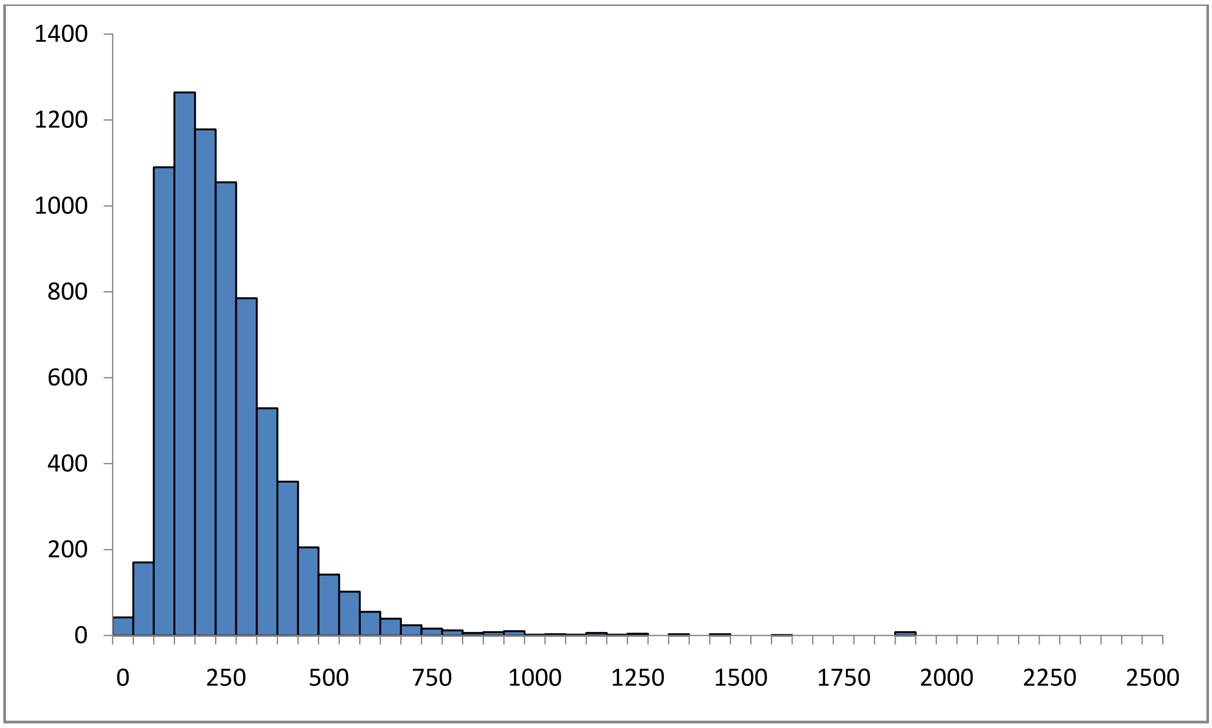

Figure 2 Histogram of real equivalised household disposable income ($'00) for Australia in 2001

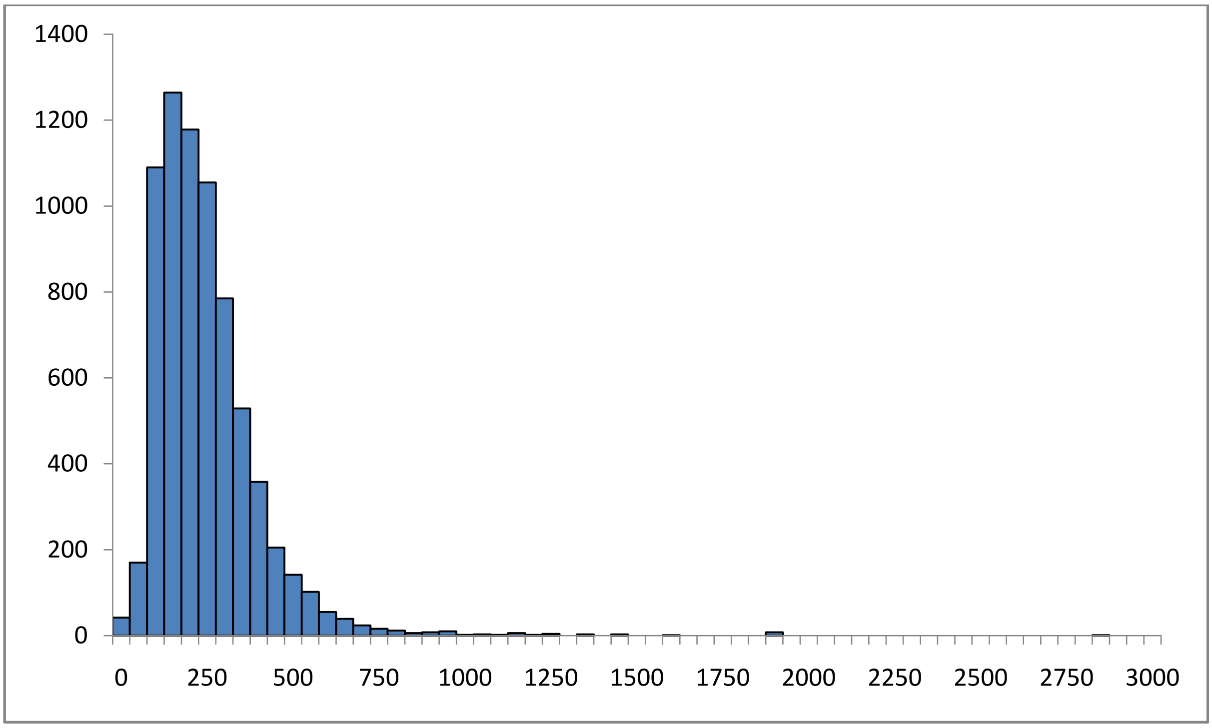

Figure 3 Histogram of real equivalised household disposable income ($'00) for Australia in 2005

	2001	**2005**
Mean	206.9265	223.2122
Median	177.9394	192.2828
Maximum	2298.615	2820.013
Minimum	0.054808	0.449236
Std. Dev.	146.6253	157.3059
Skewness	4.00632	3.623183
Kurtosis	41.47192	33.33139
Observations	7614	7083

Table 1 Descriptive statistics of real equivalised household disposable income ($'00) for Australia in 2001 and 2005

Figure 2 and Figure 3 illustrate the sample distribution of real equivalised household disposable income in Australia for the years 2001 and 2005 respectively. It can be seen that both sample distributions exhibit positive skewness typical of income distributions. The data for 2001 comprises a sample of 7614 observations, with a sample mean income of $20,692.65 and a sample median income of $17,793.94 (as shown in Table 1). The data for 2001 comprises a sample of 7083 observations, with a sample mean income of $22,321.22 and a sample median income of $19,228.28.

6. EMPIRICAL ANALYSIS

This section provides a discussion of the results from the analysis of the real equivalised disposable income data after estimating each of the proposed models using FIML. Firstly, the estimated parameters of each bivariate distribution, applied with both Singh-Maddala and Dagum marginal distributions, and their associated density functions are compared to parameter estimates obtained under the univariate case. Secondly, the models are tested for independence using standard hypothesis tests. Finally, estimates of the welfare measures, their differences and associated standard errors are compared to examine changes in inequality and poverty between the two years.

Table 2 presents the parameter estimates and corresponding standard errors for each of the bivariate income distributions where both marginal densities follow a Singh-Maddala distribution. Parameter estimates and standard errors for the bivariate copulas, in which both marginal densities follow the Dagum distribution, are contained in Table 3. In addition, the marginal income distribution for each year was estimated independently, under both the Singh-Maddala and Dagum specifications, with the results also presented in Table 2 and Table 3 accordingly. It can be seen from the standard errors that the Takahasi bivariate Singh-Maddala distribution and the Gumbel copula have produced more efficient estimates of the parameters than those obtained under the univariate approach, compared to the Gaussian and Clayton copulas, for the case of two Singh-Maddala marginals. However, it is interesting to note that both the log-likelihood of each model and the Akaike information criterion (AIC) suggest that the Gaussian and Clayton copulas outperform Takahasi’s bivariate Singh-Maddala distribution. With respect to the case of two Dagum marginals, it appears that the Gaussian and Gumbel copulas have produced more efficient estimates of the parameters than those obtained under the univariate approach, with standard errors for some of the parameters of the Clayton copula exceeding those of the univariate distributions.

In terms of overall model comparison, it can be seen that each of the bivariate income distributions have log-likelihood values greater than the values achieved under the univariate approach, providing evidence in favour of the use of bivariate income distributions. Overall, the Gumbel copula appears to provide the best fit for the partially dependent income data, producing the highest log-likelihood value and lowest AIC. This is observed for both forms of the marginal income distributions. As the Gumbel copula captures strong right tail dependence, this seems to suggest that there is a relatively higher degree of correlation within high income earners. That is, if a household earned a high income in 2001, then they are also expected to earn a high income in 2005. This coincides with a relatively higher degree of income mobility amongst low income earners, with the Gumbel copula capturing relatively weak left tail dependence. In addition, the Gumbel copula with two Dagum marginals is slightly favoured over the Gumbel copula with two Singh-Maddala marginals, as it produced a slightly higher AIC, although both have produced identical log-likelihood values.

The estimated density functions for income under each distribution given in Figures A.1 to A.4, which are contained in Appendix A. It can be seen that each model has resulted in a unimodal and positively skewed density, which are typical of income distributions, and closely resemble the sample histograms presented in Section 5. For each year and marginal specification it appears that the Gaussian and Gumbel copulas produce relatively similar income distributions in comparison to the Clayton copula which produces a relatively lower

	2001			2005					
Distribution	**a**	**b**	**q**	**a**	**b**	**q**	**θ**	**LogL**	**AIC**
Univariate	2.326763	248.3544	1.849938	2.354199	261.8718	1.786583	-	-90296.74	-
	(0.030517)	(7.237804)	(0.084418)	(0.033820)	(8.013253)	(0.086050)	-		
Takahasi	2.659385	187.2049	1.180077	2.674124	201.5115	1.180077	-	-89663.21	15.01100
	(0.027174)	(3.096971)	(0.034375)	(0.028875)	(3.382508)	(0.034375)	-		
Gaussian	2.306171	258.6003	1.961395	2.333232	274.1514	1.924729	0.654493	-89517.05	14.98687
	(0.028768)	(7.343034)	(0.085785)	(0.031184)	(8.032825)	(0.087742)	(0.006603)		
Clayton	2.226330	282.2625	2.209808	2.273241	292.7240	2.083788	1.214559	-89806.08	15.03525
	(0.027424)	(8.716143)	(0.103483)	(0.029347)	(9.209966)	(0.100780)	(0.029937)		
Gumbel	2.404412	225.8673	1.582509	2.426464	240.7493	1.556804	1.956940	-89412.60	14.96827
	(0.030011)	(5.302110)	(0.058131)	(0.032845)	(5.946110)	(0.060727)	(0.030163)		

Table 2 FIML estimates and standard errors in parentheses for the bivariate distributions with Singh-Maddala marginals

	2001			2005					
Distribution	**a**	**b**	**p**	**a**	**b**	**p**	**θ**	**LogL**	**AIC**
Univariate	3.297944	217.3304	0.639739	3.301757	233.3335	0.645083	-	-90324.99	-
	(0.062788)	(3.258864)	(0.019846)	(0.065267)	(3.803521)	(0.021809)	-		
Gaussian	3.407191	225.0946	0.599126	3.440949	241.8563	0.597565	0.666582	-89527.85	14.98868
	(0.060955)	(3.062192)	(0.016737)	(0.064473)	(3.525197)	(0.018351)	(0.006754)		
Clayton	3.616174	242.4160	0.517617	3.603839	257.9431	0.532217	1.369025	-89772.37	15.02961
	(0.070194)	(3.283488)	(0.015807)	(0.072298)	(3.703287)	(0.017204)	(0.038702)		
Gumbel	3.179721	210.1159	0.687982	3.172679	224.2632	0.700321	1.982700	-89412.60	14.96938
	(0.051635)	(2.985036)	(0.018667)	(0.053183)	(3.445819)	(0.020654)	(0.030422)		

Table 3 FIML estimates and standard errors in parentheses for the bivariate distributions with Dagum marginals

mode and fatter right tail. The lower mode and fatter right tail of the Clayton copula suggests a slightly greater number of households are earning a higher disposable income than what is suggested under the other bivariate income distributions. In comparing the marginal distributions, it appears that there is little difference between the Singh-Maddala and Dagum distributions, which is also shown by the close log-likelihood and AIC values across Tables 2 and 3.

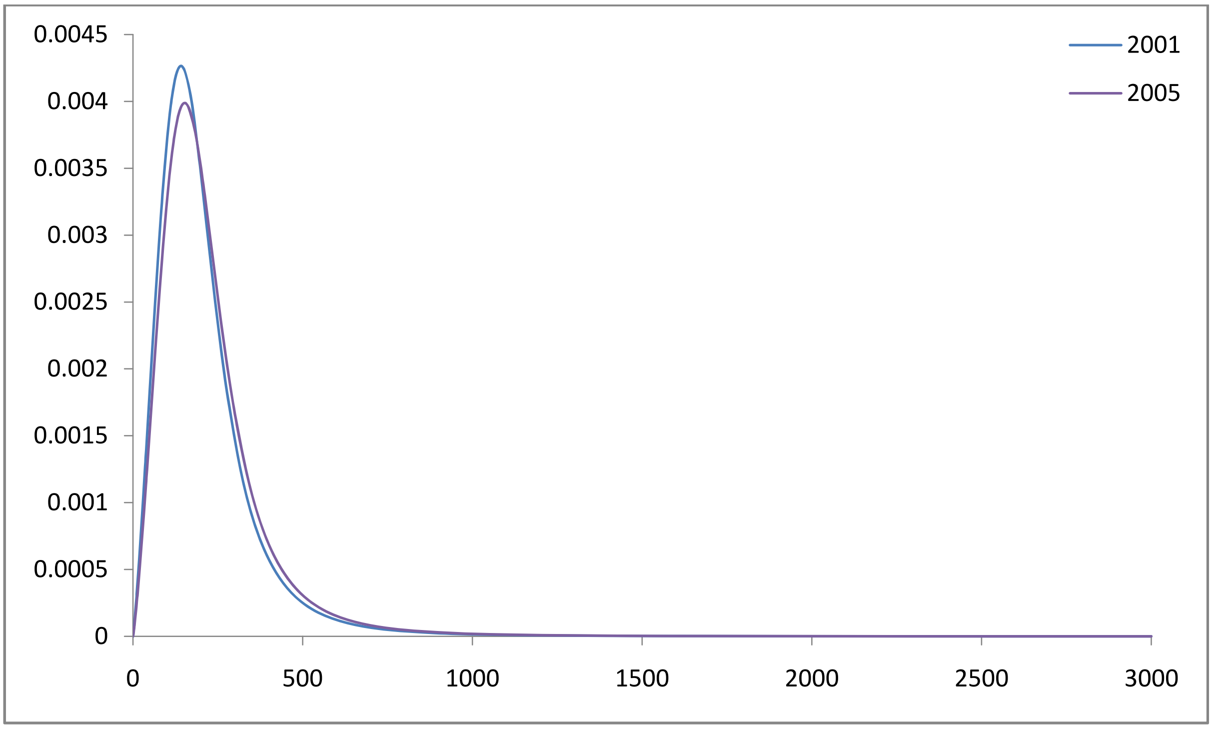

Figure 4 Estimated income densities of real equivalised disposable income ($'00) for Australia in 2001 and 2005 under the Gumbel copula with two Dagum marginals

The estimated density functions for income under the Gumbel copula with two Dagum marginals for 2001 and 2005 are given in Figure 4, showing the shift in income distribution between the two years. Although it appears from the income densities that there has been little change in the distribution of income in Australia, the income distribution for 2005 shows a slightly fatter right tail along with a lower peak at the mode of the distribution than the income distribution for 2001. This suggests that more households are earning higher disposable incomes in 2005 compared to 2001, and that in general, households are wealthier in 2005 than in 2001. A similar shift in the distribution of real equivalised disposable income between 2001 and 2005 was captured by each of the proposed bivariate distributions.

It was previously recognised that each of the bivariate income distributions provided a better fit to the partially dependent income data than the univariate income distributions, based on values for the log-likelihood and AIC of each model. However, the case for independence can be formally tested by conducting standard hypothesis tests on the dependence parameter for each copula, given the asymptotic properties of the maximum likelihood estimates. Recall that for the Gaussian copula, a dependence parameter with a value of 0 indicates independence. For the Gumbel copula, independence is given under a dependence parameter equal to 1. Whilst independence cannot be achieved for a particular value of the dependence parameter under the Clayton copula, as it approaches 0 the Clayton copula approaches independence. Therefore, this restriction can be tested to indicate independence under the Clayton copula. Note that as the Takahasi bivariate Singh-Maddala distribution does not contain a dependence parameter, testing for independence is not as straightforward.

From the estimation output for each copula model, it was observed that each of the dependence parameters for the Gaussian and Clayton copulas produced an extremely low p-value along with large test statistics. This was also observed for the dependence parameter of the Gumbel copulas when calculated according to the restriction being tested. The results suggest that there is sufficient evidence of a significant degree of dependence between the two marginal income distributions and that independence is not supported by the data.

In order to examine the extent of income inequality and poverty present in Australia, estimates of mean income, modal income, the Gini coefficient and the headcount ratio were obtained by substituting each estimated parameter into the expressions outlined in Section 3. These are presented in Table 4 along with their standard errors, which were estimated using the delta method. Also included are the nonparametric values observed from the sample data. The sample Gini coefficient was calculated from the unit record data using the formula,

$$G = \frac{1}{n}\left[n + 1 - 2\,\frac{\sum_{i=1}^{n}(n+1-i)y_i}{\sum_{i=1}^{n} y_i}\right], \tag{30}$$

where y_i is the income of the i^{th} household after arranging the data in ascending order according to income. The poverty line used to calculate the headcount ratio was half the sample median income for each respective year.

	2001				2005			
Distribution	**Mean**	**Mode**	**Gini**	**HCR**	**Mean**	**Mode**	**Gini**	**HCR**
Sample	206.9265	-	0.334001	0.165222	223.2122	-	0.334005	0.158125
Singh-Maddala Marginals								
Univariate	206.2660	136.9027	0.334300	0.149908	222.5971	147.7996	0.334053	0.149006
	(1.57754)	(1.41951)	(0.00299)	(0.00322)	(1.77170)	(1.57692)	(0.00313)	(0.00338)
Takahasi	208.9075	132.7646	0.346500	0.141751	224.4695	143.4309	0.344649	0.141679
	(1.89535)	(1.20561)	(0.00383)	(0.00297)	(2.04301)	(1.34788)	(0.00385)	(0.00311)
Gaussian	206.5632	138.3875	0.331692	0.148449	221.8006	149.4590	0.329849	0.147936
	(1.46863)	(1.39318)	(0.00283)	(0.00305)	(1.61203)	(1.53331)	(0.00289)	(0.00320)
Clayton	207.9688	139.1738	0.332425	0.150330	224.5462	150.9608	0.330881	0.147474
	(1.53322)	(1.44212)	(0.00270)	(0.00309)	(1.70770)	(1.57359)	(0.00282)	(0.00313)
Gumbel	208.3347	135.4187	0.340225	0.147926	224.3702	146.2933	0.339206	0.147345
	(1.54575)	(1.30321)	(0.00300)	(0.00306)	(1.68869)	(1.45529)	(0.00312)	(0.00323)
Dagum Marginals								
Univariate	208.9561	144.1536	0.341680	0.147031	225.1684	155.6433	0.340452	0.146296
	(1.75898)	(1.75716)	(0.00372)	(0.00312)	(1.95200)	(1.95421)	(0.00380)	(0.00330)
Gaussian	208.9155	147.3901	0.338300	0.146658	223.9281	159.3271	0.335500	0.146407
	(1.59275)	(1.73790)	(0.00350)	(0.00301)	(1.73896)	(1.92538)	(0.00352)	(0.00318)
Clayton	209.1139	152.8924	0.336465	0.151098	225.4435	164.8970	0.334246	0.148391
	(1.63794)	(1.87701)	(0.00334)	(0.00314)	(1.80975)	(2.05805)	(0.00340)	(0.00320)
Gumbel	210.4714	141.4473	0.346134	0.146124	226.6301	152.2794	0.345114	0.145430
	(1.64247)	(1.58109)	(0.00348)	(0.00298)	(1.79466)	(1.74932)	(0.00357)	(0.00314)

Table 4 Estimates of the inequality and poverty measures and their standard errors in parentheses

From Table 4, it can be seen that each of the estimated summary measures appear to be very close to the observed sample values. In addition, the relatively small standard errors for each measure indicate that each was measured with a high level of precision. However, the results show that a different model provides a more accurate estimate for each summary measure. For example, the Gaussian copula with two Singh-Maddala marginals has produced an estimate for mean income in 2001 closest to the corresponding sample value and with the lowest standard error. However, with respect to modal income, Takahasi's bivariate Singh-Maddala distribution has produced estimates with the lowest standard errors for both year. Interestingly, the univariate Singh-Maddala distribution produces estimates of the Gini coefficient and headcount ratio closest to their sample values in both years, though not each has achieved the lowest standard error. What are of particular interest however, are the differences in each measure between the two years, and their corresponding standard errors which allow the analysis of changes in inequality and poverty over time. These results are contained in Table 5 below.

Distribution	Mean	Mode	Gini	HCR
Sample	16.2857	-	0.000004	-0.007097
Singh-Maddala Marginals				
Univariate	16.3311	10.8969	-0.000248	-0.000902
	(2.37225)	(2.12172)	(0.004327)	(0.004667)
Takahasi	15.5620	10.6663	-0.001853	-0.000072
	(2.17076)	(1.76814)	(0.004020)	(0.004422)
Gaussian	15.2374	11.0715	-0.001843	-0.000512
	(1.87752)	(1.82960)	(0.003766)	(0.003528)
Clayton	16.5775	11.7871	-0.001543	-0.002857
	(2.30518)	(1.83449)	(0.003912)	(0.003115)
Gumbel	16.0356	10.8746	-0.001019	-0.000581
	(1.60216)	(1.70618)	(0.003321)	(0.003943)
Dagum Marginals				
Univariate	16.2123	11.4897	-0.001228	-0.000735
	(2.62760)	(2.62803)	(0.005315)	(0.004543)
Gaussian	15.0126	11.9370	-0.002796	-0.000252
	(2.02217)	(2.56127)	(0.004724)	(0.003499)
Clayton	16.3296	12.0046	-0.002220	-0.002707
	(2.42845)	(2.72812)	(0.004893)	(0.002967)
Gumbel	16.1587	10.8321	-0.001021	-0.000695
	(1.67349)	(2.13190)	(0.003892)	(0.003821)

Table 5 Differences of the estimated inequality and poverty measures and their standard errors in parentheses

It can be seen from Table 5, that each of the bivariate income distributions have produced more efficient estimates than the univariate distributions for each of the differences in the summary measures, with the exception of the Clayton copula containing two Dagum marginals, with respect to the difference in modal income. This provides evidence in favour of the earlier argument that using bivariate distributions is necessary for capturing dependence amongst panel income data and to allow the production of more efficient estimates for the differences of various inequality and poverty measures of interest. It is also interesting to note that the Gumbel copula with two Singh-Maddala marginals has produced the most efficient estimates of the differences in mean income, modal income and the Gini coefficient, with standard errors smaller than any other model considered. However, in terms of the headcount ratio, the Clayton copula with two Dagum marginals has produced the most efficient estimate.

The estimates of each difference can also be used to analyse trends in income inequality and poverty. Those for mean and modal income indicate that on average, Australian households are better off, with estimates observing an increase between 2001 and 2005. According to the Gumbel copula with two Singh-Maddala marginals, differences indicate that mean income has increased by approximately $16,035, or 7.7 percent, and that modal income has increased by approximately $10,875, or 8 percent, after adjusting for inflation. In analysing the trend in inequality, the sample Gini coefficients have remained virtually unchanged between 2001 and 2005, with an extremely small increase of 0.000004 points indicating that there has been no improvement over the five-year period. This would appear to suggest that the overall increase in income has been proportionately distributed among households. However, the differences obtained for each of the distributions considered suggest that there has been a slight reduction in inequality, with the Gumbel copula containing two Singh-Maddala marginals showing the Gini coefficient decreasing by approximately 0.3 percent. With respect to trends in poverty, the sample headcount ratio and difference estimates for each distribution indicate a slight decrease in the proportion of the population considered to be living in poverty, over the five-year period between 2001 and 2005. However, the extent of the decrease appears to have been slightly underestimated, with a decrease in the headcount ratios, under the Clayton copula with two Dagum marginals, of approximately 1.8 percent, as opposed to a 4.3 percent decrease observed from the sample headcount ratio.

7. CONCLUSION

When analysing the incidence of inequality and poverty through the estimation of income distributions, the presence of dependence in a sample of income taken from a panel study has been left largely unaddressed in the literature. The main objective of this paper was to suggest various bivariate distributions for modelling a partially dependent sample of household income, from which inequality and poverty measures could be produced to examine changes in such measures over time. Of particular interest were the standard errors for the differences of each welfare measure between two years of a panel study, as they are incorrectly estimated under the univariate approach.

Maximum likelihood was used to fit the Gaussian, Clayton and Gumbel copulas, each with either two Singh-Maddala marginals or two Dagum marginals, along with a bivariate Singh-

Maddala distribution, to real equivalised household income in Australia. In comparing the parameter estimates of the bivariate income distributions to those obtained for the univariate income distributions, it was found that the former performed better overall as they produced higher log-likelihood values. This was consolidated by tests for independence conducted on the dependence parameter for each copula. On a comparison of the bivariate models, it was found that the Gumbel copula with two Dagum marginals best approximated the distribution of income in Australia.

In addition, the parameter estimates were used to obtain estimates for mean income, modal income, the Gini inequality measure, and the headcount ratio poverty measure for each marginal distribution. Each measure was shown to be close to its respective nonparametric estimate, with small standard errors also indicating that each was estimated with a high degree of precision. However, it was found that a different model provided a more accurate estimate for each measure. In terms of the differences for each summary measure, all bivariate distributions considered produced more efficient estimates than those obtained from the univariate distributions. It was shown that the Gumbel copula with two Singh-Maddala marginals produced estimates for the differences in mean income, modal income and the Gini coefficient with the smallest standard errors. However, for the headcount ratio, the Clayton copula with two Dagum marginals produced estimates of the difference with the smallest standard errors.

When using the differences of each summary measure to analyse trends in income inequality and poverty, it was found that estimates of mean and modal income indicated that on average, Australian households were better off in 2005 than in 2001. In analysing the trend in inequality, the sample Gini coefficients suggested that there had been no improvement over the five-year period. Conversely, the estimates obtained from the bivariate distributions suggested that there had been a slight reduction in inequality. With respect to trends in poverty, both the sample headcount ratio and associated estimates from the bivariate distributions indicated a slight decrease in the proportion of the population considered to be living in poverty between 2001 and 2005.

The methodology proposed in this paper can be easily extended to multivariate case, which would allow the use of data from each available year of a panel study, rather than only two years. Various multivariate copulas are available in the literature, including multivariate extensions of the Gaussian, Clayton and Gumbel copulas. This would enable the researcher to gain a larger picture of the trends in inequality and poverty in each year. Further extensions of the proposed methodology also include the Bayesian analysis of such multivariate income distributions. The application of Bayesian inference methods is gaining interest in the field of income inequality and poverty analysis at the univariate level. This would facilitate the production of posterior densities for not only the parameters, but also the inequality and poverty measures of interest.

APPENDIX A

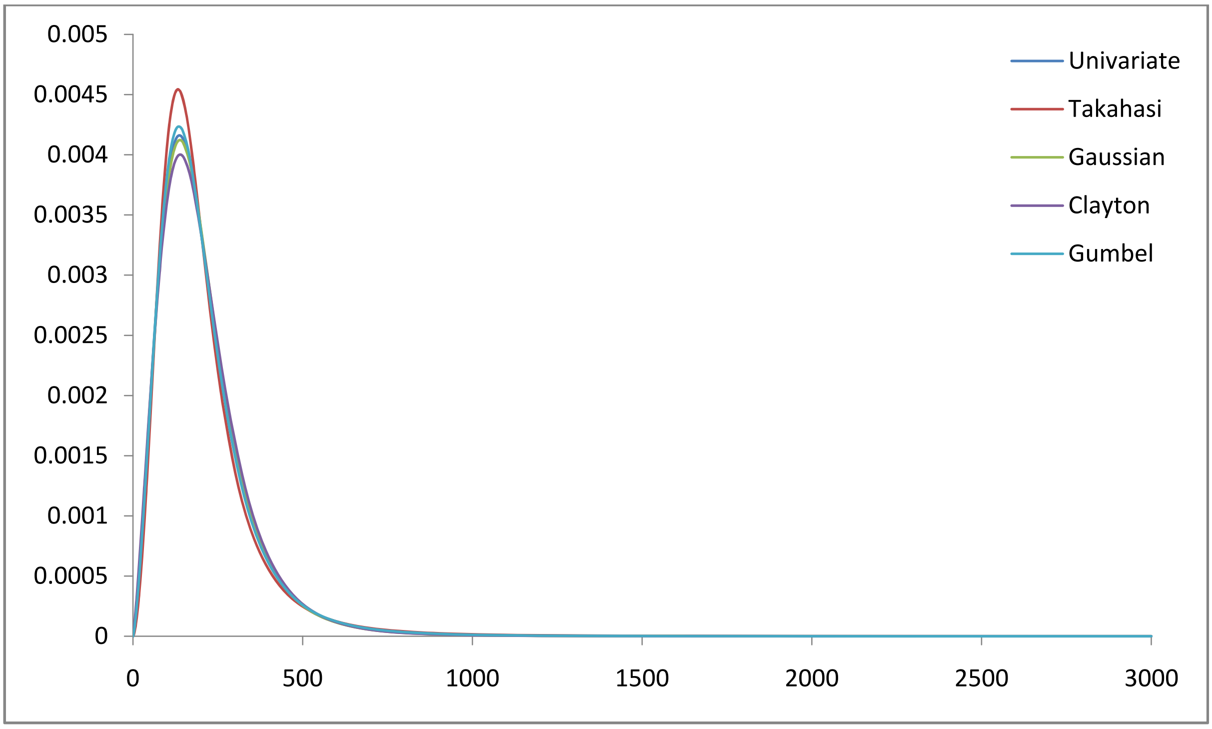

Figure A.1 Estimated income densities of real equivalised disposable income ($'00) for Australia in 2001 for bivariate distributions with two Singh-Maddala marginals

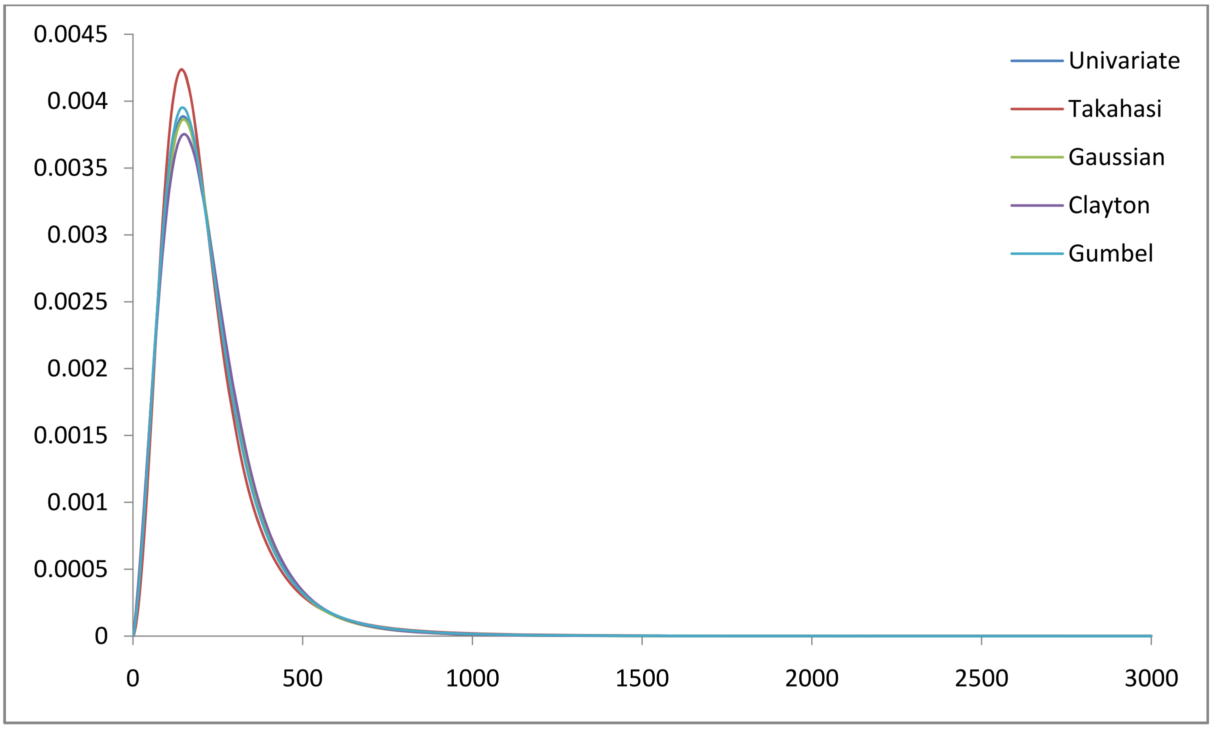

Figure A.2 Estimated income densities of real equivalised disposable income ($'00) for Australia in 2005 for bivariate distributions with two Singh-Maddala marginals

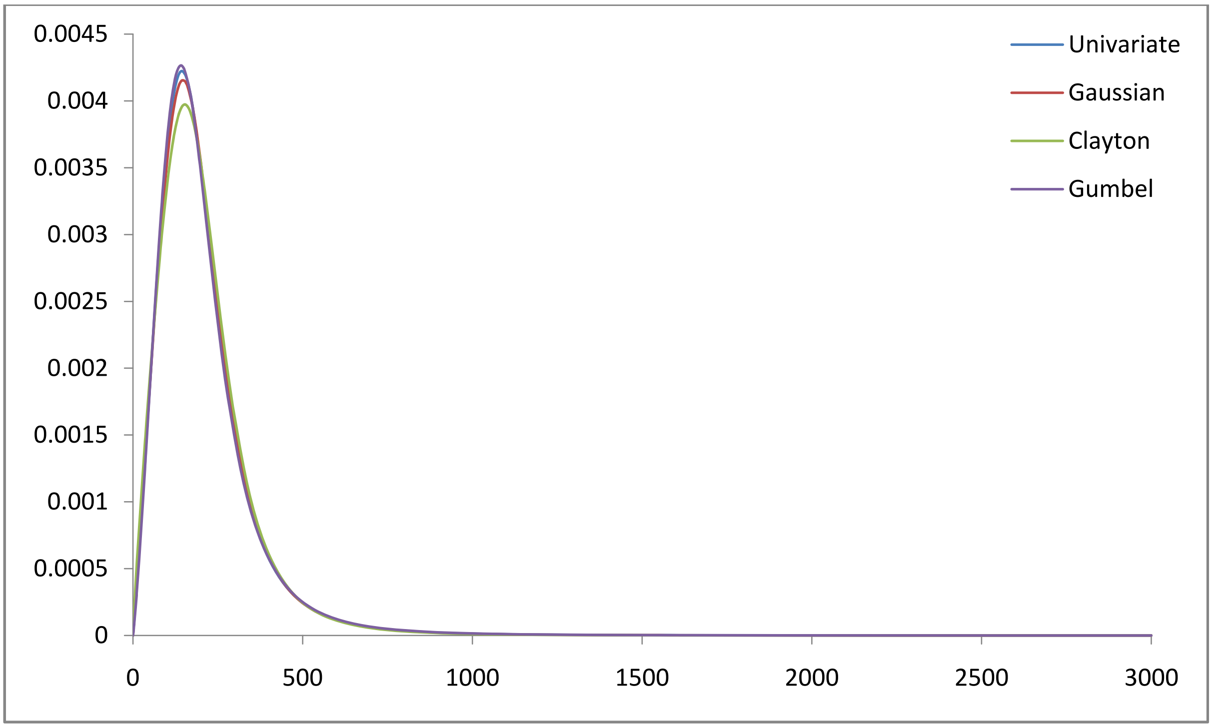

Figure A.3 Estimated income densities of real equivalised disposable income ($'00) for Australia in 2001 for bivariate distributions with two Dagum marginals

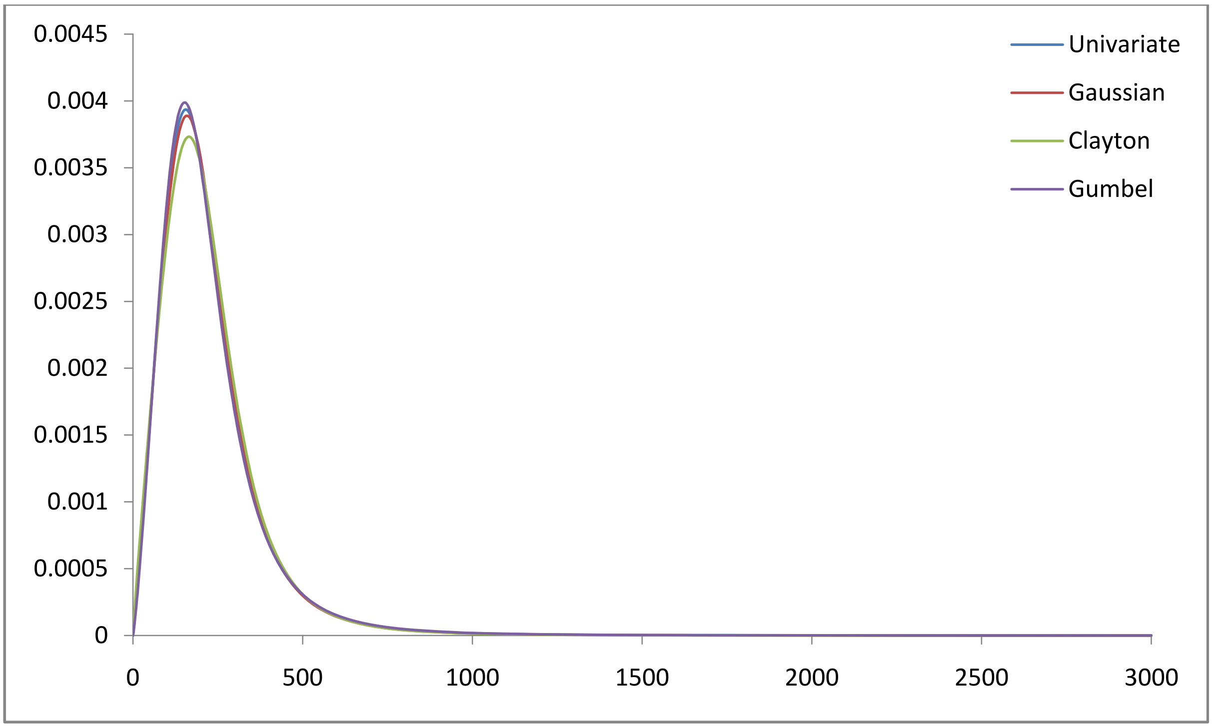

Figure A.4 Estimated income densities of real equivalised disposable income ($'00) for Australia in 2005 for bivariate distributions with two Dagum marginals

REFERENCES

Burr, I. W., 1942, "Cumulative frequency functions", *Annals of Mathematical Statistics*, Vol. 13, pp. 215–232

Clayton, D. G., 1978, "A model for association in bivariate life tables and its application in epidemiological studies of familial tendency in chronic disease incidence", *Biometrika*, Vol. 65, pp. 141–151

Dagum, C., 1977, "A new model of personal income distribution: Specification and estimation", *Economie Appliquée*, Vol. 30, pp. 413–437

Gumbel, E. J., 1960, "Distributions des valeurs extrêmes en plusieurs dimensions", *Publications de l'Institut de Statistique de L'Université de Paris*, Vol. 9, pp. 171-173

Kleiber, C., 1996, "Dagum vs. Singh-Maddala income distributions", *Economics Letters*, Vol. 53, pp. 265-268

Kleiber, C. and Kotz, S., 2003, *Statistical Size Distributions in Economics and Actuarial Sciences*, John Wiley & Sons, New Jersey

Kmietowicz, Z. W., 1984, "The Bivariate Lognormal Model for the Distribution of Household Size and Income", *The Manchester School of Economics and Social Studies*, Vol. 52, pp. 196-210

McDonald, J.B. and Ransom, M.R., 1979, "Functional Forms, Estimation Techniques and the Distribution of Income", *Econometrica*, Vol. 47, No. 6, pp. 1513-1525

McDonald, J.B., 1984, "Some Generalized Functions for the Size Distribution of Income", *Econometrica*, Vol. 52, No. 3, pp. 647-665

Salem, A. B. Z. and Mount, T. D., 1974, "A Convenient Descriptive Model for Income Distribution: The Gamma Density", *Econometrica*, Vol. 42, No. 6, pp. 1115-1127

Sarabia, J. M., Castillo, E., Pascual, M. and Sarabia, M., 2005, “Bivariate Income Distributions with Lognormal Conditionals”, *International Conference in Memory of Two Eminent Social Scientists: C. Gini and M. O. Lorenz*, University of Sienna, Italy

Singh, S.K. and Maddala, G.S., 1976, “A Function for Size Distribution of Incomes”, *Econometrica*, Vol. 44, No. 5, pp. 963-970

Sklar, A., 1959, "Fonctions de répartition à in dimensions et leurs marges", *Publications de l'Institut de Statistique de L'Université de Paris*, Vol. 8, pp. 229-231

Takahasi, K., 1965, “Note on the Multivariate Burr’s Distribution”, *Annals of the Institute of Statistical Mathematics*, Vol. 17, pp. 257–260

Trivedi, P. K. and Zimmer, D. M., 2005, “Copula Modeling: An Introduction for Practitioners”, *Foundations and Trends in Econometrics*, Vol. 1, No. 1, pp. 1-111