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Abstract

This paper looks carefully at situations in which public and private protection are complementary, that is, when private protection must be coordinated with public protection to be effective. For example, home alarms deter theft by being connected to a local police station: if the police do not respond to a home alarm, the home alarm on its own is virtually useless in halting a crime in action. We make a distinction between gross and net complementarity and substitution, where the latter takes into account the effect on the crime rate. We show that when public and private protection are complements the optimal provision of public protection trades offs the manipulation effect of encouraging private protection with the compensatory effect of providing protection to households that do not privately invest. We discuss the implications of our results for policy and empirical research in this area.

JEL classification: H41, H42, K42

Keywords: crime, private protection, policing, externalities

1 Introduction

How should we deploy scarce public resources if public protection of property and private protection of property are complements? Building on previous studies, this paper makes an inquiry into situations of complementarity in public and private protection. By complementarity we mean that the effectiveness of each type of protection is increasing in the presence of the other. We show that very different optimal policy recommendations are generated when public and private protection are substitutes versus when they are complements. As we discuss in detail below, the provision of both public and private protection ultimately creates four types of goods: 1) the public sector provides a public good by lowering crime through means of increased policing - a publicly provided public good, but 2) the public sector also provides a private good by directly protecting households from crime - publicly provided private good. 3) Each household may invest in private protection to protect their own home - a privately provided private good but 4) this may also have an impact on the overall return to crime - a privately provided public good. The optimal deployment of scarce resources depends critically on the technology of protective interventions and trading off the externalities.

Acknowledging that different types of private protection interact differently with public protection, this paper presents a model to study how households and the government can invest according to different situations for the best results. For example, bars on windows or specialty locks do not require police intervention to deter crime - they are substitutable forms of protection. On the other hand, a private home or business alarm is a complement for public protection services. For instance, in the well-publicized April 2015 Hatton Garden gem heist, 200 million GBP worth of jewels were stolen. The police dismissed the burglar alarm at the time of the heist, however, had they responded, they may have been able to catch the robbers in action: the alarm on its own was ineffective without police response. Interestingly, the theoretical literature on the interaction between private and public protection focuses exclusively on situations where public and private protection are technologically substitutes[1].

[1] Another example of strong complementarity in technology is documented in the Ayres and Levitt (1998) study of the impact of Lojack, a system designed for recovering stolen cars. A secret radio transmitter is placed in a car to enable the police to track it once it has been stolen. Because thieves cannot discover the transmitters, 95% of the Lojack-equipped stolen cars are recovered. As a result, this system reduces the profitability of car thefts along with the incidence of them.

There has been a long record of literature leading up to our study of complementarity. Tulkens and Jacquemin (1971) and Clotfelter (1977) were the first to discuss the optimal combination of public and private protection when public protection is a substitute for private protection. Clotfelter (1978), Hotte and van Ypersele (2008), and Baumann and Friehe (2013) investigate externalities generated by private protection and its impact on public policy but they both do not consider what would happen if public and private protection were complements. Helsley and Strange (2005) introduced the concept of a political incentive externality- how private protection influences the objective function of the government. They argue that increased local private policing diverts crime to other targets and reduces the aggregate expenditure on traditional policing, revealing another form of substitutability. Helsley and Strange (1999) explore the competition between gated communities and highlights the potential for strategic complementarity or substitutability. However, they abstract from public protection investment. The only theoretical contribution found to date that formally considers complementarity is Galiani et al. (2018) which considers the decision of land developers to invest in private security under both uniform and targeted public security deployment. Their model does not produce investments in private security when public security is targeted, but when it is uniform high income earners invest in private security to displace crime to unprotected households. Our research complements the Galiani et al. (2018) contribution by considering the specific mechanisms through which externalities generated through public and private investments in the absence of residential location decisions or directed criminal search. Similarly our work complements Fu et al. (2018). They use a general equilibrium model to analyze the effect of police on crime in the absence of private protection choice. They show that the predicted optimal allocation of police across Metro areas in the US depends on unobserved, Metro-specific attributes. They demonstrate a gap in our understanding optimal policy that may be partially filled by the incidence of private security investments.

Lee and Pinto (2009) explore the interaction between private and public protection in a multi-jurisdictions setting. They use a general model with an appropriation technology that could allow for complementarity, but only explore the case where private and public protection are substitutes. Ben-Shahar and Harel (1995) look at the question of incentivizing private protection effort. They acknowledge the possibility that private protection may be a substitute or complement for government effort, but they also assume the lack of complementarity.

Historically, the empirical literature has found support for both substitutability and complementarity. In support of substitutability, Bartel (1975) finds that increased public protective expenditures reduces the demand for private protection. Friedman, Hakim, and Spiegel (1987) also discuss the shifts between public protection and private security by emphasizing the role of community size. Philipson and Posner (1996) find that when public anti-crime activities improved, the proportion of homes with burglar alarms drop significantly. There has also been evidence of complementarity between public and private protection. Focusing on environmental regulation, Langpap and Shimshack (2010) find that private enforcement crowds in public monitoring (complementarity), but crowds out public sanctions (substitutability).

More recently, several studies have highlighted the importance of complementarity in the effectiveness of public good provision, specifically when crime is involved. Albouy et al.(2018) show that public safety, a private good purchased through residential choice decisions is a complement to environmental amenities like public parks: safe parks are a public good, while unsafe parks are a public bad as confirmed by their data from US cities. Blattman et al (2017) presents evidence from randomly assigned police officers and investments in municipal services that targeted public "hot spot" policing itself is not an effective crime reduction policy, but that when paired with local improvements in lighting and cleanliness of streets crime is significantly reduced.

Also related to our contribution are recent studies of crime spillovers that highlight the role of innovative crime-reducing technology adoption and deployment. Amodio (2017) studies spillovers from investment in protection in Buenos Aires. Using a household's knowledge of victimizations outside of an immediate neighborhood as an instrument for individual investment in private security he finds that if neighbors invest in alarms or CCTV households are more likely to also adopt these private security measures. However, if neighbors invest in bars on windows or specialty locks, there is no effect on household decisions to adopt. While this highlights the displacement effect of deterrent activity, it also demonstrates that the response to the externalities generated are specific to the technology used. Our analysis predicts this finding. More private investments in CCTV increases the benefits of public investments in policing, which in turn would induce others to adopt CCTV. With alarms on the other hand, no such positive feedback would be present. Muayo and Rosi (2019) find that installation of police monitored cameras in Montevideo reduce crime in the area of coverage by 20%, and that neighboring areas also benefitted from the reduction in crime, demonstrating the public good nature of specific investments in protection that reduce the value of crime and hence incentives for criminals to target even the unprotected households. Besley and Mueller (2018) use a calibrated model of firm investment in private protection against predation to demonstrate that public and private protection are not subsitutes. In a similar setting, DeAngelo et al (2017) use data from the National Hockey League to study the relationship between public rule enforcement by referees and private rule enforcement in the form of fighting (vigilante justice). They show that fighting only deters rule breaking in the absence of public rule enforcement, while there is complementarity in that referees deter rule breaking with and without private rule enforcement.

We construct a theoretical model to generate new insights, and to impose some structure for the empirical work. The model is a general equilibrium search model in the spirit of Decreuse, Mongrain and van Ypersele (2014). On the supply side of the market of crime, potential perpetrators of criminal activities choose whether or not to commit a crime. On the demand side, households invest in private protection by choosing both the size of their investment and potentially the type of protection. Types of protection vary with respect to their degree of substitutability or complementarity with public protection. The provision of public protection then further depends on the factors that might affect government decisions.

The public sector is composed of an enforcement authority who decides on the allocation of resources. There are potentially two externalities: 1) a diversion externality (negative) where the investment by one household diverts crime to other households. This generally leads to over-investment in private protection, and excessively low crime rates. Then we have 2) a deterrence externality (positive) where households do not consider the effect of their investment on criminal's decisions. This generally leads to under-investment in private protection, as well as an excessively high crime rate. We concentrate on the second type of externality- the deterrence externality- but all results are symmetrically opposite when compared to the first type of externality.

In section two, we develop the theoretical model with one form of public protection and one form of private protection, allowing for either gross complementarity or gross substitutability. We define the relationship between public and private protection as gross substitutes when public investment helps households that are not investing in private protection relatively more than those who are investing in private protection. Similarly, if public investment helps households who are investing in private protection relatively more than those who are not investing, then public and private protection are gross complements. In this case, an increase in public protection increases the incentive to invest in private protection. We also identify when public and private protection can be net substitutes or net complements. That is, when government increases public protection, it reduces the crime rate. With a lower crime rate, there is less incentive to invest in any form of private protection. This endogenously introduces substitutability between private and public investment: forms of protection that are gross substitutes are also net substitutes, but some forms of protection that are gross complements could be net substitutes.

We characterize the constrained first best (CFB) outcome, where government chooses both the levels of private and public protection investment. The first best outcome is trivial as it features no crime at all. We then compare the CFB outcome with a second best (SB) outcome, where the government chooses the level of investment in public protection and households choose their own private investment. In the SB outcome, we show that the crime rate is too high because the deterrence externality is not taken into account by households. This changes government incentives in two ways: 1) The "manipulation" incentives prompt the government to take actions intended to influence the level of investment in private protection to reduce crime. If public and private protection are net complements, the government over-provides public enforcement to push households into providing more private investment. If, instead, protections are substitutes, the government under-provides public protection. 2) The "adjustment" incentives come from the fact that the returns on public investment now differs because there are fewer protected houses. This is similar to portfolio adjustment, when the rates of return change, the optimal investment strategy also changes. As an example, imagine that private and public protections display strong net complementarity. On one hand, the government may over-provide public protection to encourage private investment by exploiting complementarity. This is a way to encourage households to provide more private protection and consequently lowering crime. On the other hand, the return on public investment is lower. Complementarity implies that public investment is more effective for protected houses. The problem is that there are too few of them, so public investment is less effective that it should be. When the return on the publicly provided private good is low, there not much incentive to invest. When the manipulation incentives is stronger, governments should over-provide public protection when investment are net complement and under-provide it when they are net substitute.

In the third section, we look at a richer environment where two forms of private protection are available at the same time - one complement and one substitute. In order to link the theoretical model to some empirical facts, we look at two particular forms of protection which are the most prevalent as recorded in the victimization survey of the 2014 Canadian General Social Survey (GSS): private alarms, which are gross complements for public protection, and bars on windows- which are gross substitutes. This will allow us to look at the distributional impact of public policy as these two forms of private protection are more intensively used by households of different income brackets. All proofs are in the Appendix.

2 Basic Model

The economy is composed of two types of agents and a government. There is a unitary mass of individuals who may be inclined to commit property crime. Each of these individuals chooses between attempting robbery or remaining honest. For expositional reasons, we refer to those individuals as dishonest, even if a proportion choose to commit no crime. Dishonest individuals have heterogenous cost r of committing a crime. We assume that r is uniformly distributed between zero and one. The wealth of dishonest individuals is normalized to zero. Let c be the proportion of dishonest individuals that actually commit crime. Two components determine a criminal's payoff: first, matching between a thief and a potential victim must happen for any crime to take place. We assume a constant return matching technology, where a criminal finds a victim with constant probability ρ. Second, thieves who are successfully matched with a victim steal a certain fraction of the household's wealth. For every dollar stolen the criminal only gets to enjoy $\lambda \leq 1$ of it. Stolen goods are traditionally discounted on the market, and there is the consideration that goods may have sentimental value. There is also the fact that some goods or infrastructure might be broken during a robbery or victimization may impose some psychic cost. The total amount stolen is determined in part by the household's investment in private protection. Thieves who are unsuccessful at finding a suitable victim get a payoff normalized to zero. On the other side of the matching market are the households that are the potential victims. There is a mass H of households for whom the cost of committing a crime is arbitrarily large. Households are heterogenous in wealth w, which is uniformly distributed between zero and H. Each household is matched with a criminal with probability $\rho c/H$. A household loses a proportion $\ell_i(u)$ of their wealth, where $i = 0$ if the household is unprotected and $i = 1$ if the household invested in private protection. Private protection reduces potential losses, where the difference in expected losses is given by $\Delta(u) = [\ell_0(u) - \ell_1(u)] > 0$. Losses are also affected by public protection u. Public enforcement reduces private losses, so $\ell_i'(u) < 0$. We assume that $\ell_i''(u) > 0$. For expositional purposes, we assume that $\ell_i'(1) \to 0$, so that $u \in [0, 1]$.

Definition 1: *Public and private protection are gross complement (resp. gross substitute if* $\Delta'(u) >$ *(resp.* $<$*)*0

When $\Delta'(u) > 0$ public protection help relatively more privately protected households than non-protected ones. This generates gross complementarity between public and private protection. When $\Delta'(u) < 0$, unprotected households benefit relatively more, so public and private protection are gross substitutes. Obviously, the neutral case is given when $\Delta'(u) = 0$.

Private protection carries a fixed cost F. Denote by η the proportion of households who make the investment in private protection.

Let the government choose the level of public protection, u. There are two components to the cost of investment in public protection, where the total cost is given by Γu. We assume that the government maximizes the sum of honest agents' welfare. The timing is simple: the government chooses u, and then households choose whether or not to invest in private protection. Given these parameters, dishonest individuals choose whether or not to commit a crime. We now proceed to looking at the component of household choice when considering investment.

2.1 Private Protection Investment

A household with wealth w invests in private protection if the marginal benefit is larger or equal to the investment cost F. Since the marginal benefit is strictly increasing with wealth, households with wealth $w \geq \bar{w}(c, u)$ will invest in private protection where:

$$\bar{w}(c, u) = \frac{FH}{\rho c \Delta(u)}. \tag{1}$$

An increase in the matching probability ρ, or an increase in the number of active criminals c increases investment in private protection. The sign of $\bar{w}_u(c, u)$ depends on the sign of $\Delta'(u)$. When $\Delta'(u) > 0$, public enforcement promotes investment in private protection since $\bar{w}_u(c,u) < 0$, which means that public and private protection are gross complements. When $\Delta'(u) < 0$, public enforcement reduces private protection, $\bar{w}_u(c,u) > 0$: public and private protection are gross substitutes. Note that these gross effects are for a given crime rate. At equilibrium, the crime rate will depend on public protection so the net effect may be different.

2.2 Criminality Decision

Dishonest individuals attempt to commit a robbery if and only if the expected return is larger than the cost r. Let $R(\bar{w}(c,u),u)$ be the expected expected amount stolen from a random household. Dishonest individuals with cost $r < \bar{r}$ attempt to commit a robbery, where $\bar{r} = \rho\lambda R(\bar{w}(c,u),u)$. This amounts to:

$$R\big(\bar{w}(c,u),u\big) = \ell_0(u)\int_0^{\bar{w}(c,u)} \frac{w}{H}dw + \ell_1(u)\int_{\bar{w}(c,u)}^{H} \frac{w}{H}dw. \tag{2}$$

Lemma 1 describes the equilibrium investment in private protection and the dishonest individuals' criminality decisions.

Lemma 1: *The equilibrium number of dishonest individuals attempting a robbery $c(u)$ and the equilibrium protection income threshold $\bar{w}(u)$ are determined by the following two equations:*

$$c(u) = \rho\lambda R\big(\bar{w}(u),u\big) = \rho\lambda\ell_0(u)\int_0^{\bar{w}(u)} \frac{w}{H}dw + \rho\lambda\ell_1(u)\int_{\bar{w}(u)}^{H} \frac{w}{H}dw; \tag{3}$$

$$\bar{w}(u) = \frac{FH}{\rho c(u)\Delta(u)}. \tag{4}$$

Crime rate is then given by $\rho c(u)/H$.

The proportion of households who invest in private protection is then given by $\eta(u) = \int_{\bar{w}(u)}^{H}(1/H)dw$. A change in public protection has an ambiguous effect on private protection. As stated in Proposition 1 below, public protection displaces private protection when the elasticity of $\Delta(u)$ is smaller than the elasticity of $R(\bar{w}(u),u)$. This condition differs from the gross substitution condition $\Delta'(u) < 0$ stated before because crime rate is no longer given. An increase in public protection reduces crime, which reduces the incentive to invest in private protection. The net substitutability is a condition on the equilibrium it is formally defined as follows:

Definition 2: *Public and private protection are net complements (resp. gross substitutes) if* $\frac{d\bar{w}}{du} <$ *(resp.* $>$*)*0 Only if public and private protections are sufficiently gross complements will private protection respond positively to public protection.

Proposition 1: *Let*

$$\varepsilon(\Delta|u) = \frac{u\ \Delta'(u)}{\Delta(u)} \text{ and } \varepsilon(R|u) = \frac{-u}{R(\bar{w}(u), u)} \frac{\partial R(\bar{w}(u), u)}{\partial u},$$

and denote by $\bar{\Delta}'(u)$*, the value of* $\Delta'(u)$ *such that* $\varepsilon(\Delta|u) = \varepsilon(R|u)$*. Then public and private protections are net complements iff* $\varepsilon(\Delta|u) > \varepsilon(R|u)$ *otherwise, both forms of protections as net substitutes.*

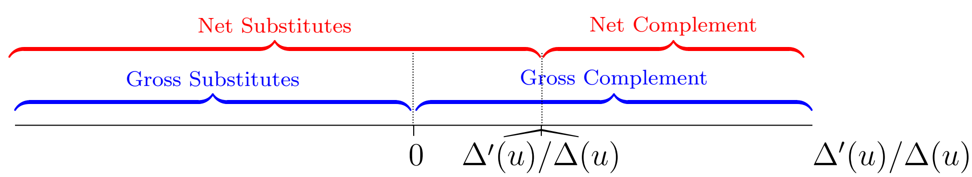


Figure 1: Net and Gross Substitute/Complement

When both forms of protection are gross substitutes, they are also net substitutes. Figure 1 highlights the difference between net and gross substitutes/complements. The opposite is not necessarily true. Intuitively, Proposition 1 comes down how the expected reduction in losses generated by private protection changes with u.[2] For example, if $\rho c(u)\Delta(u)$ is independent of u, then public and private protection are net neutral. The reduction in the probability of victimization is exactly compensated by the increase in private protection effectiveness. When the latter is smaller than the effect on crime rate, then both form of protections are net substitutes and vice versa.

[2]Proposition 1 takes as given that crime rate is decreasing with public enforcement, which is true in most cases. If private and public protections were strong substitutes, an increase in public enforcement could potentially lead to a large reduction in private protection such that the return to robbery increases. While conceptually interesting, this special case is not realistically relevant. We assume away this possibility. A sufficient condition for the equilibrium number of dishonest individuals attempting a robbery $c(u)$ to be decreasing with public protection is that $-\ell_1'(u) > -\ell_0'(u)/2$.

2.3 Constrained First Best

Obviously, the best possible outcome would be a world without crime and no protection spending. For practical reasons, we will not assume such a case. We define instead the constrained first best (CFB) as the outcome obtained by a government who maximizes the sum of all honest agents utility by choosing the level of public protection u and the set of households who invest in private protection $\tilde{w}$. Note that a government could potentially manipulate $\bar{w}(u)$ via taxes or subsidies to get to the desired $\tilde{w}$. This objective function is most commonly used in the literatures of law and economics.[3] The CFB problem face by the government is:

$$\max_{\{u,\tilde{w}\}} \left\{ \frac{H^2}{2} - \rho c(\tilde{w},u)\ell_0(u) \int_0^{\tilde{w}} \frac{w}{H} dw - \rho c(\tilde{w},u)\ell_1(u) \int_{\tilde{w}}^{H} \frac{w}{H} dw \right.$$
$$\left. - HF \int_{\tilde{w}}^{H} \frac{1}{H} dw - \Gamma u \right\}, \tag{5}$$

where $c(\tilde{w},u) = \rho\lambda R(\tilde{w},u)$ is the number of dishonest individuals attempting a robbery given $\tilde{w}$ and public protection u.

Definition 3: *The CFB investment in private protection $\tilde{w}^\star$ and in public enforcement $u^\star$ are respectively given by:*

$$\underbrace{\frac{\rho c(\tilde{w}^\star,u^\star)\Delta(u^\star)}{H}\left[\tilde{w}^\star - \frac{FH}{\rho c(\tilde{w}^\star,u^\star)\Delta(u^\star)}\right]}_{\text{Net private MB of lower losses(PvPPvG)}} + \underbrace{\rho R(\tilde{w}^\star,u^\star)\frac{\partial c(\tilde{w}^\star,u^\star)}{\partial \tilde{w}}}_{\text{Social MB of lower crime(PvPPuG)}} = 0; \tag{6}$$

$$\underbrace{-\rho c(\tilde{w}^\star,u^\star)\frac{\partial R(\tilde{w}^\star,u^\star)}{\partial u}}_{\text{Private MB of lower losses(PuPPvG)}} - \underbrace{\rho R(\tilde{w}^\star,u^\star)\frac{\partial c(\tilde{w}^\star,u^\star)}{\partial u}}_{\text{Social MB of lower crime(PuPPuG)}} = \underbrace{\Gamma.}_{\text{MC of public protection}} \tag{7}$$

The first equation represents the optimal allocation of private protection. On the left hand side, we find the sum of all net private marginal benefits associated with lower losses net of the cost of investment in private protection. This is a privately-provided private good (PvPPvG). We must also add the public good aspect of private protection. More investment in private protection leads to lower crime rates. This is commonly referred to in the literature as the deterrence effect of private protection and is a privately-provided public good (PvPPuG).

[3]See Hotte and van Ypersele (2008) for a discussion.

The investment in public protection is also governed by public and private benefits. On the left hand side of the second equation is the private benefit of public investment, or the publicly-provided private good (PuPPvG). When public investment increases, all households face lower expected losses for a given crime rate. We can also find on the left hand side, the public good aspect of the public enforcement arising from a lower crime rate, and is a classic publicly-provided public good (PuPPvG). On the right hand side is the marginal cost of public protection.

Proposition 2: *The decentralized income threshold $\bar{w}(u^\star)$ for investing in private protection is higher than the CFB level $\tilde{w}^\star$. Consequently, the decentralized equilibrium features under-provision of private protection.*

When comparing the decentralized outcome with the optimal one, an important externality must be considered. Households do not take in consideration the effect of their own protection decision on the crime rate. At the optimum, external marginal benefit must equal the marginal cost of providing private protection F.

2.4 Second Best

We now look at public enforcement when private protection is not controlled by the government, but chosen directly by households. In such a case, the government problem is:

$$\max_{\{u\}} \left\{ \frac{H^2}{2} - \rho c\big(\bar{w}(u), u\big)\ell_0(u) \int_0^{\bar{w}(u)} \frac{w}{H} dw - \rho c\big(\bar{w}(u), u\big)\ell_1(u) \int_{\bar{w}(u)}^{H} \frac{w}{H} dw - F \int_{\bar{w}(u)}^{H} dw - \Gamma u. \right\} \tag{8}$$

The solution to this problem will be referred to as a second best (SB) solution. The goal is to understand in what manner this solution differs from the CFB solution and what is the role of complementarity and substitutability of the private and public investment.

Definition 4: *The SB investment in public enforcement, u^*, is given by:*

$$\underbrace{-\rho c(u^*)\frac{\partial R\big(\bar{w}(u^*),u^*\big)}{\partial u}}_{\text{Private MB of lower losses at } \bar{w}(u^*)>\tilde{w}\text{(PuPPvG)}} - \underbrace{\rho R\big(\bar{w}(u^*),u^*\big)\frac{\partial c\big(\bar{w}(u^*),u^*\big)}{\partial u}}_{\text{Social MB of lower crime for } \bar{w}(u^*)>\tilde{w}\text{(PuPPuG)}}$$
$$- \underbrace{F\frac{d\bar{w}(u^*)}{du}}_{\text{Social benefit of manipulating private protection (PvPPuG)}} = \underbrace{\Gamma.}_{\text{MC of public protection}} \tag{9}$$

The first two terms in Definition 4 represent the same reasons for investing in public protection as in the first best situation. However, it is evaluated at a sub-optimal investment in private protection. The last term on the left hand side of Definition 4, represents the fact that the government may alter public enforcement to manipulate private investments in a more desirable way. The marginal benefit of promoting private protection must equal to F, its marginal cost. On the right hand side of Definition 4, we have the marginal cost of public enforcement.

Using Definition 4, we can characterize differences in public enforcement, private protection and crime rates between the constrained first best and the second best. We start by looking at one expression derived from the proof of Proposition 2. It simply describes the difference between the SB first order condition on u and the similar first order condition for the CFB, both evaluated at the CFB solution $u^\star$. The expression is composed of two important part:

$$\underbrace{2\rho^2\lambda\left[R\big(\bar{w}(u^\star),u^\star\big)\frac{-\partial R\big(\bar{w}(u^\star),u^\star\big)}{\partial u} - R(\tilde{w}^\star,u^\star)\frac{-\partial R(\tilde{w}^\star,u^\star)}{\partial u}\right]}_{\text{Adjustment for under-investment in private protection (PuPPvG and PuPPuG)}}$$
$$\underbrace{-F\ \frac{d\bar{w}\big(u^\star\big)}{du}}_{\text{Net social MB of manipulating private protection (PvPPuG - fiscal externality)}} \lesseqgtr 0. \tag{10}$$

Whenever the expression above is positive, the government should provide more public enforcement in the second best environment compared to what should be done in the CFB. That is, the government should over-invest in public enforcement. When the expression is negative, the government should under-invest. The term on the second line represents the "manipulation" incentives faced by the government: the incentive to manipulate the private protection decision of households. Crime rate is affected by both private and public protections. Because households do not internalized the effect of private protection on crime, the government wish to stimulate private protection so to reduce crime rate. The marginal benefit additional investments in private protection is simply equal to its marginal cost F. Isolating this channel, when public and private protection are net complements, the government would like to over-invest in protection to stimulate private protection. When public and private protection are net substitutes, the government would like to under-invest in protection to force households to invest more in private protection on their own.

The term on the first line represent the adjustments made by the government to account for the under-investments in private protection. The government must adapt public investment to take into consideration the fact that there are fewer protected houses. The marginal benefit of the publicly provided private good is different because of the difference between protected and non-protected houses. Do the adjustment incentives call for more or less protection? Because there are fewer protected houses, the sign of $\Delta'(u)$ matters for whether the government desires more or less public protection. With gross complementarity $\Delta'(u) > 0$, protected houses benefit relatively more than unprotected one. The return on public investment under the SB is then lower than under the CFB. Public protection help relatively more the protected households, but the problem is that there are fewer protected households. Consequently, the government would be inclined to invest less, because public protection is not as effective. On the contrary, if $\Delta'(u) < 0$, unprotected households benefit relatively more from public investment, so government should over-invest in public protection to help all the additional unprotected households.

At the same time, when the crime rate is higher, so is the probability of victimization. This implies naturally that public investment is more desirable. Consequently, even if $\Delta'(u) > 0$, the government may want to over provide public enforcement. Yes, there are few protected houses to help, so return on u is lower. But at the same time, the probability of victimization is higher, so public protection is more desirable.

Proposition 3: *With a first degree approximation, when public and private protection are net neutral, the optimal investments in public protection are the same in the SB and in the CFB, but crime rate is higher for the CFB. Whenever,* $1 > 2\left[1 + \epsilon(R|w)\right] \frac{\bar{w}(u^\star) - \tilde{w}^\star}{\bar{w}(u^\star)}$*, then SB would call for over-investment in public protection if public and private protection are net complements, while asking for under-investment in public protection if both forms of protection are net substitutes. If the sign of the condition above is reverse, the opposite applies. In both cases, crime rate would be higher.*

Given the complexity of (10) it is difficult to obtain clear insights without taking additional steps. In Proposition 3, we take a first degree approximation of the adjustment incentives. Later on, we investigate the bias it creates.

With net neutrality, the government is unable to manipulate behaviour. With the linearization, however, the marginal return on public protection effort is independent of private protection investment level. In this case, there is simply nothing to do differently for the government. Crime rate is higher because private protection is lower, but public investment remain the same. Starting from net neutrality, if investments becomes more complementary, the manipulation incentives call for additional public investments, but the adjustment incentives suggests to shy away from public investment because the return is too low. The value of manipulating private investments relative to adjusting for the under-provision of private investments determines the appropriate response from the government.

If we concentrate on the first order effects, Proposition 3 shows that the manipulation incentives dominate whenever the condition highlighted is satisfied. This requires that $\epsilon(R|w)$ is sufficiently small. In such a case, private response to public investment is large, leading to strong manipulation incentives. It must also be the case that $\bar{w}(u^\star) - \tilde{w}^\star$ is sufficiently small, so that the need to adjust for the lack of private investments is limited.

Corollary to Proposition 3: *When taking into account the bias introduced by the first degree approximation, there should be over-provision of public enforcement when public and private protections are net neutral. Furthermore, if* $1 < 2\left[1 + \epsilon(R|w)\right] \frac{\bar{w}(u^\star) - \tilde{w}^\star}{\bar{w}(u^\star)}$, *there should definitively be the over-provision of public protection when protection is gross neutral.*

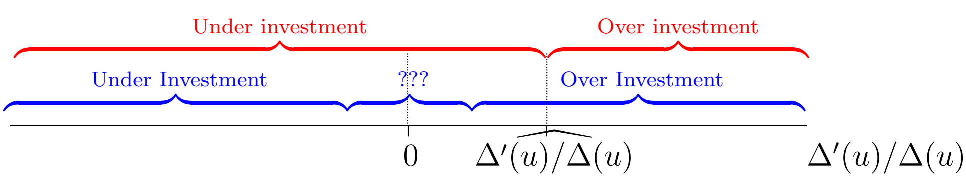


Figure 2: Investment in public protection for $1 > 2\left[1 + \epsilon(R|w)\right] \frac{\bar{w}(u^\star) - \tilde{w}^\star}{\bar{w}(u^\star)}$

The corollary to Proposition 3 assesses the direction of the bias created by the first order approximation used in Proposition 3. Put simply, the first order approximation neglects some of the impacts of an excessively high crime rate. The direction of the bias depends on whether public and private enforcements are complement or substitute. Where both forms of investment are net neutral, obviously the government cannot alter private behaviour by over or under investing. Whether the return on public protection investment is higher or lower is less obvious. When investments are net neutral, they are gross complement, meaning that public investment help relatively more the protected households. Since there are fewer protected houses in the SP, public investments is less desirable relative to the CFB. Put simply, the return on the publicly provided private good is lower. However, crime rate is higher, and more protection is desirable because it is more often necessary. The need for the publicly provided public good is then stronger. With linearization, those two effects exactly cancel out. Because the effects on crime rates works thru feed back channels, some of its value is ignored with the approximation.

Figures 2 and 3 also help understand further implications of the bias. The red brackets show the public investment decisions under the linear approximation, while the blue brackets represent the same decisions when taking the bias into consideration. Take a case where the manipulation incentives are dominant. Figure 2 shows that the actual transition (in blue) from under-investment to over-investment happen when private and public protection are net substitute and even potentially gross substitute. When $\Delta'(u) = 0$, the gross return on public enforcement is independent of the number of protected houses. That leaves only two considerations. On one hand, the government would like stimulate private protection by investing less and exploiting net substitutability. On the other hand, crime rate is too high and government wants to invest more in public protection because it is more often required. When the adjustment incentives dominates, government may want to over-provide public provision even if there are fewer houses that are protected as we can see in Figure 3. Again, an excessively high crime rate justifies higher public investment.

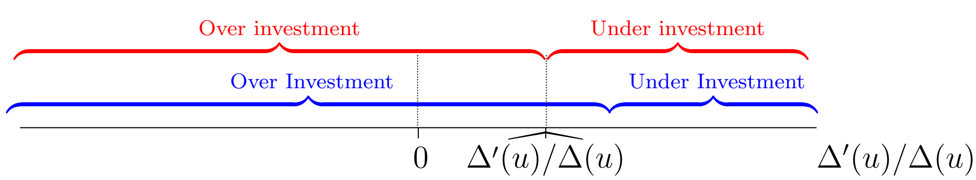


Figure 3: Investment in public protection for $1 < 2\left[1 + \epsilon(R|w)\right] \frac{\bar{w}(u^\star) - \tilde{w}^\star}{\bar{w}(u^\star)}$

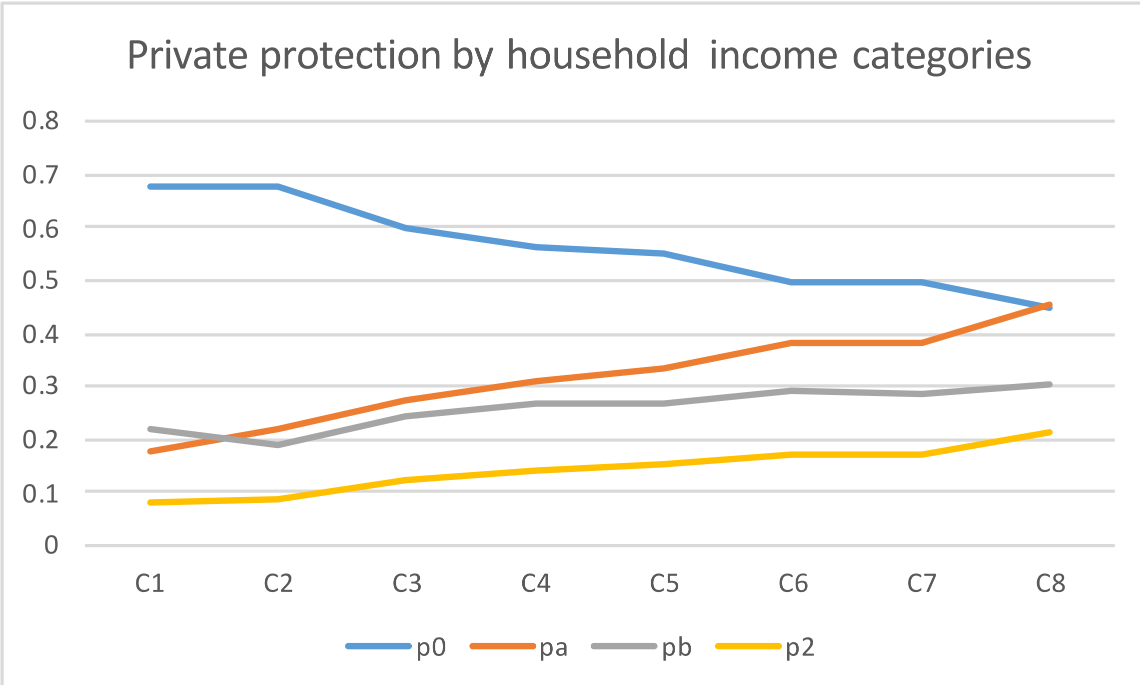
3 Multiple Forms of Private Investments

The preceding exercise allows us to derive some results about optimal public enforcement policy in a tractable fashion. In this section, we link the different forms of substitutability and complementarity with observable forms of private protection. To fully capture the interaction between public and private enforcement, we now introduce two different forms of private protection. Because of the technological differences, but also because of their prevalence and the information available in victimization survey of the Canadian General Social Survey (GSS), we look at alarms as one of the private investments, with the other being bars on windows or specialty locks.

The Canadian GSS provide us with cross-sectional individual data on victimization. We used the last available survey, cycle 28 done in 2014. The survey samples dwellings, and the respondent may be anyone belonging to the household. We only keep the following types of households: singles or couples with or without children. This excludes less traditional households with additional relatives and non relatives at home. Such households lead to never-ending questions as to the identity of wage-earners and the respondent's statute in the household. We also exclude observations where the respondent is a child. Household income is not well reported in such cases. There are four category of investments in private protection. We abstract from the one where individuals are asked if they are changing their habits to lower their exposure to crime. Not only is the question vague, but the variable is almost independent of an individual's characteristics. We also abstract from dogs as a method of protection because of their limited use. In the survey, respondents declare whether they have ever installed an alarm and if they have ever installed locks or bars. Picture 1 shows the investment by income categories used in the survey.

With the two forms of private protection, we end-up with four types of households: (p0) are the households who have no protection at all, (pb) are the households with only bars/locks as a form of protection, (pa) are the households with only alarms and (p2) are the households with both form of protections. Below, we present a model with reasonable assumptions that generates results consistent with the relationship we observe in the data between household income and protection.

A household that does not invest in any form of protection suffers a loss of $\ell_0(u)$ when targeted as in the previous section. Technology A requires police response to be effective. An alarm system is the perfect example of such technology. The indicator



function $a \in \{0, 1\}$ determines if a household should invest in such a technology. If a household invests in protection A, the loss is then given by $\ell_0(u) - \Delta_a(u)$, where $\Delta_a'(u) > 0$. Public enforcement reduces expected losses for all households, but alarm-protected households receive an additional benefit from public responses to alarms. The second form of private protection B, we assume that the effectiveness of such form of private protection is independent of public protection. Bars on windows or specialty locks would be a good examples. The indicator function $b \in \{0, 1\}$ determines if the individual invests in this second technology. Investments in this form of protection reduces losses by $\ell_0(u) - \Delta_b$.[4] Investment in technology $i \in \{a, b\}$ costs F_i. We assume that alarms are more costly, so $F_a > F_b$. A thief matched with a

[4]One could argue that bars or locks may in fact help unprotected houses relatively more. As we saw in the previous section however, gross substitutes and gross neutral are both cases of net substitutability.

household imposes a loss $L(a,b;u)w$. The loss function is defined the following way:

$$L(a,b;u)=\begin{cases}\ell_0(u) & \text{if } a=0 \text{ and } b=0;\\ \ell_0(u)-\Delta_a(u) & \text{if } a=1 \text{ and } b=0;\\ \ell_0(u)-\Delta_b & \text{if } a=0 \text{ and } b=1;\\ \ell_0(u)-\Delta_a(u)-\Delta_b+D & \text{if } a=1 \text{ and } b=1,\end{cases} \tag{11}$$

where $0\leq D<\Delta_b$ represents the fact that the marginal benefit of installing a given protection measure may be lower when the alternative measure is already installed. In other words, there may be redundancy between the two forms of protection. We assume that $\Delta_a(u)\in\left[\Delta_b\frac{F_a}{F_b}-D\frac{F_a-F_b}{F_b},\Delta_b\frac{F_a}{F_b}\right]$ for all relevant values of u. Whenever this condition is not satisfied, no household invests solely in protection A or solely in protection B. Since these cases are empirically irrelevant, we ignore such possibility.

Denote by $N_i(c,u)$ the mass of individuals who invest a positive amount in technology $i\in\{a,b\}$ to be determined later.

3.1 Investments in Private Protection

A household with wage w, maximizes utility $W(a,b;w)$, by choosing investment a and b, where:

$$W(a,b;w)=w-\frac{\rho c}{H}L(a,b;u)w-aF_a-bF_b. \tag{12}$$

Define $\bar{w}_{\{0,a\}}(c,u)$ as the revenue such that an individual is indifferent between having protection of type A and having no protection. Define $\bar{w}_{\{b,ab\}}(c,u)>\bar{w}_{\{0,a\}}(c,u)$ as the revenue such that an individual is indifferent between having both from of protection and having only protection B. Similarly, we define $\bar{w}_{\{0,b\}}(c)$ and $\bar{w}_{\{a,ab\}}(c)>\bar{w}_{\{0,b\}}(c)$ for the investment in protection B. The different revenues cut-offs are given by:

$$\bar{w}_{\{0,a\}}(c,u)=\frac{HF_a}{\rho c\Delta_a(u)} \quad\text{and}\quad \bar{w}_{\{b,ab\}}(c,u)=\frac{HF_a}{\rho c[\Delta_a(u)-D]};$$

$$\bar{w}_{\{0,b\}}(c)=\frac{HF_b}{\rho c\Delta_b(u)} \quad\text{and}\quad \bar{w}_{\{a,ab\}}(c)=\frac{HF_b}{\rho c(\Delta_b(u)-D)};$$

Define $\bar{w}_{\{a,b\}}(c,u)=\frac{H[F_a-F_b]}{\rho c[\Delta_a(u)-\Delta_b(u)]}$ as the income level such that investing only in A is equivalent to investing only in B.

Lemma 2: *Investment in private protection as a function of income is given by:*

$$\begin{cases} \text{none} & \text{if } w \leq \bar{w}_{\{0,b\}}(c); \\ \text{only } B & \text{if } \bar{w}_{\{0,b\}}(c) < w \leq \bar{w}_{\{a,b\}}(c,u); \\ \text{only } A & \text{if } \bar{w}_{\{a,b\}}(c,u) < w \leq \bar{w}_{\{a,ab\}}(c); \\ \text{both } A \text{ and } B & \text{if } w > \bar{w}_{\{a,ab\}}(c). \end{cases}$$

Total investments are $N_A(c,u) = \frac{1-\bar{w}_{\{a,b\}}(c,u)}{H}$, *and* $N_B(c,u) = \frac{\bar{w}_{\{a,b\}}(c,u)-\bar{w}_{\{0,b\}}(c)+1-\bar{w}_{\{a,ab\}}(c)}{H}$.

To understand Lemma 2, we refer to Figure 4. The red line corresponds to the lower contour of the sum of losses associated with theft net of expenditures on private protection. Naturally, low income individuals do not invest in protection at all. Above a certain point, individuals are wealthy enough to invest in the cheaper form of investment. Above that, individuals are ready to chose the more efficient, but more expensive investment A. The wealthiest individuals invest in both.

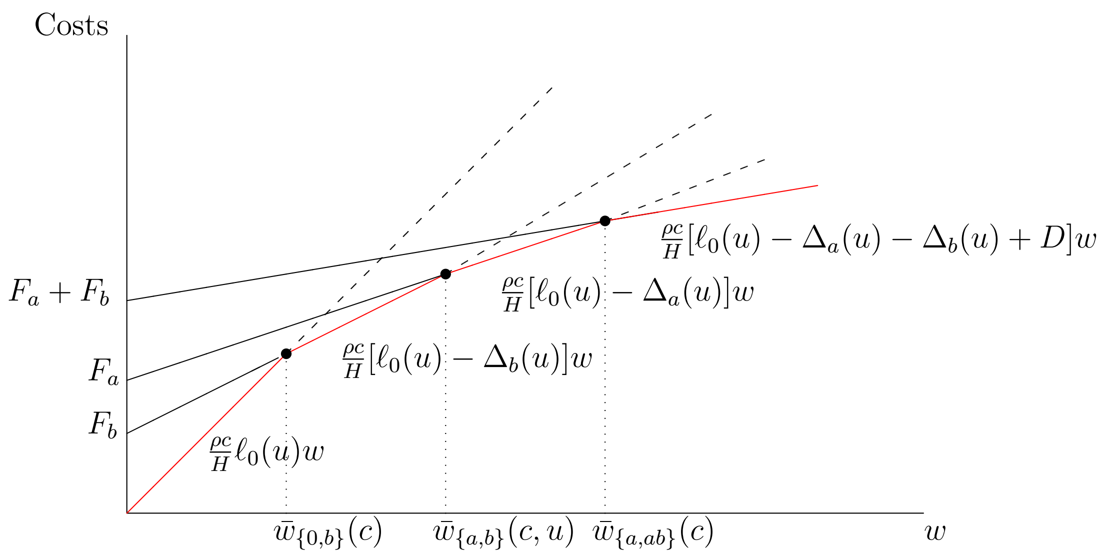


Figure 4: Private Protection Investments.

Taking crime rate as constant, public protection and private investment in protection A are gross complements. The driving force of this complementarity is the fact that public spending benefits relatively more individuals who invest in A relative to the ones who do not, or simply $\Delta'_a(u) > 0$. Interestingly, public protection and private investment in B are gross substitutes despite $\Delta'_b(u) = 0$. Investments in public protection u displaces private investment in protection B, even if it does not directly influence the return on such investments. An increase in u increases investment in protection A relative to protection B. As public investment on protection increases, $\bar{w}_{\{a,b\}}(c,u)$ moves to the left on Figure 4. Lemma 3 below summarizes the effect of a change in u on the prevalence of both types of private protections.

Lemma 3: *For a given crime rate, the effects of a change in public protection u on the prevalence of private protection A and B is the following:*

$$\frac{\partial N_A(c,u)}{\partial u} = \frac{(F_a - F_b)\Delta_a'(u)}{\rho c[\Delta_a(u) - \Delta_b]^2} > 0 \text{ and } \frac{\partial N_B(c,u)}{\partial u} = -\frac{(F_a - F_b)\Delta_a'(u)}{\rho c[\Delta_a(u) - \Delta_b]^2} < 0.$$

We will now discuss the impact of a change in the number of criminals, c, on investment on private protection A and B. An increase in the number of criminals augments the expected benefit of investing in both forms of private protection, but it also increases the relative expected benefit of investing in protection A.

Lemma 4: *Higher crime rate ($\rho c/H$) stimulates investments in private protection A for all parameters value and in B if and only if*

$$\frac{F_a}{F_b} < 1 + \left[\frac{\Delta_a(u) - \Delta_b}{\Delta_b}\right]\left[\frac{2\Delta_b - D}{\Delta_b - D}\right]. \tag{13}$$

Low-to-mid income individuals invest only in protection B, while mid-to-high income individuals invest only in protection A. High income individuals invest in both. An increase in the number of criminals induces additional low income households, who were previously not investing in protection, to investment in B and more high income individuals, who were previously investing only in A, to invest in both. However, additional mid-income individuals invest in A instead of B. The investment in protection A is more productive than B, but more costly. As the probability of victimization increases, the benefit increases relative to the high cost. Regardless, total private protection increases. All households that switch from B to A choose a more productive, but more costly form of protection.

3.2 Equilibrium Crime Rate and Private Protection

From Lemma 2, we can define $a(w;c,u) \in \{0,1\}$ and $b(w;c,u) \in \{0,1\}$ as the private protection investment choices by an individual with wealth w, given public policy u

and crime intensity c. A dishonest individual who chooses thievery has an expected payoff of $\rho\lambda R\left(\bar{W}(c,u),u\right)$ where $\bar{W}(c,u)$ is a vector of investment decisions in private protection A and B. The expected returns is given by:

$$R\left(\bar{W}(c,u),u\right) = \ell_0(u)\frac{H}{2} - \Delta_a(u)\int_{\bar{w}_{\{a,b\}}(c,u)}^{H}\frac{w}{H}dw - \Delta_b\int_{\bar{w}_{\{0,b\}}(c,u)}^{\bar{w}_{\{a,b\}}(c,u)}\frac{w}{H}dw - (\Delta_b - D)\int_{\bar{w}_{\{a,ab\}}(c,u)}^{H}\frac{w}{H}dw \tag{14}$$

Lemma 5: *The equilibrium number of dishonest individuals attempting a robbery $c(u)$ and the equilibrium protection income threshold vector $\bar{W}(u)$ are determined by the following four conditions:*

$$c(u) = \rho\lambda R\big(\bar{W}(u),u\big);$$

$$\bar{w}_{\{0,b\}}(u) = \frac{HF_b}{\rho c(u)\Delta_b},\ \ \bar{w}_{\{a,b\}}(u) = \frac{H[F_a - F_b]}{\rho c(u)[\Delta_a(u) - \Delta_b]}\ \text{and}\ \bar{w}_{\{a,ab\}}(u) = \frac{HF_b}{\rho c(u)(\Delta_b - D)}.$$

Crime rate is then given by $\rho c(u)/H$.

Since public protection and private protection B are gross substitutes, it is not surprising to learn that they are also net substitutes. Below is the condition for net complementarity for public protection and private protection A. The condition is similar to the condition with only one form of protection, except that net complementarity is now easier to achieve. The right hand side of the condition below shows that as public protection increases, crime rate is lower and so there is less incentive to invest in any form of protection. The right hand side is gross elasticity of technology A multiplied by a number larger than one, making it easier for gross complementarity to dominate the net effect associated with an increase in crime rate. With only one form of protection, net neutrality implies that the effect of a lower crime rate exactly offsets the effect of more effective private protection. With two forms of private protection the relative merit of both investment also enter into the decision making process. With additional public protection spending, not only protection A is more effective, but protection B become relatively less desirable. Consequently, a technology that would be net substitute (or net neutral) taken in isolation can be net complement when a gross substitute technology is also available. As we can see in the condition stated in Proposition 4, if we render protection B irrelevant ($\Delta_b = 0$ and $D = 0$), then the condition would be the same as in Proposition 1.

Proposition 4: *Public protection and private protection B are always net substitutes, while public protection and private protection A are net complements if and only if:*

$$\epsilon(\Delta_a|u)\frac{\Delta_a(u)}{\Delta_a(u)-\Delta_b}\left[1+\frac{R\left(\bar{W}(u),u\right)}{H}\left[\Delta_b\bar{w}_{\{0,b\}}(u)^2+(\Delta_b-D)\bar{w}_{\{a,ab\}}(u)^2\right]\right]>\epsilon(R|u). \tag{15}$$

4 Transfers and Private Protection

With one form of protection, the constrained first best could be implemented if transfers were available. Imagine a transfer T was given to households who invested in private protection. In a decentralized market, the marginal adopter would be such that the household wealth equals $\frac{(F-T)H}{\rho c(u)\Delta(u)}$. From the first order conditions stated in Definition 4, we can see that a transfer equivalent to $T=\left[1-\frac{\tilde{w}^*}{\bar{w}(u^*)}\right]F$ would generate a constrained first best private investments. Simply put, there one externality that is not internalized and there is one instrument to align private behaviour with the desirable one.

With two types of private investments, one could expect the same type of logic to apply. It is not the case however. Investment in both A and B are not aligned with optimal decisions. Because households do not take in consideration the public good aspect of their investments, there is not enough private protection. All three margins are distorted: i) some low-to-mid income household do not have investment B, when they should, ii) some high income households who only have investment A, should have both, and iii) some mid income houses are protected by investment, but should be protected by investment A instead. Imagine that the government could grant two distinct transfer T_a and T_b to subsidize private protection A and B respectively. This is not sufficient to implement the constraint first best outcome. Having two instruments is not enough, because there are three decisions distorted, $\bar{w}_{\{0,b\}}(u^\star)$, $\bar{w}_{\{a,b\}}(u^\star)$ and $\bar{w}_{\{a,ab\}}(u^\star)$.

Proposition 5: *No set of transfers T_a and T_b can guarantee that investment decisions $\bar{w}_{\{0,b\}}(u^\star)$, $\bar{w}_{\{a,b\}}(u^\star)$ and $\bar{w}_{\{a,ab\}}(u^\star)$ match the CFB investment decisions.*

Proposition 5 implies that second best public investment decision will be higher or lower than the constraint first best decisions. Over or under provision is determined by a condition similar, but obviously more complex, to the condition stated in Proposition 3 and its corollary.

5 Conclusion

Photons themselves do not deter criminal activity. Rather criminal acts are made more difficult as photons of light reflect off of their criminal acts and are received by complementary objects likes the eyes of witnesses or CCTV cameras. In contrast, metal bars and fences provide a barrier or obstacle that must be overcome if one wishes to acquire the protected property regardless of the presence of other forms of protection. Our analysis above describes how optimal decisions on the protection of property by private citizens and public providers are affected by the potential interactions of technologies. Complementarities matter for research on the economics of crime when scholars wish to estimate the effect of police on crime. If criminal activity is affected by private protection, and private protection interacts public protection then any regression of crime rates on measure of police protection will yield biased and inconsistent estimates when private protection is unmeasured. Greater efforts are required to accurately measure private protection.

We build on a simple depiction of rational household and criminal behavior, and introduce complexity by considering security goods markets of varying composition. We also demonstrate how wealth transfers are unable to match CFB investment decisions, proving the robustness of returning to the CFB model. In considering effects of gross and net complementarity and substitutability, this paper contributes to the discussion on how investment can be optimally allocated between public and private goods. With complementarity of public and private protection the optimal policy trades off the manipulation incentives of affecting private investment decisions with the adjustment incentives of addressing unprotected property.

We concerned ourselves primarily with households but our analysis could be extended to consider firms, neighborhoods, or regions. The insights generated from our analysis can be applied to any scenario in which economic actor chooses protection in the presence of vertical externalities. The distribution or composition of exposure to various forms of protection is a fruitful avenue of future research. A highlight of our research is that even when a public good is uniformly provided, when interacting in a complementary way with private goods the distribution of the benefits are not uniform. This implies that we should consider the equitable distribution of police resources and the incentives that we provide to police. If police are increasingly given performance based pay for example, this would more greatly skew police resources towards households with complementary technologies.

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7 Appendix

Proof of Lemma 1: The number of criminals $c(u)$ is given by $c(u) = \int_0^{\bar{r}} dr$, and so $c(u) = \rho\lambda R(\bar{w}(c,u),u)$. By substituting (1) in (2), we obtain equation (3) that determined $c(u)$. We can then substitute back to get $\bar{w}(u) = \frac{FH}{\rho c(u)\Delta(u)}$. ■

Proof of Proposition 1: Comparative static using Cramer's rule on implicit equations (3) and (4) reveals that:

$$\frac{d\bar{w}(u)}{du} = -\left(\frac{\bar{w}(u)}{u}\right)\left[\frac{\frac{u}{R(\bar{w}(u),u)}\frac{\partial R(\bar{w}(u),u)}{\partial u} + \frac{u\Delta'(u)}{\Delta(u)}}{1 + \frac{\bar{w}(u)}{R(\bar{w}(u),u)}\frac{\partial R(\bar{w}(u),u)}{\partial w}}\right]. \tag{16}$$

Consequently, $\frac{d\bar{w}(u)}{du} < 0$ if and only if $\frac{u\Delta'(u)}{\Delta(u)} > \frac{-u}{R(\bar{w}(u),u)}\frac{\partial R(\bar{w}(u),u)}{\partial u}$. ■

Proof of Proposition 2: We can rewrite (6) as the following:

$$\left[\frac{FH}{\rho c(\tilde{w}^{\star}, u^{\star})\Delta(u^{\star})} - \tilde{w}^{\star}\right] = \tilde{w}^{\star}. \tag{17}$$

Note that $\frac{FH}{\rho c(\tilde{w}^{\star},u^{\star})\Delta(u^{\star})}$ is equivalent to $\bar{w}(u^{\star})$, but where $c(w,u^{\star})$ is evaluated at $\tilde{w}$. We then have that $\bar{w}(u^{\star}) > \tilde{w}^{\star}$. ■

Origin of Definition 2: The first order condition for the SB problem with respect to u is given by:

$$-\rho^2\lambda R\big(\bar{w}(u^*),u^*\big)\frac{\partial R\big(\bar{w}(u^*),u^*\big)}{\partial u} - \rho^2\lambda R\big(\bar{w}(u^*),u^*\big)\frac{dR\big(\bar{w}(u^*),u^*\big)}{du}$$
$$-\frac{\rho\lambda R\big(\bar{w}(u^*),u^*\big)\Delta(u^*)}{H}\left[\bar{w}(u^*) - \frac{FH}{\rho\lambda R\big(\bar{w}(u^*),u^*\big)\Delta(u^*)}\right]\frac{d\bar{w}(u^*)}{du} = \Gamma. \tag{18}$$

Since $\bar{w}(u^*) = \frac{FH}{\rho\lambda R(\bar{w}(u^*),u^*)\Delta(u^*)}$, the second term on the second line is zero. We can rewriting $\frac{dR(\bar{w}(u^*),u^*)}{du}$ as $\frac{\partial R(\bar{w}(u^*),u^*)}{\partial u} + \frac{\partial R(\bar{w}(u^*),u^*)}{\partial w}\frac{d\bar{w}(u^*)}{du}$. Given that $\frac{\partial R(\bar{w}(u^*),u^*)}{\partial w} = \frac{\bar{w}(u^*)\Delta(u^*)}{\rho H}$, we get equation (9). Note that for expositional clarity, we replace $c(\bar{w}(u^*),u^*)$ with $c(u^*)$ when appropriate. ■

Proof of Proposition 3: We start by evaluating the SB first order condition at $u^{\star}$. Note that the equality is no longer guaranteed:

$$-2\rho c(u^{\star})\frac{\partial R\big(\bar{w}(u^{\star}),u^{\star}\big)}{\partial u} - \rho c(u^{\star})\frac{\partial R\big(\bar{w}(u^{\star}),u^{\star}\big)}{\partial \bar{w}}\frac{d\bar{w}\big(u^{\star}\big)}{du} - \Gamma \lesseqgtr 0. \tag{19}$$

To properly differentiate between $\bar{w}(u^\star)$ and $\tilde{w}^\star$, we replace $c(u)$ by $\rho\lambda R(w,u)$. Also note that $\rho c(u^\star)\frac{\partial R\left(\bar{w}(u^\star),u^\star\right)}{\partial \bar{w}} = FH$. Subtracting the first order condition for the CFB problem we get equation (10) in the main text. Rewriting equation (10) we get that:

$$R\big(\bar{w}(u^\star),u^\star\big)\frac{-\partial R\big(\bar{w}(u^\star),u^\star\big)}{\partial u} - R(\tilde{w}^\star,u^\star)\frac{-\partial R(\tilde{w}^\star,u^\star)}{\partial u} - \frac{F}{2\rho^2\lambda}\frac{d\bar{w}(u^\star)}{du} \lesseqgtr 0. \quad (20)$$

Using a linear approximation of $R(\tilde{w}^\star,u^\star)\frac{-\partial R(\tilde{w}^\star,u^\star)}{\partial u}$ evaluate at $\bar{w}(u^\star)$, we get:

$$-\frac{\rho c(u^\star)\Delta(u^\star)}{H}\left[\bar{w}(u^\star)-\tilde{w}^\star\right]\frac{\bar{w}(u^\star)}{u^\star}\left[\varepsilon(\Delta|u)-\varepsilon(R|u)\right] - \frac{F}{2}\,\frac{d\bar{w}(u^\star)}{du} \lesseqgtr 0. \quad (21)$$

Using Proposition 1, the expression above becomes:

$$\left[\varepsilon(\Delta|u)-\varepsilon(R|u)\right]\left[1-2\left[1+\varepsilon(R|w)\right]\frac{\bar{w}(u^\star)-\tilde{w}^\star}{\bar{w}(u^\star)}\right] \lesseqgtr\ 0, \quad (22)$$

where $\varepsilon(R|w) = \frac{\bar{w}(u^\star)}{R(\bar{w}(u^\star),u^\star)}\frac{\partial R(\bar{w}(u^\star),u^\star)}{\partial w}$. When $1 > 2\left[1+\epsilon(R|w)\right]\frac{\bar{w}(u^\star)-\tilde{w}^\star}{\bar{w}(u^\star)}$ is positive, if $\frac{u^\star\Delta'(u^\star)}{\Delta(u^\star)} - \varepsilon(R)_u < 0$, the SB investment is lower than the CFB investment. When $\frac{u^\star\Delta'(u^\star)}{\Delta(u^\star)} - \varepsilon(R)_u > 0$, the SB investment in public protection is then larger than the CFB investment. The exact opposite applies when the term in bracket is negative. ■

Proof of Corollary to Proposition 3: We first derive all higher order Taylor's approximation terms and look for signs at net and gross neutrality. The second order approximation term is

$$\frac{\Delta(u^\star)}{H}\left[\frac{R(\bar{w}(u^\star),u^\star)}{u^\star}\Big[\varepsilon(\Delta|u)-\varepsilon(R|u)\Big] + 2\frac{\bar{w}(u^\star)^2}{H}\Delta'(u^\star)\right]\frac{\left[\bar{w}(u^\star)-\tilde{w}^\star\right]^2}{2}. \quad (23)$$

When $\varepsilon(\Delta|u)-\varepsilon(R|u)=0$, then $\Delta'(u^\star)>0$. Consequently, the second order approximation term is positive. When $\Delta'(u^\star)=0$, then $\varepsilon(\Delta|u)-\varepsilon(R|u)<0$ and the second order approximation term is negative. The third order Taylor's approximation terms is

$$\bar{w}(u^\star)\Delta'(u^\star)\Delta(u^\star)\frac{\left[\bar{w}(u^\star)-\tilde{w}^\star\right]^3}{H^2}. \quad (24)$$

Obviously, when $\Delta'(u^\star)=0$ the term is nil. When $\varepsilon(\Delta|u)-\varepsilon(R|u)=0$, then it is positive because $\Delta'(u^\star)>0$. The fourth order behaves exactly like the third one, since it is simply given by:

$$\Delta'(u^\star)\Delta(u^\star)\frac{\left[\bar{w}(u^\star)-\tilde{w}^\star\right]^4}{24H^2}. \quad (25)$$

All higher order approximation terms are zero. Consequently, when $\varepsilon(\Delta|u)-\varepsilon(R|u) = 0$, expression (22) should be negative, meaning that there should be over-provision of public protection under the SB. When $\Delta'(u^\star) = 0$, there is an additional positive term in expression (22). If $1 < 2\left[1+\epsilon(R|w)\right]\frac{\bar{w}(u^\star)-\tilde{w}^\star}{\bar{w}(u^\star)}$, then there should definitively be under-provision of public protection under the SB at $\Delta'(u^\star) = 0$. If $1 > 2\left[1+\epsilon(R|w)\right]\frac{\bar{w}(u^\star)-\tilde{w}^\star}{\bar{w}(u^\star)}$, we cannot confirm the sign of (22) at $\Delta'(u^\star) = 0$. ■

Proof of Lemma 2: Looking at Figure 1 is helpful in understanding the proof. Given the assumption made about $\Delta_a(u)$, we have that $\bar{w}_{\{0,b\}}(c) < \bar{w}_{\{0,a\}}(c,u) < \bar{w}_{\{a,ab\}}(c)$. Moreover, it must be the case that $\bar{w}_{\{a,b\}}(c,u) \in [\bar{w}_{\{0,b\}}(c), \bar{w}_{\{a,ab\}}(c)]$. Individuals with $w < \bar{w}_{\{0,b\}}(c)$ do not invest in any protection. Individuals with income between $\bar{w}_{\{0,b\}}(c)$ and $\bar{w}_{\{a,b\}}(c,u)$ prefer to invest in protection B over protection A and they prefer investing in protection B as opposed to not investing at all. This implies that $\bar{w}_{\{0,a\}}(c,u)$ is irrelevant in the decision making. Above $\bar{w}_{\{a,b\}}(c,u)$, $F_b + \frac{\rho c}{H}[\ell_0(u) - \Delta_b]w$ is dominated by $F_a + \frac{\rho c}{H}[\ell_0(R) - \Delta_a(u)]w$. Consequently, individuals between $\bar{w}_{\{a,b\}}(c,u)$ and $\bar{w}_{\{a,ab\}}(c)$ invest only in A. Individuals with income above $\bar{w}_{\{a,ab\}}(c)$ invest in both. ■

Proof of Lemma 3: All results are simple comparative statics of $N_A(c,u)$ and $N_B(c,u)$. ■

Proof of Lemma 4: The effect of c on $N_B(c,u)$ is:

$$\frac{\partial N_B(c,u)}{\partial c} = \frac{1}{\rho c^2}\left[\frac{F_b}{\Delta_b} + \frac{F_b}{\Delta_b - D} - \frac{F_a - F_b}{\Delta_a(u) - \Delta_b}\right].$$

The expression above is positive if an only if condition (13) is satisfied. ■

Proof of Lemma 5: The proof is identical to proof of Lemma 1. ■

Proof of Proposition 4: We start by looking at the total effect of u on the vector $\bar{W}(u)$ using Cramer's rule.

$$\frac{d\bar{w}_{\{0,b\}}(u)}{du} = \frac{\frac{\rho H \bar{w}_{\{0,b\}}(u)}{2c(u)}\left[\Delta_a'(u)\left(1+\bar{w}_{\{a,b\}}(u)^2/H^2\right) - \ell_0'(u)\right]}{1+\frac{\rho}{Hc(u)}\left[\Delta_b \bar{w}_{\{0,b\}}(u)^2 + [\Delta_a(u)-\Delta_b]\bar{w}_{\{a,b\}}(u)^2 + (\Delta_b - D)\bar{w}_{\{a,ab\}}(u)^2\right]} > 0;$$

$$\frac{d\bar{w}_{\{a,b\}}(u)}{du} = \bar{w}_{\{a,b\}}(u)\frac{\frac{\Delta_a'(u)}{\Delta_a(u)-\Delta_b}\left[1+\frac{\rho}{Hc(u)}\left[\Delta_b\bar{w}_{\{0,b\}}^2+(\Delta_b-D)\bar{w}_{\{a,ab\}}^2\right]\right]+\frac{\rho}{c(u)}\frac{\partial R\big(\bar{W}(u),u\big)}{\partial u}}{1+\frac{\rho}{Hc(u)}\left[\Delta_b\bar{w}_{\{0,b\}}(u)^2+[\Delta_a(u)-\Delta_b]\bar{w}_{\{a,b\}}(u)^2+(\Delta_b-D)\bar{w}_{\{a,ab\}}(u)^2\right]};$$

$$\frac{d\bar{w}_{\{a,ab\}}(u)}{du} = \frac{\frac{\rho H\bar{w}_{\{a,ab\}}(u)}{2c(u)}\left[\Delta_a'(u)\left(1+\bar{w}_{\{a,b\}}(u)^2/H^2\right)-\ell_0'(u)\right]}{1+\frac{\rho}{Hc(u)}\left[\Delta_b\bar{w}_{\{0,b\}}(u)^2+[\Delta_a(u)-\Delta_b]\bar{w}_{\{a,b\}}(u)^2+(\Delta_b-D)\bar{w}_{\{a,ab\}}(u)^2\right]} > 0.$$

The impact of public investment on private protection A is give by $\frac{dN_A(u)}{du} = -\frac{1}{H}\frac{d\bar{w}_{\{a,b\}}(u)}{du}$, which is positive if and only if (15) is satisfied. The impact of public investment in private protection B is give by $\frac{dN_B(u)}{du} = \frac{1}{H}\left[\frac{d\bar{w}_{\{a,b\}}(u)}{du} - \frac{d\bar{w}_{\{0,b\}}(u)}{du} - \frac{d\bar{w}_{\{a,ab\}}(u)}{du}\right]$, which is always negative. ■

Proof of Proposition 5: We define the vector of private protections $\tilde{W}$ a government would choose if it was able to force private investment decisions. The CFB problem face by the government is:

$$\begin{aligned}\max_{\{u,\tilde{w}\}}\ &\left[H-\rho c(\tilde{W},u)\ell_0(u)\right]\frac{H}{2}+\rho c(\tilde{W},u)\Delta_a(u)\int_{\tilde{w}_{\{a,b\}}}^{H}\frac{w}{H}dw\\ &+\rho c(\tilde{W},u)\Delta_b\int_{\tilde{w}_{\{0,b\}}}^{\tilde{w}_{\{a,b\}}}\frac{w}{H}dw+\rho c(\tilde{W},u)(\Delta_b-D)\int_{\tilde{w}_{\{a,ab\}}}^{H}\frac{w}{H}dw\\ &-\int_{\tilde{w}_{\{a,b\}}}^{H}F_a dw-\int_{\tilde{w}_{\{0,b\}}}^{\tilde{w}_{\{a,b\}}}F_b dw-\int_{\tilde{w}_{\{a,ab\}}}^{H}F_b dw-u.\end{aligned}$$

The CFB investment in private protection vector $\tilde{W}^\star$ and in public enforcement $u^\star$ are respectively given by:

$$\left[\tilde{w}_{\{0,b\}}^\star-\frac{F_bH}{\rho c(\tilde{W}^\star,u^\star)\Delta_b}\right]=\frac{\Delta_b-F_b}{\Delta_b}\tilde{w}_{\{0,b\}}; \tag{26}$$

$$\left[\tilde{w}_{\{a,b\}}^\star-\frac{(F_a-F_b)H}{\rho c(\tilde{W}^\star,u^\star)[\Delta_a(u)-\Delta_b]}\right]=\frac{[\Delta_a(u)-\Delta_b]-[F_a-F_b]}{[\Delta_a(u)-\Delta_b]}\tilde{w}_{\{a,b\}}; \tag{27}$$

$$\left[\tilde{w}_{\{a,ab\}}^\star-\frac{F_bH}{\rho c(\tilde{W}^\star,u^\star)B}\right]=\frac{\Delta_b-D-F_b}{\Delta_b-D}\tilde{w}_{\{a,ab\}}; \tag{28}$$

$$-2\rho c(\tilde{W}^\star,u^\star)\frac{\partial R(\tilde{W}^\star,u^\star)}{\partial u}=1; \tag{29}$$

Without transfers, first order conditions (26), (27) and (28) suggest that $\bar{w}_{\{0,b\}}(u^\star)$, $\bar{w}_{\{a,b\}}(u^\star)$ and $\bar{w}_{\{a,ab\}}(u^\star)$ are higher than the CFB investment level. Call T_a and

T_b the transfers given to households who buy private protection A and B respectively. Private investments would then given by $\bar{w}_{\{0,b\}}(u^\star) = \frac{H[F_b - T_b]}{\rho c(\tilde{W}^\star, u^\star)\Delta_b}$, $\bar{w}_{\{b,a\}}(u^\star) = \frac{H[F_a - T_b - F_a + T_a]}{\rho c(\tilde{W}^\star, u^\star)[\Delta_a(u^\star) - \Delta_b]}$ and $\bar{w}_{\{a,ab\}}(u^\star) = \frac{H[F_a - T_a]}{\rho c(\tilde{W}^\star, u^\star)[\Delta_b - D]}$. No set of transfers T_a and T_b can guarantee that $\bar{w}_{\{0,b\}}(u^\star)$, $\bar{w}_{\{a,b\}}(u^\star)$ and $\bar{w}_{\{a,ab\}}(u^\star)$ all equals $\tilde{w}^\star_{\{0,b\}}$, $\tilde{w}^\star_{\{a,b\}}$ and $\tilde{w}^\star_{\{a,ab\}}$ given by conditions (26), (27) and (28). ■