Behavioral Expectations under Indeterminacy: An Empirical Evaluation

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Roadmap

Background

- Standard DSGE models are typically solved under the assumption of rational expectations (RE)
  - **But** the empirical plausibility of RE is being questioned
  - A growing literature that relaxes RE in favor of behavioral expectations (BE): adaptive expectations; overextrapolation; diagnostic expectations; cognitive discounting
- Most DSGE analyses focus exclusively on cases where equilibrium is determinate
  - The possibility of equilibrium indeterminacy in theory and in practice
  - In Japan, the policy rate remained near zero for a quarter of a century until August 2024
  - A lack of empirical research based on DSGE models that allow for indeterminacy
- **Solutions for BE models under indeterminacy are not yet established**

What We Do

First, establish quantitative implications of BE models under indeterminacy:

- In addition to RE, consider two types of BE:
  - Diagnostic expectations (DE)
  - Cognitive discounting (CD)
- Characterize solutions for BE models under indeterminacy both analytically and numerically by
  - representing the linearized system of equations using the RE operator
  - considering how forecast errors are defined (RE or BE)
- Each combination of expectation formation and forecast error specification yields a different equilibrium law of motion

What We Do (cont.)

Next, assess the empirical relevance of BE specifications under indeterminacy:

- Estimate medium-scale DSGE models for Japan's zero interest rate period
  - Likely episode of indeterminacy: Failure of the Taylor principle
  - Compare the empirical performance of the models with different expectation formation and forecast error specifications
- DE model with RE forecast errors delivers the best empirical performance
  - DE model captures realistic short-run dynamics with overreactions of aggregate variables
  - Variance and historical decompositions of inflation highlight the role of sunspot shocks

Related Literature

- Departures from Full Information Rational Expectations
  Marcet & Sargent (1989); Evans & Honkapohja (2001); Coibion & Gorodnichenko (2015); Eusepi & Preston (2018); Gabaix (2018); Azeredo da Silveira & Woodford (2019); Farhi & Werning (2019); Garcia-Schmidt & Woodford (2019); Angeletos, Huo & Sastry (2020)

- Diagnostic expectations in macro and finance:
  Kahneman & Tversky (1972); Kahneman et al. (1982); Kahana (2012); Bordalo et al. (2018); Choi & Mertens (2019); Bordalo et al. (2020); Bordalo et al. (2021); Bordalo et al. (2022); Chodorow-Reich et al. (2022); Afrouzi et al. (2023); Bianchi et al. (2024); Bianchi et al. (2024a); L'Huillier et al. (2024); Maxted (2024); Na & Yoo (2025)

- Cognitive discounting:
  Gabaix (2020); Ilabaca et al. (2020); Meggiorini & Milani (2021); Meggiorini (2023); Afsar et al. (2024); Hirose et al. (2024); Benchimol et al. (2025)

- Solution methods:
  Blanchard & Kahn (1980); Klein (2000); Sims (2002); Lubik & Schorfheide (2003); Adams (2023)

Roadmap

Steps for solving BE models

1. Represent a linearized system of equations using the RE operator
   - Augmented with a parameter that captures the degree of deviation from RE
   - Diagnostic expectations (DE):
     - "Representativeness heuristic" (Kahneman & Tversky, 1972):
       $\mathbb{E}_t^{DE} x_{t+1} = \mathbb{E}_t x_{t+1} + \theta \left(\mathbb{E}_t x_{t+1} - \mathbb{E}_{t-1} x_{t+1} \right)$, where θ is the degree of diagnosticity
   - Cognitive discounting (CD)
     - "Partial myopia" toward distant event (Gabaix, 2020):
       $\mathbb{E}_t^{CD} x_{t+1} = \phi \mathbb{E}_t x_{t+1}$, where ϕ is the degree of cognitive discounting, $0 < \phi \leq 1$
2. Apply a linear RE solution algorithm
   - Blanchard & Kahn (1980); Klein (2000); Sims (2002)
3. Derive the equilibrium law of motion for endogenous variables

Sims' GENSYS Algorithm

- Canonical form:

$$\Gamma_0 s_t = \Gamma_1 s_{t-1} + \Psi \varepsilon_t + \Pi \eta_t$$

  - s_t: vector of endogenous variables; ε_t: vector of exogenous shocks; η_t: vector of forecast errors

- Two possible definitions for η_t:

  1) RE-based forecast errors:

$$\eta_t = x_t - \mathbb{E}_{t-1} x_t$$

  2) BE-consistent forecast errors:

$$\eta_t = x_t - \mathbb{E}^*_{t-1} x_t$$

- Existing studies implicitly assume RE-based forecast errors

Simple Univariate Example: RE Model

$$\text{Model:} \quad y_t = \frac{1}{\alpha}\mathbb{E}_t y_{t+1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

$$\text{Forecast error:} \quad \eta_t = y_t - \mathbb{E}_{t-1} y_t$$

- $|\alpha| > 1 \Rightarrow$ Determinacy
  - The unique solution:

$$y_t = \varepsilon_t$$

- $|\alpha| < 1 \Rightarrow$ Indeterminacy
  - The solution exhibits richer dynamics with the following ARMA (1,1) representation:

$$y_t = \alpha y_{t-1} + \tilde{M}\varepsilon_t - \alpha\varepsilon_{t-1} + M_\zeta \zeta_t,$$

where $\tilde{M}$ and M_ζ are arbitrary parameters; and ζ_t is a sunspot shock

Simple Univariate Example: Two BE Models

DIAGNOSTIC EXPECTATIONS

$$y_t = \frac{1}{\alpha}\mathbb{E}_t^{DE} y_{t+1} + \varepsilon_t$$

$\Rightarrow$ RE representation:

$$y_t = \frac{1+\theta}{\alpha}\mathbb{E}_t y_{t+1} - \frac{\theta}{\alpha}\mathbb{E}_{t-1} y_{t+1} + \varepsilon_t$$

COGNITIVE DISCOUNTING

$$y_t = \frac{1}{\alpha}\mathbb{E}_t^{CD} y_{t+1} + \varepsilon_t$$

$\Rightarrow$ RE representation:

$$y_t = \frac{\phi}{\alpha}\mathbb{E}_t y_{t+1} + \varepsilon_t$$

Specifying Forecast Errors in BE Systems

- BE systems admit two possible definitions of forecast errors:
  1. RE-based forecast errors (forecast error $\eta_{1,t}$; forecast revision $\eta_{2,t}$):

$$\begin{aligned} \eta_{1,t} &= y_t - \mathbb{E}_{t-1} y_t \\ \eta_{2,t} &= \mathbb{E}_t y_{t+1} - \mathbb{E}_{t-1} y_{t+1} \end{aligned}$$

  2. BE-consistent forecast errors:

$$\begin{aligned} \text{Under DE:} \quad \eta_{1,t} &= y_t - \mathbb{E}^{DE}_{t-1} y_t \\ &= y_t - (1+\theta)\mathbb{E}_{t-1} y_t + \theta \mathbb{E}_{t-2} y_t \\ \text{Under CD:} \quad \eta_{1,t} &= y_t - \mathbb{E}^{CD}_{t-1} y_t \\ &= y_t - \phi \mathbb{E}_{t-1} y_t \end{aligned}$$

Solutions for BE Models under Determinacy

- For all combinations of BE formation and forecast error specifications, the unique solution is

$$y_t = \varepsilon_t$$

- Identical to the RE solution

Solutions for BE Models under Indeterminacy

- DE model with RE forecast errors:

$$\begin{bmatrix} y_t \\ \mathbb{E}_t y_{t+1} \\ \mathbb{E}_t y_{t+2} \end{bmatrix} = A(\alpha,\theta) \begin{bmatrix} y_{t-1} \\ \mathbb{E}_{t-1} y_t \\ \mathbb{E}_{t-1} y_{t+1} \end{bmatrix} + B(\alpha,\theta,\tilde{M})\varepsilon_t + C(\alpha,\theta,M_\zeta)\zeta_t$$

- DE model with DE forecast errors:

$$\begin{bmatrix} y_t \\ \mathbb{E}_t y_{t+1} \\ \mathbb{E}_t y_{t+2} \\ \mathbb{E}_{t-1} y_{t+1} \end{bmatrix} = A(\alpha,\theta) \begin{bmatrix} y_{t-1} \\ \mathbb{E}_{t-1} y_t \\ \mathbb{E}_{t-1} y_{t+1} \\ \mathbb{E}_{t-2} y_t \end{bmatrix} + B(\alpha,\theta,\tilde{M})\varepsilon_t + C(\alpha,\theta,M_\zeta)\zeta_t$$

- CD model with RE forecast errors:

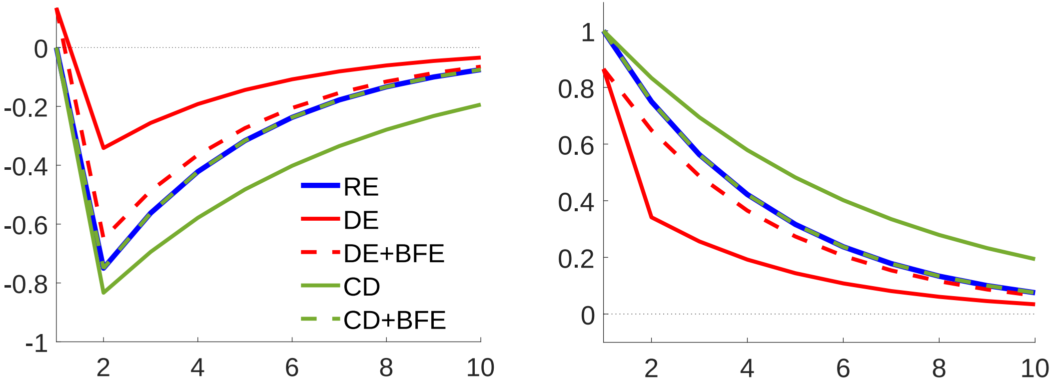
$$y_t = \frac{\alpha}{\phi} y_{t-1} + \tilde{M}\varepsilon_t - \frac{\alpha}{\phi}\varepsilon_{t-1} + M_\zeta \zeta_t$$

- CD model with CD forecast errors:

$$y_t = \alpha y_{t-1} + \tilde{M}\varepsilon_t - \alpha\varepsilon_{t-1} + M_\zeta \zeta_t$$

Numerical Illustration: $\alpha = 0.75$; $\theta = 0.9$; $\phi = 0.9$; $\tilde{M} = 0$; $M_\zeta = 1$

(a) Fundamental shock (b) Sunspot shock



1. Forecast error specifications critically affect dynamics properties in BE models
2. Responses under CD+BFE are identical to those under RE
3. DE models exhibit overreaction features

Roadmap

Empirical Assessment

- Which BE formation and forecast error specification delivers the best fit to the data under indeterminacy?

    1. RE model
    2. DE model with RE forecast errors
    3. DE model with DE forecast errors
    4. CD model with RE forecast errors
    5. CD model with CD forecast errors

- What makes differences in their empirical performance?

- What changes in the interpretation of the data?

A Medium-Scale DEGE Model for Japan

- Based on the framework of Hirose (2020)
  - Along the lines of Christiano, Eichenbaum, and Evans (2005); Smets and Wouters (2007); Justiniano, Primiceri, and Tambalotti (2010)

- The policy rate is set exogenously:

$$\tilde{R}_t^n = \psi_r \tilde{R}_{t-1}^n + \varepsilon_t^r$$

  - Monetary policy does not satisfy the Taylor principle $\Rightarrow$ Equilibrium indeterminacy

- Exogenous shocks:
  - Technology; Preference; MEI; External demand; Wage markup; Price markup; Monetary policy; Sunspot

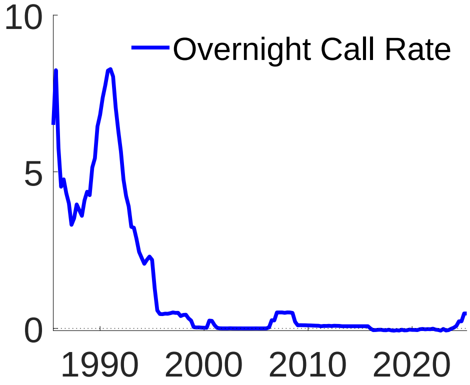
◂ Model Description

Estimation Strategy and Data

- Estimate the model using a Bayesian approach allowing for indeterminacy
  - As implemented by Hirose, Kurozumi, and Van Zandweghe (2020, 2023)
  - Sequential Monte Carlo (SMC) algorithm

- Sample: 1999:I to 2024:I
  1. Real GDP growth
  2. Real consumption growth
  3. Real investment growth
  4. Hours worked
  5. Inflation: GDP deflator
  6. Real wage growth
  7. Overnight call rate

- Zero interest rate policy **except** Aug 2000–Mar 2001/Jul 2006–Dec 2008



Prior Distributions of Parameters

Parameter		Distribution	Mean	S.D.
θ	Degree of diagnosticity	Normal	1.000	0.300
ϕ	Degree of cognitive discounting	Beta	0.850	0.050
$\overline{z}$	Steady-state output growth rate	Gamma	0.129	0.050
$\overline{\pi}$	Steady-state inflation rate	Normal	-0.039	0.050
$\bar{l}$	Steady-state hours worked	Normal	0.000	0.050
σ	Relative risk aversion	Gamma	1.500	0.100
γ	Habit persistence	Beta	0.500	0.100
φ_w	Wage adjustment cost	Gamma	20.00	5.000
χ	Inverse elasticity of labor supply	Gamma	2.000	0.750
μ	Capital utilization adjustment cost	Gamma	1.000	0.500
φ_i	Investment adjustment cost	Gamma	4.000	1.000
λ_p	Steady-state price markup	Gamma	0.150	0.050
φ_p	Investment adjustment cost	Gamma	30.00	5.000
γ_p	Price indexation	Beta	0.500	0.150
ψ_r	Interest rate smoothing	Beta	0.750	0.150
ρ_x's	Persistence of shocks	Beta	0.500	0.150
σ_x's	Standard deviation of shocks	Inverse gamma	0.500	4.000
$M_x{}'s$	Arbitrary parameters	Normal	0.000	1.000

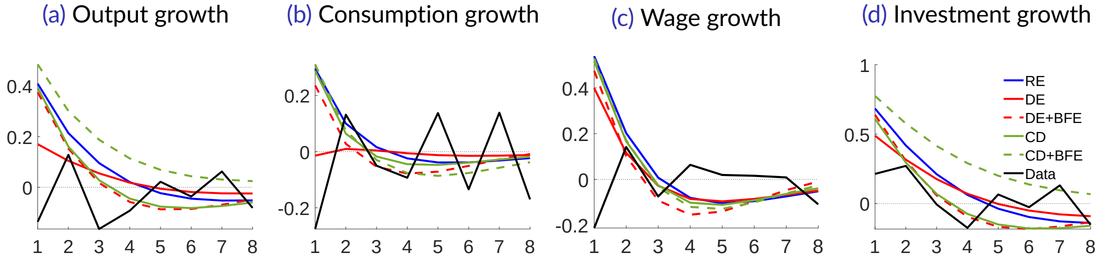
Posterior Estimates of BE Parameters and Marginal Data Densities

	RE	DE	DE+BFE	CD	CD+BFE
θ (DE)	-	1.228	0.045	-	-
ϕ (CD)	-	-	-	0.976	0.911
log P(Y^T)	-876.5	-805.7	-967.9	-916.0	-852.9

- DE with RE forecast errors gives the highest MDD with larger θ
- CD with CD forecast errors fits the data well with smaller ϕ

Posterior estimates

Autocorrelation Functions



- DE with RE forecast errors gives the highest MDD by

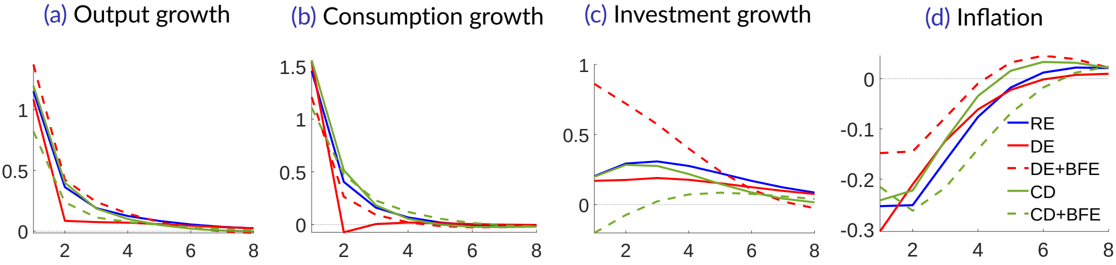
1. matching first-lag autocorrelations in the data comparably well
2. replicating the near-zero autocorrelations at longer lags

Squared Differences Between the Model-Implied and Sample Variances

	$\Delta \log Y_t$	$\Delta \log C_t$	$\Delta \log I_t$	$\Delta \log W_t$	$\log l_t$	$\Delta \log P_t$	$\log R_t^n$
RE	1.73	0.51	44.80	5.13	144.42	0.48	0.00
DE	0.08	0.36	0.75	3.05	26.04	0.26	0.00
DE+BFE	4.55	0.24	44.67	3.86	979.52	0.88	0.00
CD	2.17	2.41	42.58	5.90	84.25	0.65	0.00
CD+BFE	4.46	1.59	138.73	7.16	11890.24	0.99	0.00

- DE with RE forecast errors yields the smallest squared differences
- Large differences in the variance of hours worked are due to their excessive persistence generated by the model under indeterminacy.

Impulse Response Functions: Technology Shocks

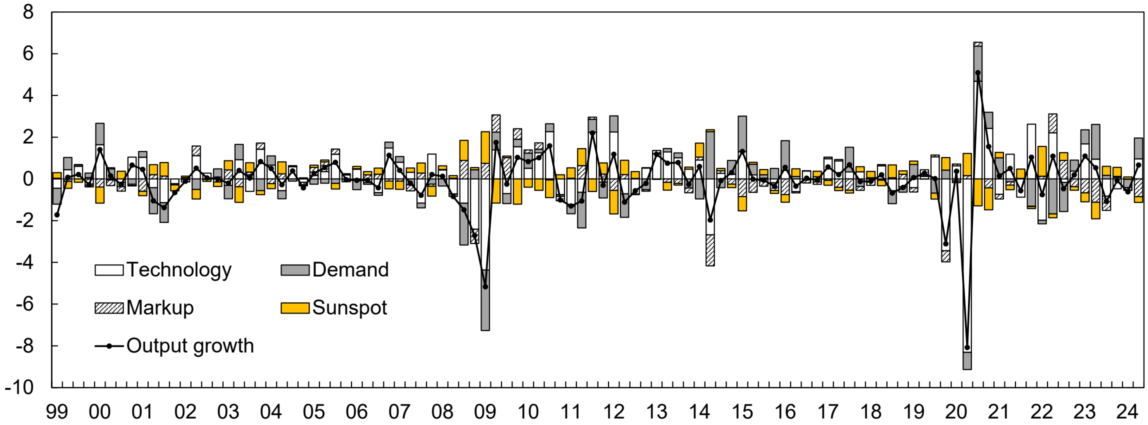


- In general, DE models generate overreaction and systematic reversals
- This feature is also present under indeterminacy

Variance Decomposition: RE vs. DE

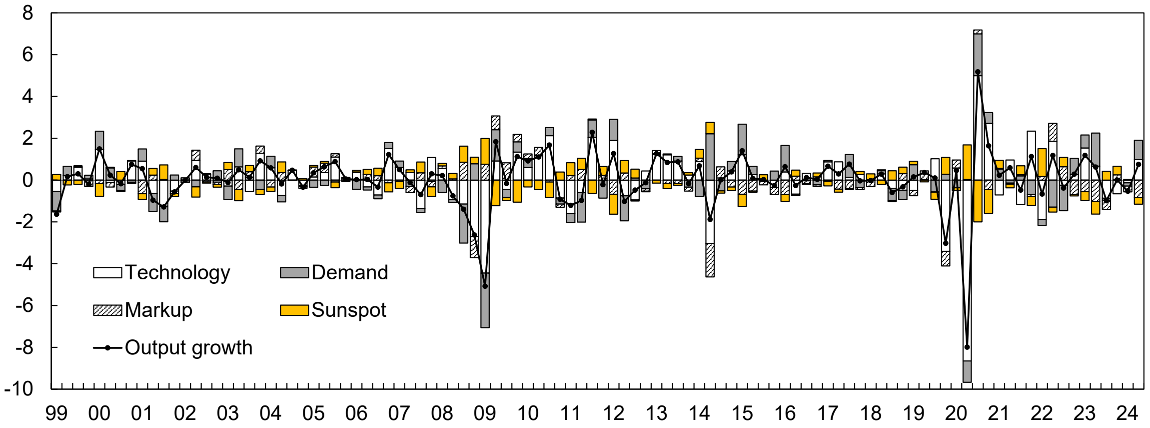
	$\Delta \log Y_t$	$\Delta \log C_t$	$\Delta \log I_t$	$\Delta \log W_t$	$\log l_t$	$\Delta \log P_t$	$\log R_t^n$
RE model:							
Technology	50.9	69.5	6.6	33.2	4.5	2.0	0.0
Preference	0.5	2.7	0.0	0.3	0.1	0.0	0.0
MEI	16.7	1.8	67.9	1.9	48.7	8.8	0.0
External demand	10.0	0.1	0.8	0.5	3.9	0.5	0.0
Wage markup	1.5	0.6	2.8	22.4	12.5	3.7	0.0
Price markup	8.1	4.8	12.5	29.7	13.7	30.1	0.0
Monetary policy	0.6	1.1	0.4	0.4	0.7	0.9	100.0
Sunspot	11.7	19.5	9.0	11.6	15.9	54.0	0.0
DE model:							
Technology	58.0	75.7	8.3	35.8	7.4	3.2	0.0
Preference	0.2	1.2	0.3	0.5	0.2	0.3	0.0
MEI	8.5	1.6	58.7	1.6	32.2	8.3	0.0
External demand	11.9	0.2	2.1	1.1	6.5	1.2	0.0
Wage markup	1.6	0.5	4.0	17.4	16.5	4.3	0.0
Price markup	6.7	2.8	15.1	24.1	12.5	25.7	0.0
Monetary policy	1.2	1.6	1.0	1.5	2.2	2.5	100.0
Sunspot	12.0	16.3	10.4	18.1	22.6	54.6	0.0

Historical Decomposition of Output: RE



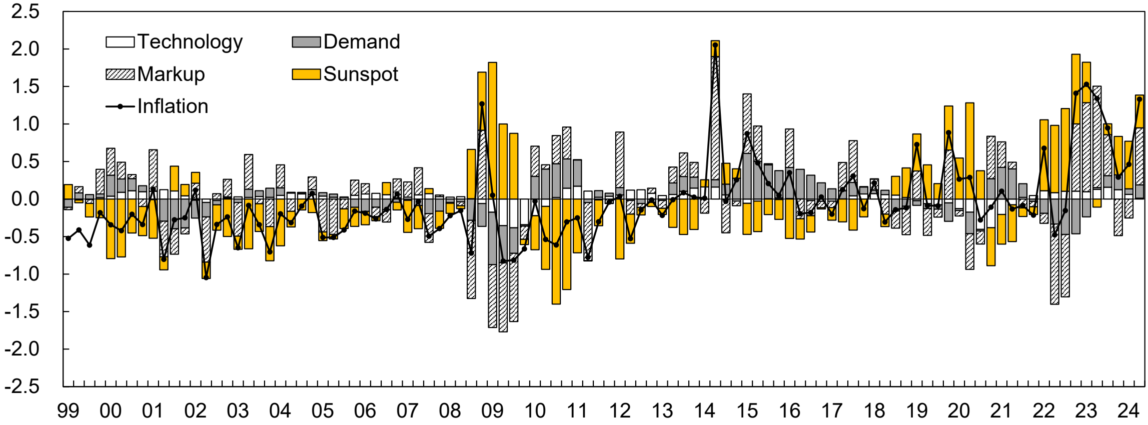
- Technology shocks are the primary driver of fluctuations
- Demand shocks also contribute substantially, particularly during GFC
- Sunspot shocks help offset major swings during/after GFC and COVID-19 pandemic

Historical Decomposition of Output: DE



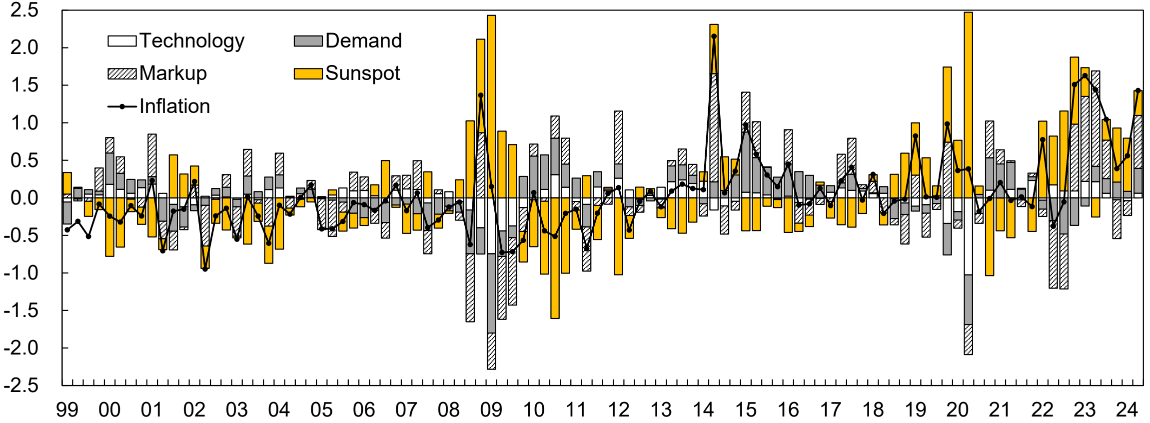
- Technology shocks remain the dominant driver of output fluctuations
- Sunspot shocks continue to play a stabilizing role

Historical Decomposition of Inflation: RE



- Markup shocks mainly drive inflation
- Sunspot shocks largely drive inflation dynamics:
  - Amplifying inflation, especially during GFC and COVID-19 pandemic

Historical Decomposition of Inflation: DE



- Technology shocks contribute more during the GFC and COVID-19 compared to RE
- Contributions of sunspot shocks are more pronounced during these periods

Roadmap

1. Introduction

2. Solution for BE Models under Indeterminacy

3. Empirical Assessment

4. Conclusion

Concluding Remarks: Key Findings

- Investigated the quantitative implications and empirical relevance of BE under indeterminacy
- Characterized solutions for BE models under indeterminacy
  - Considering two possible definitions of forecast errors: RE or BE
- Estimated a medium-scale DSGE model for Japan's zero interest rate period
  - DE with RE forecast errors best fits the data
    - Captures overreaction of aggregate variables to shocks
  - Technology and markup shocks mainly drive output and inflation, respectively
  - Sunspot shocks stabilize output but amplify inflation volatility
- Overall: Highlighted the role of BE and indeterminacy in macroeconomic fluctuations

Concluding Remarks: Future Directions

- Extend analysis to other BE specifications under indeterminacy:
  - Adaptive expectations (Cagan, 1956; Friedman, 1957); overextrapolation (Angeletos, Huo, and Sastry, 2020)
- Determinacy conditions differ across BE formulations (Adams, 2023)
  - Our estimation approach accommodates both determinacy and indeterminacy
- Generalized DE:
  - Incorporating distant past as a reference set (Bianchi, Ilut, and Saijo, 2024a)
  - Smoothed variants linking uncertainty and overreaction (Bianchi, Ilut, and Saijo, 2024b)
  - Under indeterminacy, these mechanisms would further amplify the internal propagation of shocks

Additional Slides

Households

Each household maximizes

$$e^{z_t^b}\left(\frac{\left(C_t(h)-\gamma C_{t-1}(h)\right)^{1-\sigma}}{1-\sigma}-\frac{Z_t^{1-\sigma}l_t(h)^{1+\chi}}{1+\chi}\right)$$
$$+\mathbb{E}_t^*\sum_{j=1}^{\infty}\beta^j e^{z_{t+j}^b}\left\{\frac{\left(C_{t+j}(h)-\gamma C_{t+j-1}(h)\right)^{1-\sigma}}{1-\sigma}-\frac{Z_{t+j}^{1-\sigma}l_{t+j}(h)^{1+\chi}}{1+\chi}\right\}$$

subject to

$$C_t(h)+I_t(h)+\frac{B_t(h)}{P_t}=W_t(h)l_t(h)+R_t^k(h)u_t(h)K_{t-1}(h)+R_{t-1}^n\frac{B_{t-1}(h)}{P_t}+T_t(h)$$
$$-\frac{\varphi_w}{2}\left\{\frac{P_tW_t(h)}{P_{t-1}W_{t-1}(h)\pi z}-1\right\}^2 W_t(h)l_t(h),$$

$$K_t(h)=\{1-\delta(u_t(h))\}K_{t-1}(h)+e^{z_t^i}\left\{1-S\left(\frac{I_t(h)}{I_{t-1}(h)z}\right)\right\}I_t(h)$$

Final-Good Firm

Maximizes

$$P_t Y_t - \int_0^1 P_t(f) Y_t(f) df,$$

subject to

$$Y_t = \left\{ \int_0^1 Y_t(f)^{1/(1+\lambda_t^p)} df \right\}^{1+\lambda_t^p}$$

The market clearing condition for the final good:

$$Y_t = C_t + I_t + dZ_t e^{z_t^d} + \frac{\varphi_p}{2} \left(\frac{\pi_t}{\pi_{t-1}^{\gamma_p} \pi^{1-\gamma_p}} - 1 \right)^2 Y_t$$

Back

Intermediate-Good Firms

Production function: $Y_t(f) = \left(Z_t l_t(f)\right)^{1-\alpha} \left(u_t K_{t-1}(f)\right)^{\alpha} - \varphi_y Z_t$

Each intermediate-good firm maximizes

$$\max_{P_t(f)} \left\{ \frac{P_t(f)}{P_t} - mc_t - \frac{\varphi_p}{2} \left(\frac{\frac{P_t(f)}{P_{t-1}(f)}}{\pi_{t-1}^{\gamma_p} \pi^{1-\gamma_p}} - 1 \right)^2 \right\} Y_t(f) +$$

$$\mathbb{E}_t^* \sum_{j=1}^{\infty} \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} \left\{ \frac{P_{t+j}(f)}{P_{t+j}} - mc_{t+j} - \frac{\varphi_p}{2} \left(\frac{\frac{P_{t+j}(f)}{P_{t+j-1}(f)}}{\pi_{t+j-1}^{\gamma_p} \pi^{1-\gamma_p}} - 1 \right)^2 \right\} Y_{t+j}(f)$$

subject to

$$Y_t(f) = Y_t \left(\frac{P_t(f)}{P_t} \right)^{-\frac{1+\lambda_t^p}{\lambda_t^p}}$$

◂ Back

Posterior Distributions of Parameters: RE and DE

	RE		DE+RE forecast errors		DE+DE forecast errors	
Parameter	Mean	90% interval	Mean	90% interval	Mean	90% interval
θ	-	-	1.228	[0.953, 1.489]	0.045	[-0.032, 0.135]
$\bar{z}$	0.174	[0.090, 0.265]	0.086	[0.039, 0.130]	0.115	[0.064, 0.164]
$\bar{\pi}$	0.008	[-0.062, 0.086]	-0.091	[-0.147, -0.031]	-0.101	[-0.195, -0.003]
$\bar{l}$	-0.047	[-0.105, 0.015]	0.011	[-0.063, 0.083]	0.022	[-0.079, 0.109]
σ	1.384	[1.238, 1.527]	1.339	[1.193, 1.477]	1.422	[1.268, 1.560]
γ	0.184	[0.120, 0.248]	0.435	[0.363, 0.511]	0.229	[0.157, 0.298]
φ_w	28.464	[19.309, 36.817]	26.512	[20.358, 33.122]	33.487	[26.681, 39.657]
χ	1.645	[0.761, 2.467]	2.265	[1.256, 3.246]	0.997	[0.491, 1.494]
μ	1.752	[1.098, 2.481]	1.344	[0.603, 2.033]	0.127	[0.025, 0.212]
φ_i	3.262	[2.306, 4.228]	4.342	[3.053, 5.558]	1.888	[1.330, 2.428]
λ_p	0.221	[0.149, 0.293]	0.227	[0.157, 0.303]	0.236	[0.165, 0.315]
φ_p	29.803	[22.898, 36.238]	32.819	[24.931, 39.344]	33.285	[25.986, 40.304]
γ_p	0.471	[0.215, 0.721]	0.375	[0.198, 0.578]	0.681	[0.483, 0.883]
ψ_r	0.803	[0.694, 0.933]	0.814	[0.732, 0.896]	0.888	[0.813, 0.961]
ρ_z	0.200	[0.103, 0.295]	0.139	[0.053, 0.218]	0.121	[0.035, 0.199]
ρ_b	0.476	[0.221, 0.729]	0.699	[0.467, 0.924]	0.871	[0.818, 0.921]
ρ_i	0.874	[0.784, 0.979]	0.908	[0.867, 0.958]	0.367	[0.151, 0.579]
ρ_d	0.862	[0.798, 0.923]	0.845	[0.784, 0.905]	0.836	[0.774, 0.896]

Posterior Distributions of Parameters: RE and DE (cont.)

	RE		DE+RE forecast errors		DE+DE forecast errors	
Parameter	Mean	90% interval	Mean	90% interval	Mean	90% interval
ρ_w	0.578	[0.443, 0.712]	0.587	[0.413, 0.735]	0.461	[0.325, 0.608]
ρ_p	0.647	[0.495, 0.820]	0.653	[0.501, 0.807]	0.362	[0.190, 0.546]
σ_z	1.932	[1.696, 2.143]	1.589	[1.272, 1.919]	2.351	[2.115, 2.578]
σ_b	0.595	[0.293, 0.901]	0.699	[0.322, 1.057]	2.373	[1.905, 2.797]
σ_i	2.934	[2.178, 3.713]	1.896	[1.546, 2.198]	0.569	[0.294, 0.842]
σ_d	2.892	[2.643, 3.163]	3.123	[2.794, 3.440]	2.581	[2.354, 2.819]
σ_w	1.219	[0.966, 1.447]	1.230	[0.904, 1.547]	1.116	[0.949, 1.273]
σ_p	0.416	[0.338, 0.482]	0.352	[0.260, 0.433]	0.521	[0.415, 0.617]
σ_r	0.098	[0.087, 0.109]	0.102	[0.091, 0.113]	0.097	[0.084, 0.110]
σ_s	0.543	[0.425, 0.678]	0.533	[0.380, 0.680]	0.735	[0.496, 0.970]
M_z	-0.623	[-0.841, -0.420]	-1.445	[-1.798, -1.067]	-0.773	[-0.957, -0.566]
M_b	-0.410	[-0.873, 0.023]	-0.322	[-0.857, 0.327]	-0.601	[-0.725, -0.480]
M_i	-0.107	[-0.177, -0.028]	-0.228	[-0.330, -0.120]	0.314	[-0.525, 1.295]
M_d	-0.134	[-0.180, -0.089]	-0.222	[-0.276, -0.165]	-0.107	[-0.176, -0.043]
M_w	-0.606	[-0.765, -0.437]	-0.548	[-0.706, -0.369]	-0.909	[-1.156, -0.665]
M_p	-0.074	[-0.369, 0.232]	0.916	[0.405, 1.468]	-0.830	[-1.316, -0.319]
M_r	0.162	[-1.333, 1.908]	0.199	[-1.267, 1.560]	-0.146	[-1.630, 1.487]
$\log p(\mathbf{Y}^T)$	-876.519		-805.719		-967.915	

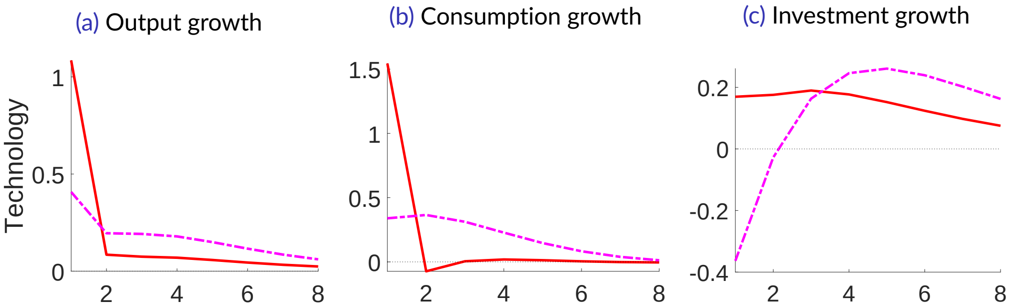
Posterior Distributions of Parameters: CD

	CD+RE forecast errors		CD+CD forecast errors	
Parameter	Mean	90% interval	Mean	90% interval
ϕ	0.976	[0.953, 0.996]	0.911	[0.891, 0.935]
$\bar{z}$	0.085	[0.029, 0.144]	0.089	[0.047, 0.131]
$\bar{\pi}$	-0.019	[-0.094, 0.067]	-0.120	[-0.174, -0.063]
$\bar{l}$	0.022	[-0.052, 0.090]	0.015	[-0.055, 0.090]
σ	1.323	[1.172, 1.466]	1.317	[1.165, 1.476]
γ	0.234	[0.136, 0.319]	0.152	[0.091, 0.207]
φ_w	32.005	[23.644, 39.575]	27.964	[20.005, 36.365]
χ	1.696	[1.038, 2.373]	0.592	[0.227, 0.978]
μ	1.745	[1.260, 2.297]	3.432	[2.299, 4.348]
φ_i	2.226	[1.640, 2.737]	3.858	[3.230, 4.526]
λ_p	0.170	[0.105, 0.237]	0.294	[0.218, 0.376]
φ_p	30.363	[24.237, 36.095]	33.644	[27.518, 39.732]
γ_p	0.502	[0.274, 0.734]	0.655	[0.467, 0.876]
ψ_r	0.889	[0.820, 0.957]	0.817	[0.686, 0.940]
ρ_z	0.183	[0.094, 0.283]	0.158	[0.057, 0.252]
ρ_b	0.590	[0.318, 0.908]	0.375	[0.180, 0.564]
ρ_i	0.792	[0.698, 0.894]	0.754	[0.656, 0.847]
ρ_d	0.825	[0.758, 0.894]	0.837	[0.776, 0.910]

Posterior Distributions of Parameters: CD (cont.)

	CD+RE forecast errors		CD+CD forecast errors	
Parameter	Mean	90% interval	Mean	90% interval
ρ_w	0.480	[0.324, 0.636]	0.475	[0.337, 0.623]
ρ_p	0.588	[0.426, 0.755]	0.384	[0.223, 0.542]
σ_z	1.984	[1.734, 2.222]	1.764	[1.405, 2.137]
σ_b	1.370	[0.692, 2.023]	0.459	[0.254, 0.691]
σ_i	2.703	[2.059, 3.345]	3.374	[2.681, 4.041]
σ_d	2.587	[2.344, 2.794]	2.630	[2.295, 2.994]
σ_w	1.134	[0.940, 1.324]	1.142	[0.950, 1.337]
σ_p	0.508	[0.402, 0.602]	0.430	[0.367, 0.493]
σ_r	0.098	[0.083, 0.111]	0.101	[0.089, 0.112]
σ_s	0.518	[0.359, 0.670]	0.625	[0.462, 0.803]
M_z	-0.562	[-0.790, -0.339]	-0.600	[-0.804, -0.403]
M_b	-0.285	[-0.413, -0.158]	0.319	[-0.474, 1.094]
M_i	-0.202	[-0.325, -0.081]	0.252	[0.144, 0.353]
M_d	-0.133	[-0.193, -0.076]	-0.099	[-0.150, -0.051]
M_w	-0.542	[-0.675, -0.385]	-0.926	[-1.206, -0.657]
M_p	-0.213	[-0.639, 0.232]	-0.649	[-0.996, -0.302]
M_r	0.540	[-0.922, 2.001]	0.128	[-1.360, 1.407]
$\log p(\mathbf{Y}^T)$	-915.986		-852.880	

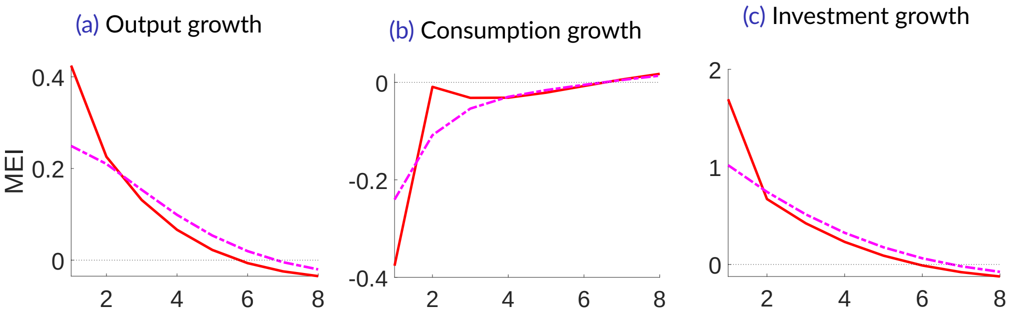
IRFs to Technology Shocks: DE Versus RE Counterfactual



Note: DE in red, RE counterfactual in magenta

- DE generate amplified responses on impact, followed by rapid reversals

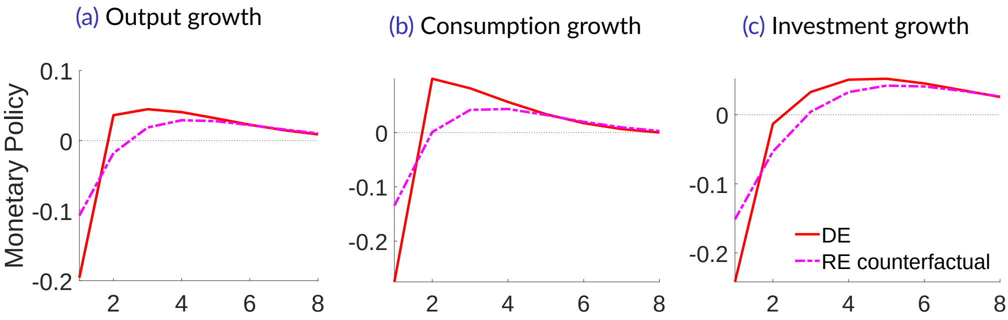
IRFs to MEI Shocks: DE Versus RE counterfactual



Note: DE in red, RE counterfactual in magenta

- Strong investment response crowds out consumption

IRFs to Monetary Policy Shocks: DE Versus RE Counterfactual



Note: DE in red, RE counterfactual in magenta

- Effects of monetary policy shocks are stronger on impact but quickly reverse