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Abstract

Many key macroeconomic variables such as the NAIRU, potential GDP, and the neutral real rate of interest—which are needed for policy analysis—are latent. Collectively, these latent variables are known as 'stars' and are typically estimated using the Kalman filter or smoother from models that can be expressed in State Space form. When these models contain more shocks than observed variables, they are 'short', and potentially create issues in recovering the star variable of interest from the observed data. Recovery issues can occur when the model is correctly specified and its parameters are known. In this paper, we summarize the literature on shock recovery and demonstrate its implications for estimating stars in a number of widely used models in policy analysis. The ability of many popular and recent models to recover stars is shown to be limited. We suggest ways this can be addressed.

JEL classification: C22, C32, E58

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1 Introduction

Stars were historically used as a navigational tool to guide a journey. Today, stars play a similar role in the conduct of macroeconomic policy. When an asterisk is attached to variables such as output, interest rates, or inflation, these variables are collectively known as '*stars*' and refer to an equilibrium state towards which the economy is expected to adjust. Potential output, the neutral real rate of interest and the Non-Accelerating Inflation Rate of Unemployment (NAIRU) are prominent stars.[1]

Stars are effectively the steady-state values that exist in a theoretical model. Since they are functions of the model's parameters, any changes in the star variable itself would necessitate changes in the parameters of the model. Often, such changes are difficult to account for, because stars are likely to be complex, non-linear functions of the model's parameters. A simpler and more commonly used alternative approach is to treat the star variable as a latent exogenous process, with a popular choice being a driftless random walk. As the star variable is not directly observed, it needs to be estimated. This is typically done with a State Space model, using the Kalman filter and smoother to extract a measure of the latent star variable. In almost all such implementations, the number of shocks in the model exceeds the number of observed variables. Adopting the description of Forni *et al.* (2019), such a system is said to be '*short*'. A key finding from the recent theoretical work on shock recovery is that it is never possible to recover all the shocks from a short system.

The objectives of this paper are two-fold. First, we demonstrate the importance of the literature on shock recovery for the modelling of star variables — and more broadly — for policy analysis. Although it is well known that stars are typically estimated imprecisely (e.g., Staiger *et al.*, 1997 and Laubach and Williams, 2003), particularly when this is done in real time (Orphanides and van Norden, 2002), and which creates practical issues for their

[1] There are many new ones. Zaman (2022), for example, lists various stars arising in blocks describing price inflation, wage inflation and interest rates.

use in policy analysis, it is not well known that these models may not be able to recover the star variable of interest to the policy maker. Second, we provide a critical review of many widely used and recent models that have been developed to provide estimates of stars for policy analysis. Specifically, we examine if the shocks driving the star variables in these models can in fact be recovered from the observed data, as this is necessary to recover the star variable itself. This is evaluated assuming the model is correct and that its true parameters are known. We further show that the extent to which a model can recover a star variable can be communicated intuitively with a correlation coefficient. This, we believe, will be useful to policymakers who need to be able to judge the relevance of a model for the policy process.

It is our view that those presenting estimates of stars from short systems are obliged to show that the model *can* in fact recover the key shocks driving the star variable of interest. Indeed, recoverability measures such as the correlation coefficient that we propose should be routinely reported when estimating star variables in the same way that one reports standard errors or confidence intervals to gauge the level of uncertainty surrounding point estimates of parameters computed from a statistical model. Our experience is that this is rarely done. In this paper, we show how to do so and how these results can be best communicated.

The remainder of the paper is structured as follows. Section 2 defines the concept of recoverability, the implications of a short system for recoverability, and distinguishes between recoverability and statistical uncertainty. It also provides a summary of recently developed approaches to assess shock recovery and explains how these can be extended to examine recoverability of star variables.

In Section 3, several applications aimed at recovering star variables are provided. The section begins with a simple example of recovery of potential output from the widely used Hodrick-Prescott (HP) filter, which can be formulated as an Unobserved Components (UC) model. It then proceeds to analyze recovery of the neutral real rate of interest from the influential Laubach and Williams (2003) model and its later updates in Holston *et al.* (2017, 2023), and extension in McCririck and Rees (2017), which is used in the Reserve Bank of Australia's policy model, Ballantyne *et al.* (2020). In all these models, the ability to recover the star variables of interest is limited.

Section 4 proceeds to examine if recovery of stars can be improved by building on the above models or by utilizing a different modelling approach. The original Laubach and Williams (2003) model does not allow for an interest rate rule. Since including such a rule also adds a monetary policy shock to the system, the number of excess shocks and thereby the lack of recoverability of the neutral real rate remains unchanged. This is illustrated within the model of McCririck and Rees (2017). Schmitt-Grohe and Uribe (2022) propose a different type of structural model that incorporates stars. Although their setup is (partially) more successful in recovering the neutral real rate, it crucially depends on the size of one model parameter which attributes what might be thought to be an unrealistically high percentage (nearly 80) of the variation in output growth to the neutral real rate shock. Once this parameter is set to what seems a more reasonable value, the natural real rate cannot be recovered.

Section 4 subsequently assesses recoverability of star variables following an entirely different approach that avoids providing an explicit structural model for the star variable, but instead defines it via the Beveridge-Nelson decomposition. Two such approaches, namely, one by Morley, Tran and Wong (2023), and a second by Lubik and Matthes (2015) are outlined. While the former is more successful than the Laubach and Williams (2003) based approaches, neither one of them can recover the star variable of interest.

In Section 5, two increasingly popular features of macroeconomic models are examined that can potentially (and unintentionally) obscure the stars since they result in short systems. These are news about future shocks and shocks that allow for stochastic volatility. The role of expectations and the use of surveys of forecasts are also discussed. Lastly, Section 6 describes how one might model stars without adding extra shocks to a system. Such an approach has been employed by Okimoto (2019) in the trend inflation literature using smooth-transition models, and seems promising. Section 7 concludes the paper.

2 Recovering Latent Variables from Models

2.1 What is recoverability?

To answer this question, we utilize models that can be written in the following State Space Form (SSF):

$$s_t = As_{t-1} + B\varepsilon_t \tag{1}$$

$$\zeta_t = Cs_t + D\varepsilon_t, \tag{2}$$

where the shocks ε_t are standard normal distributed with a zero mean and identity covariance matrix. There may be identification and other econometric issues in estimating A, B, C and D when there are more shocks than observables. Such issues are discussed in Buncic (2021) for the model of Holston, Laubach and Williams (2017, HLW) that aims to capture the neutral real rate of interest.[2] Despite the empirical importance of such estimation problems, for the purpose of this paper we will assume that the numerical values provided in the papers of the models considered are the true values. This is done so as to abstract from estimation issues in our analysis.

There are two ways of looking at the equations (1) and (2) describing the relationship between variables, realizations and shocks. One of these makes assumptions about the *assumed* shocks ε_t and, given A, B, C, D, characteristics such as variances and covariances of the random variable ζ_t can be determined. In this form the analysis is working from the

[2] This is sometimes also called the natural real rate.

right to left of the SSF equations. As well as knowledge of the model parameters there are *auxiliary assumptions* about the nature of ε_t, for instance, that they are uncorrelated. Given these the SSF can be used to tell the investigator about the *assumed model properties of* ζ_t.

A different perspective comes from introducing the data. Now the LHS of (2) has the data ζ_t^D and this is used to define the *estimated shocks*. These will be either filtered or smoothed. We will largely work with *smoothed shocks* and denote them by $E_T\varepsilon_t$. Thus, smoothed shocks at time t are defined as the expectation of the shock ε_t using *all the* T observations in the sample. Filtered shocks are denoted by $E_t\varepsilon_t$ and are estimated using data up to time period t. Designating the data as ζ_t^D, the (Kalman smoothed) system can be expressed as:

$$E_T s_t = AE_T s_{t-1} + BE_T\varepsilon_t \tag{3}$$

$$\zeta_t^D = CE_T s_t + DE_T\varepsilon_t. \tag{4}$$

Given a set of data (and A, B, C, D), one obtains smoothed shocks from the Kalman smoother. *Recovery* is achieved when we can obtain ε_t from the data using $E_T\varepsilon_t$. *If it is possible to recover ε_t, then it is possible to recover the latent variables s_t, as these are a function of the shocks.* Recovering shocks and stars are intrinsically interrelated.

It is important to highlight here once more that recovery is *not* a model estimation issue, as we have assumed all parameters in the model to be known. It is a *recovery* issue and examines whether it is possible to recover the assumed (theoretical) shocks from the data when using the estimate $E_T\varepsilon_t$ and all the parameters of the model are known. The ability of a model to recover the latent star variable one is trying to estimate when all model parameters are known, or nearly so, is a self-evident, minimal property that any model should satisfy.

2.2 Implications of excess shocks for recoverability

When the number of shocks equal the number of observed variables, then ε_t and $E_T\varepsilon_t$ generally coincide and recovery is satisfied. Therefore, whether the shocks ε_t have their assumed properties can be directly assessed using the estimated $E_T\varepsilon_t$. Conversely, when there are more shocks than observables, the system is said to be '*short*' (Forni *et al.*, 2019), or have '*excess shocks*' (Pagan and Robinson 2022) and recovery is not ensured.

To illustrate the implications of this, it is useful to think about a '*short*' system in the simplest possible scenario where we have one observed variable, and two shocks ε_{1t} and ε_{2t}. We then obtain the following two equations corresponding to (2) and (4):

$$\zeta_t = \varepsilon_{1t} + \varepsilon_{2t} \tag{5}$$

$$\begin{aligned}\zeta_t^D &= E_T\varepsilon_{1t} + E_T\varepsilon_{2t} \qquad (6)\\ &= \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} E_T\varepsilon_{1t} \\ E_T\varepsilon_{2t} \end{bmatrix}}_{E_T\varepsilon_t} \\ &= GE_T\varepsilon_t. \qquad (7)\end{aligned}$$

From the relation in (7), it is apparent that $E_T\varepsilon_t$ cannot be recovered uniquely from ζ_t^D, because G is not a square matrix and thus does not have an inverse. If G was square then we would have a solution for $E_T\varepsilon_t = G^{-1}\zeta_t^D$, and $GG^{-1}G = G$. When it is not square G^{-1} is replaced by a generalized inverse G^+ that satisfies $GG^+G = G$. Then $E_T\varepsilon_t = G^+\zeta_t^D$. Letting $G^+ = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$, $GG^+G = G$ implies that

$$g_1 + g_2 = 1,$$

and so there are many values for g_1.

To select one, it is common to find the value that minimizes $G^{+\prime}G^{+}$. For this case it yields

$$G^{+} = \begin{bmatrix} .5 \\ .5 \end{bmatrix},$$

which implies that $E_T\varepsilon_{1t} = .5\zeta_t^D = E_T\varepsilon_{2t}$. That is, the smoothed shocks $E_T\varepsilon_{1t}$ and $E_T\varepsilon_{2t}$ are *identical* to one another, and thus cannot be separated using the data.

2.3 Assessing shock recoverability

When excess shocks exist in a model, not *all* model shocks can be recovered. However, it may be possible to recover *some* shocks from the model, potentially those of relevance to policy makers that drive the star variable of interest. There exist methods in the literature to show which shocks can be recovered.

Forni *et al.* (2019), building on Sims and Zha (2006), developed a deficiency index to determine whether in a SVAR there is sufficient information to recover a particular shock from current and past information. They find that it may be possible to do so, even when the system as a whole is not invertible. Their deficiency index fundamentally examines the recovery of the shock from its filtered estimates.

Chahrour and Jurado (2022) extend the concept of invertibility to consider an expanded information set which additionally includes *future* information. They term this *recoverability*. Pagan and Robinson (2022) discuss how this is related to the Kalman smoother. In the context of the example of the previous sub-section, one would consider the index $\phi = \mathrm{Var}(E_T\varepsilon_{1t} - \varepsilon_{1t})$. In the simple case above, ϕ would equal $\mathrm{Var}(\frac{1}{2}(\varepsilon_{1t} + \varepsilon_{2t})) = .5$, which shows that ε_{1t} cannot be recovered. What is attractive about this quantity is that the Kalman smoother provides $E_T\varepsilon_{1t}$. To practically implement this approach for assessing recoverability, the shock is added to the state vector of the SSF, and the steady-state Kalman smoother is used to find smoothed estimates of it as well as its mean squared error (denoted by $P_{t|T}$). This can be implemented for a wide range of models.

Turning to the interpretation of ϕ we note that

$$\begin{aligned}\phi &= \text{Var}(E_T\varepsilon_{1t} - \varepsilon_{1t}). \\ &= \text{Var}(E_T\varepsilon_{1t}) - 2\text{Cov}(E_T\varepsilon_{1t}, \varepsilon_{1t}) + \text{Var}(\varepsilon_{1t}) \\ &= \text{Var}(E_T\varepsilon_{1t}) - 2\rho \times \sigma(E_T\varepsilon_{1t}) + 1\end{aligned}$$

where ρ is the correlation between $E_T\varepsilon_{1t}$ and ε_{1t}, and $\sigma(E_T\varepsilon_{1t}) \equiv \sqrt{\text{Var}(E_T\varepsilon_{1t})}$ is the standard deviation of $E_T\varepsilon_{1t}$. Re-arranging this equation yields an expression for the correlation between the smoothed and actual shocks:

$$\rho = \frac{1}{2}\left(\frac{1-\phi}{\sigma(E_T\varepsilon_{1t})} + \sigma(E_T\varepsilon_{1t})\right). \tag{8}$$

When the shock is recoverable $E_T\varepsilon_{1t} = \varepsilon_t$ and this implies that $\phi = 0$. Because $\sigma(E_T\varepsilon_{1t}) = \sigma(\varepsilon_{1t}) = 1$ it is the case that $\rho = 1$. At the other extreme, when $\phi = 1$, then $\rho = \frac{\sigma(E_T\varepsilon_{1t})}{2}$, and so the correlation between $E_T\varepsilon_{1t}$ and ε_t depends on $\sigma(E_T\varepsilon_{1t})$, which can easily be computed.

Since the star variable is a function of the shocks, whether it is recoverable or not can be assessed by looking at $P_{t|T}$. Nonetheless, there are two aspects to take note of. First, in some instances, the star variable is modelled as a latent non-stationary process. In this case, it should be the (appropriately) differenced series that is assessed for recovery. Second, it is necessary to normalize the differenced star variable so that ϕ lies between 0 and 1. It is thus more convenient to report the correlation between the smoothed and actual (differenced) star variable, akin to ρ above. This is a highly useful and intuitive way of communicating the degree to which a model can recover a star variable. Indeed, our view is that one should always routinely report such a correlation measure in the same way as one reports confidence intervals to gauge the level of statistical uncertainty surrounding point estimates of parameters.

The correlation of the differenced star variable above depends on $\sigma(E_T\varepsilon_{1t})$. Instead of calculating this quantity from the smoothed shocks obtained from the data, it is more appropriate to use its population counterpart, which can be found by simulating a long sequence of data from the model, applying the Kalman filter and smoother to obtain $E_T\varepsilon_{1t}$ and then computing $\sigma(E_T\varepsilon_{1t})$ based on the sample standard deviation from the smoothed series $E_T\varepsilon_{1t}$. The correlation ρ, and its counterpart for the differenced star variable, can thus be thought of as a population quantity.

An alternative way to assess the differences between the shocks estimated from the data and the assumed model shocks was suggested by Plagborg-Møller and Wolf (2022); namely the R^2 from the population regression of the model shock ε_{1t} against the estimated smoothed shock $E_T\varepsilon_{1t}$. An R^2 of zero means there is no correlation between them, while an R^2 of unity means that one can perfectly recover the model shock ε_{1t} from $E_T\varepsilon_{1t}$. An appealing aspect of all these approaches is that they generalize to non-linear models where the standard linear Gaussian Kalman filter cannot be applied, provided that an alternative (non-linear) filter is available to produce $E_T\varepsilon_{1t}$.

Although we have abstracted from this, note that the estimated shocks can be correlated for reasons other than the system being short, such as misspecification. Pagan and Robinson (2022) present an indirect inference approach for assessing if the correlation in the estimated shocks aligns with what was assumed by the model).

2.4 Consequences of shocks not being recoverable

The primary message of this paper is that if a star variable is modelled as a function of unrecoverable shocks, then the star variable *itself* cannot be recovered from the data.

There are various other consequences of excess shocks that can hinder our ability to interpret the economy through a model. These are discussed in Pagan and Robinson (2022). Of note here is that *variance* and *variable decompositions relate the data to the estimated shocks*. When shocks are recoverable, and they have been assumed to be uncorrelated, the variance of the data will equal the sum of the variances of the shocks. If, on the other hand, excess shocks exist, then at least some of these estimated shocks will be correlated, counter to the assumption underlying variance decompositions. Indeed, in models frequently used for estimating stars, the relationship between smoothed shocks is not a simple correlation but a complex dynamic one, making it difficult to assess the economic importance of changes in the star variables.

It is possible to compute the variance decomposition of ζ_t with respect to the ε_t, as is suggested in Plagborg-Møller and Wolf (2022). This, however, is not a variance decomposition of the data, but rather of what the assumed model and auxiliary assumptions imply. In an exactly identified SVAR that has no excess shocks, these are the same, but this is not true in cases with short systems.

3 Applications Aimed at Recovering Stars

To illustrate recoverability issues, we begin with a simple UC model which aims to recover potential output from the Hodrick Prescott (1997) filter. We then move on to the models of Laubach and Williams (2003) and their subsequent updates, Holston *et al.* (2017) and (2023), which aim to estimate the neutral real rate. Finally, we look at a model that builds upon Laubach and Williams (2003), namely that of McCririck and Rees (2017), but which estimates three stars simultaneously: the NAIRU, the neutral real rate, and potential output. In all of these cases, there are problems with recovering the star variables, particularly the neutral real rate.

3.1 Recovering Potential Output from the HP Filter

One fundamental star variable of interest is potential output. It is primarily used to construct the output gap, which is a measure of slack in the economy. In the typical setting, potential output y_t^p is driven by permanent shocks ε_{1t}, while the output gap y_t^c is assumed to be stationary, and thus can only be a function of transitory shocks ε_{2t}. There are many alternative specifications of these two components.

A popular one due to Hodrick and Prescott (1997) (HP) can be expressed as a UC model taking the form:[3]

$$y_t = y_t^p + y_t^c \tag{9a}$$

$$\Delta^2 y_t^p = \sigma^p \varepsilon_{1t} \tag{9b}$$

$$y_t^c = \sigma^c \varepsilon_{2t}, \tag{9c}$$

where ε_{1t} and ε_{2t} are uncorrelated standard normal disturbances. As shown in Pagan and Robinson (2022), the estimated smoothed permanent shocks $E_T\varepsilon_{1t}$ and the estimated smoothed output gap shocks $E_T\varepsilon_{2t}$ are *dynamically correlated* via the identity:

$$(1 - 2L + L^2)E_T\varepsilon_{1t} = (1/\lambda)E_T\varepsilon_{2t}, \tag{10}$$

where λ is the smoothing parameter of the HP filter (commonly set to 1,600 for quarterly macroeconomic data).[4] This dynamic correlation via the identity in (10) makes it impossible to disentangle the two different shocks ε_{1t} (permanent) and ε_{2t} (transitory) assumed to hold in the theoretical model in (9) when using their smoothed estimates.

[3]See Harvey and Jaeger (1993).

[4]Note that $\sqrt{\lambda} = \sigma^c/\sigma^p$ is the (inverse) signal-to-noise ratio parameter, where it is further common to assume that σ^c is equal to unity, so that $\sigma^p = 1/40$.

3.2 Recovering the Neutral Real Rate - Laubach and Williams (2003)

One the most influential models of the neutral real rate r_t^* of the past two decades is that of Laubach and Williams (2003, LW). There exist numerous alternative and/or extended versions of the LW model in the literature, and these are widely used at central banks and other policy institutions. The LW model consists of the following equations:

$$\tilde{y}_t = \alpha_1 \tilde{y}_{t-1} + \alpha_2 \tilde{y}_{t-2} + \frac{a_r}{2} \sum_{i=1}^{2} (r_{t-i} - r_{t-i}^*) + \sigma_1 \varepsilon_{1t} \tag{11a}$$

$$\pi_t = B(L)\pi_{t-1} + b_I(\pi_t^I - \pi_t) + b_o(\pi_{t-1}^o - \pi_{t-1}) + b_y \tilde{y}_{t-1} + \sigma_2 \varepsilon_{2t} \tag{11b}$$

$$\Delta z_t = \sigma_3 \varepsilon_{3t} \tag{11c}$$

$$\Delta y_t^* = g_{t-1} + \sigma_4 \varepsilon_{4t} \tag{11d}$$

$$\Delta g_t = \sigma_5 \varepsilon_{5t} \tag{11e}$$

$$r_t^* = c4 g_t + z_t, \tag{11f}$$

where $\tilde{y}_t = (y_t - y_t^*)$ is the output gap, y_t is log GDP, y_t^* is potential GDP, r_t is a real interest rate, r_t^* is the neutral real rate, and π_t, π_t^I and π_t^o are various measures of inflation. There are evolving processes for the trend growth of GDP g_t, and '*other determinants*' z_t, which affect r_t^*. There are a total of five shocks $\{\sigma_i \varepsilon_{it}\}_{i=1}^5$ with standard deviations $\{\sigma_i\}_{i=1}^5$, and the error terms $\{\varepsilon_{it}\}_{i=1}^5$ have unit variances. While the focus of LW is on estimating r_t^*, trend growth g_t is also estimated in the model.

In order to assess recoverability as defined by Chahrour and Jurado (2022), we follow the approach of Pagan and Robinson (2022) and write LW's model in (11) in SSF so that all observables are contained in ζ_t on the LHS of (1), and all shocks and remaining latent states are collected in the state vector s_t. The measurement equations are:

$$\zeta_{1t} = y_t^* - \alpha_1 y_{t-1}^* - \alpha_2 y_{t-2}^* + \frac{a_r}{2} \sum_{i=1}^{2} r_{t-i}^* + \sigma_1 \varepsilon_{1t}, \tag{12}$$

$$\zeta_{2t} = b_y y^*_{t-1} + \sigma_2 \varepsilon_{2t}, \tag{13}$$

and the relevant state dynamics are given by:

$$\Delta y^*_t = g_{t-1} + \sigma_4 \varepsilon_{4t} \tag{14}$$

$$\Delta g_t = \sigma_5 \varepsilon_{5t} \tag{15}$$

$$\Delta r^*_t = c4\sigma_5 \varepsilon_{5t} + \sigma_3 \varepsilon_{3t}. \tag{16}$$

The state vector consists of y^*_t, y^*_{t-1}, g_t, r^*_t, r^*_{t-1}, and the five shocks $\{\varepsilon_{it}\}_{i=1}^5$ to be able to assess shock recovery. The LHS observable part of ζ_t consists of:

$$y_t - \alpha_1 y_{t-1} - \alpha_2 y_{t-2} + \frac{a_r}{2} \sum_{i=1}^{2} r_{t-i} = \zeta_{1t} \tag{17}$$

$$\pi_t - B(L)\pi_{t-1} - b_I(\pi^I_t - \pi_t) - b_o(\pi^o_{t-1} - \pi_{t-1}) - b_y y_{t-1} = \zeta_{2t}. \tag{18}$$

All relevant parameter estimates are taken from LW and are reported in Appendix A for convenience.

Note here that the SSF corresponding to LW's structural model has *five shocks*, but only *two observed variables.* This means that it will not be possible to recover more than two unique shocks from this model. These could be linear combinations of all five shocks in the model, which is often termed *"packages of shocks"*, rather than any one of the five shocks in LW's model.

Given the SSF, we can determine which shocks are likely to be recoverable and which ones are not. Following again Pagan and Robinson (2022), we compute the steady state covariance matrix corresponding to the state vector containing the five shocks of interest, which we denote by $P^*_{t|T}$. A shock will be recoverable from the model if the diagonal element of $P^*_{t|T}$ contains a zero entry, and will be unrecoverable if it is equal to unity. For LW's model, the following values for $\text{diag}(P^*_{t|T})$ (corresponding to the five shocks $\{\varepsilon_{it}\}_{i=1}^5$) are obtained:

$$\text{diag}(P^*_{t|T}) = \begin{bmatrix} .71 & .02 & .98 & .36 & .94 \end{bmatrix}. \tag{19}$$

It is clear from equation (19) that the shock belonging to '*other determinants*' ε_{3t} and the trend growth shock ε_{5t} cannot be recovered from the LW model.[5] Note from equation (16) that these two shocks define the neutral rate. Hence, the neutral rate itself cannot be recovered from this model. In fact, the correlation between the smoothed estimate of the change in the neutral real rate and its true value is only 0.22.[6] This, we believe, is useful information for any policymaker considering using this model, distinct from that in the confidence intervals reflecting *statistical* uncertainty.[7]

A direct consequence of the lack of recoverability in LW's model is that the smoothed shocks of the Δy^*_t, Δg_t and Δz_t equations given in (11d), (11e) and (11c) are related through an identity. That is, defining $\eta_{it} = \sigma_i \varepsilon_{it}$, this identity involves the smoothed estimates of the trend growth shock $\Delta E_T \eta_{5t}$, the '*other determinants*' shock $\Delta E_T \eta_{3t}$, and the trend shock $E_T \eta_{4t}$:

$$\Delta E_T \eta_{5t} = 0.107 \Delta E_T \eta_{3t} - 0.028 E_T \eta_{4t}. \tag{20}$$

Therefore, LW's estimated model *cannot* distinguish which shocks are driving the real neutral interest rate r^*_t.

Holston, Laubach and Williams (2017, HLW) provide an updated version of the original LW model using a somewhat different formulation of the Phillips curve equation (11b)

[5]The cost push shock ε_{2t}, on the other hand, does seem to be recoverable.

[6]The correlation is calculated as

$$\rho = \frac{\text{Var}(E_T \Delta r^*_t) + \text{Var}(\Delta r^*_t) - \phi}{2\sigma(E_T \Delta r^*_t)\sigma(\Delta r^*_t)},$$

where $\text{Var}(\Delta r^*_t) = c^2 4^2 \sigma_5^2 + \sigma_3^2$, $\sigma(\Delta r^*_t) \equiv \sqrt{\text{Var}(\Delta r^*_t)}$, and $\text{Var}(E_T \Delta r^*_t)$ is found through simulation and application of the Kalman filter and smoother to the simulated data.

[7]ϕ for the smoothed standardized change in the neutral real rate is 0.95.

estimated over a longer sample period, and one might ask whether recoverability in HLW improves over LW. Examining again the steady state covariance matrix corresponding to the state vector it is clear that this is not the case. The $\text{diag}(P^*_{t|T})$ terms are shown below:[8]

$$\text{diag}(P^*_{t|T}) = \begin{bmatrix} .72 & .02 & .99 & .30 & .97 \end{bmatrix},$$

suggesting that the lack of recoverability of the neutral rate is unchanged. Indeed, the correlation for the estimated change in the neutral rate with the actual is 0.14.

From the smoothed states we can further establish the following two identities:[9]

$$E_T \Delta r^*_t = E_T \eta_{5t} + E_T \eta_{3t}, \tag{21}$$

and

$$\begin{aligned} E_T \Delta r^*_t = {} & E_T \Delta r^*_{t-1} - .0495 E_T \eta_{1t} + .0003 E_T \eta_{2t} \\ & - .0037 E_T \eta_{4t} + .0231 E_T \eta_{1t-1} - .0078 E_T \eta_{4t-1}. \end{aligned} \tag{22}$$

These show that whatever $E_T \Delta r^*_t$ is measuring can be equally well explained by either (21) or (22). The latter involves a dynamic combination of smoothed demand, technology and Phillips curve shocks, while the former has smoothed values of the shocks meant to explain the neutral real rate. Consequently, the presence of a short system creates interpretation difficulties.

Due to the impact of COVID-19 on the variables in HLW's model, Holston *et al.* (2023) modify the specification in HLW by allowing potential output to be impacted by govern-

[8]For the sake of brevity and to avoid repetition, the equations of the HLW model are not reproduced here but are readily available from Buncic (2022). The parameter estimates of HLW's model for the U.S. are also taken from Table 3 in Buncic (2022), and are reported once again in Appendix A of this paper.

[9]η_{5t} has been multiplied by 4, reflecting that g_t is annualized.

ment policy responses, which they measure using the Oxford policy tracker (Hale *et al.*, 2021), and they allow the variance of the shocks to temporarily increase (similar to Lenza and Primiceri, 2022). Using the parameter estimates from the post COVID-19 version of HLW, once again we find that the shocks driving the neutral rate cannot be recovered, as $\text{diag}(P^*_{t|T})$ for the five shocks yields:

$$\text{diag}(P^*_{t|T}) = \begin{bmatrix} .55 & .02 & .47 & .97 & .99 \end{bmatrix}.$$

Moreover, the correlation of the estimated change in the neutral rate with the actual remains low (0.17).

In summary, in all three variants of the Laubach and Williams (2003) model it is not possible to recover r^*_t from the data. This message can easily be communicated to policymakers using the correlation coefficient between the value implied by the model and the estimate from the data, which is never above 0.22.

To give one example of why this outcome will be an issue, consider the neutral real rate evolving as $\Delta r^*_t = \sigma \varepsilon_t$. Then the estimated neutral rate evolves as $E_T \Delta r^*_t = \sigma E_T \varepsilon_t$. Suppose, for example, that there is a zero correlation between ε_t and $E_T \varepsilon_t$. Now the model implies that r_t and r^*_t are co-integrated, so one might be led to plot r_t against $\sum_{i=1}^{t} E_T \Delta r^*_i$. It could well look as if one tracks the other but that would be a classic example of a spurious relationship; that is, the visual tracking can look close even though there is no actual relationship, simply because of the $I(1)$ property of both series.

3.3 Recovering the Neutral Real Rate and the NAIRU - McCririck and Rees (2017)

McCririck and Rees (2017, MR) is effectively an extension of LW's model, adding an equation for Okun's law to enable the determination of a number of macroeconomic stars. These are:

growth in potential GDP, the NAIRU, and the neutral real interest rate, denoted by g_t, u_t^* and r_t^*, respectively. The model takes the following form:[10]

$$\tilde{y}_t = \alpha_1 \tilde{y}_{t-1} + \alpha_2 \tilde{y}_{t-2} - \frac{a_r}{2} \sum_{i=1}^{2} (r_{t-i} - r_{t-i}^*) + \sigma_1 \varepsilon_{1t} \tag{23}$$

$$\pi_t = (1 - \beta_1)\pi_t^e + \frac{\beta_1}{3} \sum_{i=1}^{3} \pi_{t-i} + \beta_2 (u_{t-1} - u_{t-1}^*) + \sigma_2 \varepsilon_{2t} \tag{24}$$

$$\Delta z_t = \sigma_3 \varepsilon_{3t}, \tag{25}$$

$$\Delta y_t^* = g_t + \sigma_4 \varepsilon_{4t} \tag{26}$$

$$\Delta g_t = \sigma_5 \varepsilon_{5t} \tag{27}$$

$$\Delta u_t^* = \sigma_6 \varepsilon_{6t} \tag{28}$$

$$u_t = u_t^* + \beta(.4\tilde{y}_t + .3\tilde{y}_{t-1} + .2\tilde{y}_{t-2} + .1\tilde{y}_{t-3}) + \sigma_7 \varepsilon_{7t} \tag{29}$$

$$r_t^* = 4g_t + z_t, \tag{30}$$

where $\tilde{y}_t = (y_t - y_t^*)$ is the output gap, y_t is log GDP, y_t^* is potential GDP, r_t is the real interest rate, r_t^* the neutral real rate, u_t is the unemployment rate and u_t^* the NAIRU, π_t is inflation and π_t^e is measured expected inflation.

In MR's model, there are three observables — output growth, inflation and the unemployment rate — and seven shocks, so again the full set of seven shocks cannot be recovered. Casting it in a State Space Form similar to that used for LW in Section 3.2, and using the posterior means reported in Table A2 of their paper (also in Appendix A of this paper), we find the following diagonal elements of $P_{t|T}^*$:

$$\text{diag}(P_{t|T}^*) = \begin{bmatrix} .55 & .03 & .98 & .24 & .96 & .49 & .98 \end{bmatrix}. \tag{31}$$

[10]Note that in MR, g_t rather than g_{t-1} is in the potential GDP growth equation, and the sign of the interest rate variables in the IS equation has changed. Also, for ease of comparability, we use the shock numbering $\{\sigma_i \varepsilon_{it}\}_{i=1}^{7}$ as in LW, rather than the labelling used in MR.

So, while there are issues in recovering the NAIRU shock ε_{6t}, the biggest concern is still the recovery of the neutral rate, since the diag$(P^*_{t|T})$ values corresponding to the two shocks that define r^*_t (ε_{3t} and ε_{5t}) are still indicating a lack of recoverability. A dynamic correlation between the smoothed estimates of ε_{5t} and several of the other shocks is also apparent. Therefore, giving these shocks macroeconomic names or labels, and understanding what is driving the estimates of r^*_t, difficult.[11] The correlation of the estimated and actual neutral rate remains low (0.19).

4 Star Wars: Is there a Better Way to Recover Stars?

Even though one cannot recover stars from the models with excess shocks described above, perhaps one can get closer by using a different structural representation or filter. In the context of our metaphor of stars being used as a guide in a journey, it might be possible to think of this strategy as devising a better star map in order to get a more precise view of the location of the stars. To investigate this we consider two different structural models. The first introduces an explicit interest rate (or policy) rule to the LW model; something that was noticeably absent. The second model has both a monetary, as well as an interest rate rule in it, and was recently proposed for estimating neutral real rates by Schmidt-Grohe and Uribe (2022).

Instead of a different structural model one might get a clearer view of the stars with a different telescope. In particular one might define the star variable via a Beveridge-Nelson (1981, BN) decomposition and we consider two applications of this type. The first was recently advocated by Morley, Tran and Wong (MTW, 2023), and a second by Lubik and Matthes (2015, LM). The latter employ a finite-horizon version of the BN decomposition linked to a TVP VAR model for variables related to the star.

[11] The identity $E_T\eta_{1t} = -E_T\Delta\eta_{1t-1} - 456.333E_T\Delta\eta_{5t-1} - 3.771E_T\eta_{4t-2}$ exists.

4.1 Endogenous interest rates: can this change the outcome?

In the above applications involving the LW model, the policy rate was assumed to be exogenous, i.e., there was no equation to explain its evolution, such as a standard Taylor rule commonly used in macroeconomic models. As Pagan and Wickens (2022) observed, this means that the LW model has some undesirable features.

To see some of these undesirable features, it is useful to consider the time-series properties of the series implied by the LW model. By definition, r_t^* is an integrated process of order one, $I(1)$ henceforth, since both g_t and z_t are $I(1)$ processes. Because there is no equation for r_t in LW, there is no mechanism in place to ensure that r_t^* and r_t co-integrate. If they do not co-integrate, then both the output gap, $\tilde{y}_t$, and inflation π_t will be $I(1)$.[12] Since the goal of many central banks is to stabilize inflation, it is difficult to see how this can be achieved when inflation is allowed to follow an $I(1)$ process and there is no control rule to make it $I(0)$. A simple way to avoid this issue is to add a monetary rule to the LW model.

This can be examined with the MR model. As the latter was utilized in the MARTIN policy model of the Reserve Bank Australia (see Ballantyne *et al.*, 2020), it is natural to adopt their nominal interest rate rule:

$$i_t = .7i_{t-1} + .3(r_t^* + \pi_t - \bar{\pi} - 2(u_t - u_t^*)) - \Delta_2 u_t + 1.19\varepsilon_{8t}.$$

where $\bar{\pi}$ denotes the inflation target. This implies that the real interest rate r_t would be:

$$r_t = .7r_{t-1} - .7\Delta\pi_t + .3r_t^* + .3\bar{\pi} - .6(u_t - u_t^*) - \Delta_2 u_t + 1.19\varepsilon_{8t}.$$

It is important to note that we have both an additional observed variable and an additional monetary policy shock ε_{8t}.

[12] Of course the model implies that y_t^* is $I(2)$, so that y_t will be $I(2)$ as well.

So there are now four observed variables and eight shocks, again implying a short system, meaning that not all shocks will be recoverable. This is seen by computing

$$\text{diag}(P^*_{t|T}) = \begin{bmatrix} .54 & .02 & .96 & .23 & .95 & .49 & .76 & .05 \end{bmatrix}. \tag{32}$$

Recall that the neutral real interest rate in this model is driven by the innovations to Δz_t (ε_{3t}), and technology growth shock ε_{5t}. The relevant entries of $\text{diag}(P^*_{t|T})$ are 0.96 and 0.95. Evidently, these are little changed from those in (31) at 0.98 and and 0.96. Therefore, adding a policy rule does not appear to alter the lack of recoverability of the neutral real interest rate.

4.2 New Structural Models - Schmitt-Grohe and Uribe (2022)

Schmitt-Grohe and Uribe (2022, SGU) present a new structural model for estimating the neutral real rate. The model assumes that the log level of per capita output y_t is driven by two permanent stochastic components, x_t and x^r_t, which represent technology and non-monetary factors affecting the real interest rate. Inflation π_t is $I(1)$ and its permanent component is the nominal inflation target. Lastly, the nominal interest rate is $I(1)$, and it is driven by two permanent components — the inflation target and the non-monetary real rate permanent component. Denoting the transitory (gap) components in these variables with a tilde, these can be expressed as:

$$\tilde{y}_t = y_t - x_t - \delta x^r_t$$

$$\tilde{\pi}_t = \pi_t - x^m_t$$

$$\tilde{i}_t = i_t - (1+\alpha)x^m_t - x^r_t.$$

The neutral real rate is taken to be a combination of the permanent components driving inflation, x_t^m, and x_t^r, although in their final model they set $\alpha = 0$ producing

$$r_t^* = x_t^r.$$

This is different to LW's natural rate specification in equation (11f), which takes the form:

$$r_t^* = c4g_t + z_t.$$

Thus, in LW r_t^* responds to growth in potential GDP (Δg_t) coming from technology, as well as to an "other" real non-monetary shock, Δz_t. In contrast, the SGU specification has no role for technology shocks to affect r_t^*. Therefore, SGU's model is rather different to LW's.

To analyze the properties of SGU's model, we define $\Phi_t = \begin{bmatrix} \tilde{y}_t & \tilde{\pi}_t & \tilde{\imath}_t \end{bmatrix}'$ and $\xi_t = \begin{bmatrix} \Delta x_t^m & \tau_t^m & \Delta x_t & \tau_t & \Delta x_t^r \end{bmatrix}'$, where τ_t^m and τ_t are stationary monetary and real shocks. Then the dynamics for the gaps Φ_t are described by the following Vector AutoRegression (VAR) equation:

$$\Phi_t = B\Phi_{t-1} + C\xi_t,$$

while the observation equations are:

$$\Delta y_t = \Delta\tilde{y}_t + \Delta x_t + \delta\Delta x_t^r + \sigma_y \varepsilon_t^y \tag{33}$$

$$\Delta\pi_t = \Delta\tilde{\pi}_t + \Delta x_t^m + \sigma_\pi \varepsilon_t^\pi \tag{34}$$

$$\Delta i_t = \Delta\tilde{\imath}_t + (1+\alpha)\Delta x_t^m + \Delta x_t^r + \sigma_i \varepsilon_t^i, \tag{35}$$

where ε_t^y, ε_t^π and ε_t^i are measurement errors. As the observed variables are first differences, these measurement error shocks will have a permanent impact. Notice that even without the addition of measurement errors, the system is short, having three observables and five shocks.

The shocks are assumed to evolve as AR(1) processes:

$$\begin{aligned}
\Delta x_t^m &= \rho_1 \Delta x_{t-1}^m + \sigma_1 \varepsilon_{1t} \\
\tau_t^m &= \rho_2 \tau_{t-1}^m + \sigma_2 \varepsilon_{2t} \\
\Delta x_t &= \rho_3 \Delta x_{t-1} + \sigma_3 \varepsilon_{3t} \\
\tau_t &= \rho_4 \tau_{t-1} + \sigma_4 \varepsilon_{4t} \\
\Delta x_t^r &= \rho_5 \Delta x_{t-1}^r + \sigma_5 \varepsilon_{5t}.
\end{aligned} \tag{36}$$

SGU estimate the system parameters by Bayesian methods. Some of the entries in C are fixed at values needed for identification of the parameters. The posterior means of the parameters are provided in the Appendix.

It should be clear that, to recover r_t^*, one needs to be able to recover ε_{5t} in (36). Including the measurement errors, there are eight shocks and three observed variables, which means that we can only recover three shocks.[13] The last eight entries of the diagonal of the $P_{t|T}^*$ matrix are:

$$\text{diag}(P_{t|T}^*) = \begin{bmatrix} .81 & .56 & .59 & .56 & .16 & .91 & .74 & .67 \end{bmatrix}. \tag{37}$$

From (37) it appears that only the neutral real rate shock ε_{5t} might be recovered, as the value of .16 could be viewed as close to zero.

One might ask here how recovery of the real rate shock changes with the sensitivity of the output gap to it. This is captured by the parameter δ in (33) as it's value determines how important technology shocks are relative to "other" real shocks.[14] At the extreme, when $\delta = 0$, a much higher value for $P_{t|T}^*$ of .83 is found for ε_{5t}, and that would suggest that the

[13]There are 16 states in total consisting of Δx_t^m, z_t^m, Δx_t, z_t, Δx_t^r, $\tilde{y}_t$, $\tilde{\pi}_t$, $\tilde{\imath}_t$ plus the eight innovations $\{\{\varepsilon_{it}\}_{i=1}^5, \varepsilon_t^y, \varepsilon_t^\pi, \varepsilon_t^i\}$.

[14]In SGU's paper, the posterior median of δ is 8.6, so there is little difference between that value, and the posterior mean value which we use (8.3292).

shock cannot be recovered. This points to a fundamental role for δ in the recovery of shocks in this model. To examine this more closely take the equation for the output gap:

$$\begin{aligned}\tilde{y}_t &= b_{11}\tilde{y}_{t-1} + b_{12}\tilde{\pi}_{t-1} + b_{13}\tilde{\imath}_{t-1} + c_{11}\Delta x_t^m \\ &\quad + c_{13}\Delta x_t + c_{14}z_t + c_{15}\Delta x_t^r.\end{aligned}$$

There is a measurement equation involving observed output growth Δy_t that is given by:

$$\begin{aligned}\Delta y_t &= (b_{11}-1)\tilde{y}_{t-1} + b_{12}\tilde{\pi}_{t-1} + b_{13}\tilde{\imath}_{t-1} + c_{11}\Delta x_t^m \\ &\quad + c_{13}\Delta x_t + c_{14}z_t + (c_{15}+\delta)\Delta x_t^r + \sigma_y \varepsilon_t^y \\ &= \eta_t + (c_{15}+\delta)\Delta x_t^r.\end{aligned}$$

Suppose now that $\rho_5 = 0$ in equation (36). Then, η_t is uncorrelated with Δx_t^r. Moreover, δ does not affect the variance of this latter variable. Hence the variance of Δy_t would vary directly with δ, once all other parameters are set (e.g. to the posterior mean). This gives rise to two interesting observations. First, the posterior mean of c_{15} is very small ($-.0051$). If it was zero, then the model variance of Δy_t will depend on δ^2. This may explain why SGU found that there was some evidence of counter-intuitive *negative* values for δ. Indeed, setting $\delta = 8.3292$ (the posterior mean) produces standard deviations of Δy_t, Δp_t and Δi_t of $4.67,\ 1.63$ and 1.32, whereas putting $\delta = -8.3292$ we similarly get $4.65,\ 1.63$ and 1.32.

Secondly, the fraction of the variance of Δy_t explained by the real rate shock ε_{5t} will rise as δ rises. Thus, when $\delta = 8.3292$ we find that nearly 80% of the variation in GDP growth is due to neutral real rate shocks. This appears to be rather high, since these are shocks that, as Schmitt-Grohe and Uribe (2022, p. 4) write: "*could stem from, for example, secular variations in demographic variables, exogenous changes in subjective discount rates, or in other factors determining the domestic or external willingness to save*". To reduce this influence it is necessary to reduce the magnitude of δ. Indeed, if $\delta = 2$, the real neutral rate shocks explain 18% of output growth and, with that value, the diag$(P^*_{t|T})$ entry for the fifth shock ε_{5t} is .71, indicating that it cannot be recovered. Clearly, the issue here is whether we have strong opinions about the likelihood of these "other" real shocks driving so much of growth, while technology shocks determine so little, which is what a value of $\delta = 8.3292$ implies.

Why does one get such a high δ estimate from the model? Fundamentally, δ is a free parameter that enables the model to better match the data on output growth. To see this, note that the standard deviation of GDP growth is 4.89 in the empirical data. Setting $\delta = 8.3292$ leads to a model based value of the standard deviation of GDP of 4.67, and this evidently matches the data rather well. If instead, $\delta = 2$, there is a standard deviation of GDP growth of 2.37 — a rather poor match. Thus, as δ rises, a larger proportion of output growth is accounted for by the real neutral rate shock, making recovery of that shock easier from the data.

4.3 A Different Telescope - The Beveridge-Nelson Filter

The Beveridge-Nelson (BN) decomposition has been used in several ways to estimate stars. Morley *et al.* (2023, MTW) is a recent approach. They define the star variable as the permanent component of a series found with the BN decomposition. This is a sensible proposal, but there are possible short system issues which we investigate in the first sub-section that follows below. An earlier proposal using BN was Lubik and Matthes (2015, LM) who estimate a simple TVP-VAR for three variables to find the neutral real rate. They deviate from the standard BN decomposition by working with a time horizon of five years rather than an infinite one when defining the permanent component as the *'long-run'* forecast. Again, there are short system issues that we cover.

4.3.1 The MTW (2023) BN Approach

MTW's strategy consists of three steps to estimate the star variable of interest, which is the real neutral rate r_t^*. Unlike other studies, MTW treat the real rate of interest r_t as latent and define the observable real rate as $\tilde{r}_t = r_t + vm_{1t}$, where vm_{1t} is an $I(0)$ measurement error, uncorrelated with r_t.[15] There are other observable variables in the system. To briefly summarize the MTW approach, we use the data generating process of their simulation example in Section 3.3, which contains one additional observable variable $\tilde{x}_t$ that is similarly related to x_t via measurement error vm_{2t}.

First, an assumption about the behaviour of the latent variables r_t and x_t is made. In their simulations, these variables follow a VAR(1) of the form:

$$\underbrace{\begin{bmatrix} \Delta r_t \\ \Delta x_t \end{bmatrix}}_{\Delta z_t} = \underbrace{\begin{bmatrix} 0 & -.05 \\ 0 & .95 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \Delta r_{t-1} \\ \Delta x_{t-1} \end{bmatrix}}_{\Delta z_{t-1}} + \underbrace{\begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix}}_{v_t}$$

$$\Delta z_t = A \Delta z_{t-1} + v_t, \tag{38}$$

where

$$\underbrace{\begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix}}_{v_t} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \underbrace{\begin{bmatrix} .1125 & .1 \\ .1 & .1 \end{bmatrix}}_{V} \right). \tag{39}$$

The BN definition of the permanent components corresponding to (38), denoted with a su-

[15]It is unclear why the measurement error is on the level of r_t, rather than on the growth rate Δr_t, since it would become less and less important as the sample size grows. Nonetheless, the same analysis that we provide below would still apply if it was on Δr_t.

perscript p, is given by:

$$
\begin{aligned}
\Delta z_t^p &= (I - A)^{-1} v_t \qquad (40)\\
\begin{bmatrix} \Delta r_t^p \\ \Delta x_t^p \end{bmatrix} &= \begin{bmatrix} 1 & .05 \\ 0 & .05 \end{bmatrix}^{-1} \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} \\
&= \begin{bmatrix} 1 & -1 \\ 0 & 20 \end{bmatrix} \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix},
\end{aligned}
$$

yielding the individual equations:

$$\Delta r_t^p = v_{1t} - v_{2t} \qquad (41a)$$

$$\Delta x_t^p = 20 v_{2t}. \qquad (41b)$$

The relations in (41) are the permanent components of the (multivariate) BN decomposition of Δz_t, where Δr_t^p in (41a) is the BN estimate of Δr_t^*.

In their second step, because $\tilde{r}_t$ and $\tilde{x}_t$ are observables and r_t and x_t are not, all the variables are connected by measurement errors specified as $vm_t = \sqrt{0.05}\, v_t$, where v_t is defined in (39). This leads to the system:

$$
\begin{aligned}
\Delta \tilde{r}_t &= \Delta r_t + \Delta vm_{1t} \\
\Delta \tilde{x}_t &= \Delta x_t + \Delta vm_{2t},
\end{aligned}
$$

which implies

$$
\begin{aligned}
\Delta \tilde{z}_t &= \Delta z_t + \Delta vm_t \\
&= (I - AL)^{-1} v_t + \Delta vm_t.
\end{aligned}
$$

Consequently, the BN estimate of the permanent component in terms of observables $\Delta\tilde{z}_t$ is:

$$\Delta\tilde{z}_t^p = (I - A)^{-1} v_t. \tag{42}$$

Comparing (42) to (40), one can see that the shocks driving the permanent components of $\tilde{z}_t$ and z_t are the same.

MTW assume that the researcher mistakenly lets $\Delta\tilde{r}_t$ and $\Delta\tilde{x}_t$ follow a VAR(1) process, as was true of Δr_t and Δx_t, when getting a preliminary BN estimate of the permanent component r_t^*.[16] This '*preliminary BN*' estimate $\Delta\tilde{r}_t^*$ is given by:

$$\Delta\tilde{r}_t^* = 1.06\tilde{v}_{1t} - .949\tilde{v}_{2t}. \tag{43}$$

Note here that there is serial correlation in $\Delta\tilde{r}_t^*$ (its first order auto-correlation coefficient is $-.15$).

Finally, since $\Delta r_t^* = v_{1t} - v_{2t}$ (the permanent component in (41a) from the VAR(1) specification) is a white noise process, and $\Delta\tilde{r}_t^*$ in (43) is not, MTW proceed to find an estimator of Δr_t^* in the third step which has that property. They describe this as '*robust to misspecification*', where the misspecification term refers to the presence of measurement error. To produce their '*robust*' estimator of Δr_t^* ($\Delta\hat{r}_t^*$), they assume an AutoRegressive Moving Average (ARMA) process for $\Delta\tilde{r}_t^*$, and then derive the new estimate $\Delta\hat{r}_t^*$ from the BN solution for that process. Fitting an ARMA(1, 2) model to $\Delta\tilde{r}_t^*$ gives:

$$\Delta\tilde{r}_t^* = .377\Delta\tilde{r}_{t-1}^* + \omega_t - .620\omega_{t-1} + .071\omega_{t-2},$$

[16]The VAR(1) coefficient estimates are inconsistent since $\Delta\tilde{z}_t$ is a Vector Autoregressive Moving Average (VARMA) process, and not a VAR. To find the large sample estimates of the VAR(1) coefficients, we simulate 50,000 observations from their VARMA model and fit a VAR(1) to the simulated data. This gives:

$$\begin{aligned}\Delta\tilde{r}_t &= -.043\Delta\tilde{r}_{t-1} - .049\Delta\tilde{x}_{t-1} + \tilde{v}_{1t}\\ \Delta\tilde{x}_t &= -.112\Delta\tilde{r}_{t-1} + .945\Delta\tilde{x}_{t-1} + \tilde{v}_{2t}.\end{aligned}$$

where ω_t is white noise. MTW then define the robust estimate of the BN permanent component as $\Delta\hat{r}_t^* = \frac{1-.620+.071}{1-.377}\hat{\omega}_t = .72\hat{\omega}_t$, and its standard deviation is $.72 \times .152 = .11$. By construction, this approach produces an estimate with the property that $\Delta\hat{r}_t^*$ is white noise. However, $\Delta\hat{r}_t^*$ is *not* Δr_t^*. The correct BN permanent shock is $v_{1t} - v_{2t}$, which has a standard deviation of .11. Regressing this against $\hat{\omega}_t$ gives a recovery R^2 of .61. This illustrates that while MTW is more successful than Laubach and Williams (2003) and its related approaches, one cannot recover the actual permanent shock with this strategy.

For comparison, a regression of the correct BN permanent shock against the *preliminary value* $\Delta\tilde{r}_t^*$ yields an R^2 of .58, and this preliminary value is more volatile (its standard deviation is .157). This highlights that their correction improves the estimate of the variance of the correct BN permanent shock. However, its robustness is limited to producing an estimate for the change in the real neutral rate which is white noise and it does not recover Δr_t^*.

To understand why this is the case it is useful to find what it is that determines the shock driving it, $\hat{\omega}_t$. Regressing $\hat{\omega}_t$ against current and ten lags of v_{1t}, v_{2t}, vm_{1t} and vm_{2t} gives an $R^2 = .9998$, i.e., this is virtually an identity. When the terms vm_{1t} and vm_{2t} are excluded, the R^2 drops to .7, indicating that the measurement errors are very informative in the computation of $\hat{\omega}_t$, and hence the robust estimate. Contrary to this, the true BN decomposition involves only $v_{1t} - v_{2t}$, and therefore none of the lags or current values of vm_{1t} and vm_{2t} provide useful information in predicting it. The importance of measurement errors to the robust estimate Δr_t^* contributes to its recovery R^2 of .61.

4.3.2 Recovering Stars using Time-Varying Parameter Models

Another way the BN decomposition has been used to estimate stars is to couple it with a Time-Varying Parameter (TVP) model. As an example, consider the study by Lubik and Matthes (2015, LM) who estimate a simple TVP-VAR for three variables: the growth rate of real GDP, the PCE inflation rate, and the same real interest rate as in LW (2003). Their proposal is to measure the natural real rate of interest as the (conditional) long-horizon forecast of the observed real rate, so it is a variant of the BN definition of the permanent component. In their paper, the chosen time horizon is five years.

To illustrate the issues with such an approach, consider a simpler TVP model for a single equation only, the real interest rate, consisting of:

$$r_t = \rho_t r_{t-1} + \sigma_1 \varepsilon_{1t} \tag{44}$$

$$\Delta\rho_t = \sigma_2 \varepsilon_{2t}, \tag{45}$$

where ε_{1t} and ε_{2t} are mutually and serially uncorrelated. Suppose, for simplicity we define r_t^* as the prediction of r_t two periods ahead (instead of the five used in LM), that is, $r_t^* = E_t r_{t+2}$. Then, to compute $E_t r_{t+2}$, we construct the following from the relations in (44) and (45):

$$\begin{aligned} r_t^* &= E_t(\rho_{t+2} r_{t+1} + \sigma_1 \varepsilon_{1t+2}) \\ &= E_t[(\rho_t + \sigma_2 \varepsilon_{2t+2} + \sigma_2 \varepsilon_{2t+1}) r_{t+1} + \sigma_1 \varepsilon_{1t+2}] \\ &= E_t[(\rho_t + \sigma_2 \varepsilon_{2t+2} + \sigma_2 \varepsilon_{2t+1})(\rho_{t+1} r_t + \sigma_1 \varepsilon_{1t+1}) \\ &= E_t[(\rho_t + \sigma_2 \varepsilon_{2t+2} + \sigma_2 \varepsilon_{2t+1})(\rho_t + \sigma_2 \varepsilon_{2t+1}) r_t)] \\ &= E_t(\rho_t^2 + \sigma_2^2) r_t. \end{aligned} \tag{46}$$

Now, in the above all random variables observed at time t are known, but future ones are unknown and are replaced by their unconditional means of zero, i.e., $E_t(\varepsilon_{1t+i}) = 0, \forall i > 0$. It then needs to be recognized that, while r_t is known, ρ_t is not, and the expectation must be conditional on the data. The relation in (46) then leads to a star type of estimate of r_t^* having the form:

$$r_t^* = E_t(\rho_t^2) r_t + \sigma_2^2 r_t.$$

The problem then is that $E_t(\rho_t^2)$ is not computed by the Kalman filter. To proceed, Lubik and Matthes (2015) did something different. For this case their approach would be to measure r_t^* as $r_t E_t(\rho_{t+2})$, and *not* $E_t(\rho_t^2) r_t + \sigma_2^2 r_t$, as implied by the TVP model.

More generally, in any TVP VAR there will be shocks that would drive the structural equations and shocks that determine the evolution of the TVPs. So, there will be excess shocks. Consequently there will be linear relations between at least some of the filtered quantities, potentially making it again hard to know how to interpret the estimated r^*.

5 Common Model Features that can Obscure the Stars

We now turn to two more recent features of modern macroeconomic models that are widely implemented, but which feature excess shocks and whose resulting lack of shock recovery does not appear to be appreciated. As these hinder shock recovery, such features potentially obscure the stars.

The first are news shocks. These were argued by Beaudry and Portier (2004 and 2006) to be a major source of economic fluctuations. Subsequently, they have been included in many DSGE models, particularly in the specification of technology shocks. Estimates of the importance of news shocks differ dramatically; see, for example, Khan and Tsoukalas (2012) and Christiano *et al.* (2014).

The second is stochastic volatility. This was initially included in models used to summarize the data; a prominent macroeconomic example is the univariate model of U.S. inflation by Stock and Watson (2007). More recently, SV has increasingly been included in models which interpret the economy through shock estimates and impulse responses; see, for example, the SVAR of Mumtaz and Zanetti (2013).

5.1 News and the Role of Expectations

News shocks imply that the solution to the model is a VARMA rather than a VAR process. In the first sub-section below, we show in a simple example that the news shock cannot be recovered and explain why. Subsequently, we turn to examining a strategy which has been applied for avoiding a short system by expanding the number of observables through the utilization of published forecasts, and discuss some of its implications for the recovery of stars.

5.1.1 Absorbing the News

Consider the following structural system for z_t

$$z_t = \delta z_{t-1} + u_t \tag{47}$$

$$u_t = \sigma_0 \varepsilon_t^0 + \sigma_1 \varepsilon_{t-1}^1. \tag{48}$$

The shock u_t is comprised of ε_t^0, an unanticipated shock that is realized in period t, and ε_{t-1}^1, namely a news shock that is anticipated in period $t-1$ to materialize in period t. This formulation dates back at least to Beaudry and Portier (2004). It follows that

$$\begin{aligned} z_t &= \delta z_{t-1} + \sigma_0 \varepsilon_t^0 + \sigma_1 \varepsilon_{t-1}^1 \\ &= \delta z_{t-1} + \omega_t + \alpha \omega_{t-1}, \end{aligned}$$

where ω_t is a white noise process with zero mean and variance σ_ω^2. This makes z_t an ARMA$(1,1)$ and the parameters can be estimated (subject to identification checks). However,

$$\sigma_0 \varepsilon_t^0 + \sigma_1 \varepsilon_{t-1}^1 = \omega_t + \alpha \omega_{t-1},$$

and so

$$\omega_t = (1 + \alpha L)^{-1}(\sigma_0 \varepsilon_t^0 + \sigma_1 \varepsilon_{t-1}^1). \tag{49}$$

As can be seen from (49), ω_t will be a linear combination of *all* $\{\varepsilon_i^0, \varepsilon_{i-1}^1\}_{i=1}^t$ and so the shock that can be estimated, ω_t, is neither ε_t^0 nor ε_{t-1}^1.[17] A consequence of this is that it is not possible to find the separate contribution of the two shocks to the *data* variance, as explained in Section 2.3. In line with the earlier discussion, these difficulties arise because there is a single structural equation and thus only *one* observable, but there are *two* shocks that one is trying to recover. The system is thus short, unless an extra observable can be found.

5.1.2 Expanding the Observables with Forecasts

The shock ε_{t-1}^1 above is a forecast error due to news about y_t at time t using information at $t-1$. It has been suggested that one might use forecasts of the variable y_t from an outside source for this, and that constitutes an extra observable; see, for example, Hirose and Kurozumi (2021). They use a small New-Keynesian (NK) model that has three variables: output y_t, inflation π_t, and interest rates r_t (each in log deviations from their respective steady-state values).[18] Each structural equation for these variables has its shocks governed by the news structure above in (48), although Hirose and Kurozumi (2021) have news up to five periods, rather than one. This results in a total of 18 shocks, with only three observables, leading to a system that is (very) short. Hirose and Kurozumi (2021) utilize forecasts from a real-time data set of the Federal Reserve Bank of Philadelphia as measures of expected inflation, output growth and interest rates in order to alleviate the short system.

To illustrate what one might gain from this, we use the following simplified NK model

[17] This was pointed out by Nelson (1975); see also McDonald and Darroch (1983).

[18] See Section 3 on page 1446 in Hirose and Kurozumi (2021). The model here is somewhat simplified to make the point.

with news that has the structure in (48) below:

$$\begin{aligned} y_t &= \omega_j y_{t-1} + (1-\omega_j)E_t(y_{t+1}) - \tau(i_t - \pi_t) + \sigma_1\varepsilon_{1t} + \sigma_4\varepsilon_{4t-1} \qquad (50) \\ \pi_t &= \gamma\pi_{t-1} + \kappa y_t + \sigma_2\varepsilon_{2t} \\ i_t &= \phi_r i_{t-1} + (1-\phi_r)(\phi_\pi \pi_t + \phi_y y_t) + \sigma_3\varepsilon_{3t}, \end{aligned}$$

where ε_{1t}, ε_{2t} and ε_{3t} are preference, cost-push and monetary shocks, respectively, and ε_{4t} is the news shock. There are 3 observables and 4 shocks, so the system is short. Using parameter estimates for the NK model based on Hirose and Kurozumi (2021) (see Appendix A) we find:[19]

$$\text{diag}(P^*_{t|T}) = \begin{bmatrix} .22 & 0 & 0 & .78 \end{bmatrix},$$

so that the fourth shock, the shock capturing news about the future, is not recoverable. Nonetheless, two of the model shocks, ε_{2t} and ε_{3t}, *are* recoverable. Looking at the correlation of these with either $E_T\varepsilon_{1t}$ or the news shock $E_T\varepsilon_{4t}$, we find that they are close to zero, but that the correlation between $E_T\varepsilon_{1t}$ and $E_T\varepsilon_{4t}$ is .39. One can thus find the *joint* contribution of ε_{2t} and ε_{3t} to the volatility of the data on y_t, π_t and i_t, but one cannot determine the *relative* or individual contributions of $E_T\varepsilon_{1t}$ and $E_T\varepsilon_{4t}$.

Alternatively, if one has an observed forecast of y_{t+1}, one might replace $E_t(y_{t+1})$ in (50) with it to obtain the news shocks.[20] Obviously, when estimating the model parameters this would also be simplified as $E_t(y_{t+1})$ is then known. This strategy has been followed by others such as Barsky and Sims (2012).[21] Particularly noteworthy is Crump *et al.* (2019), which provides a recent, comprehensive account of estimating the NAIRU. In the baseline

[19]To do this we simulate data from the NK model with the parameter values and then find the exact VAR structure, i.e. the identity linking variables and generated shocks. This can then be placed in state space form. The method is the same as used in Liu *et al.* (2018).

[20]$E_t(y_{t+1})$ will be a linear function of the shocks in the system which will include ε_{4t-1} so it provides another structural equation whose dependent variable is observable when forecasts are used for it.

[21]Barsky and Sims (2012) use forecasts of GDP as the extra variable.

specification of Crump *et al.* (2019), the unemployment rate is taken to evolve as $u_t = (u_t - u_t^*) + z_t + \bar{u}_t$, where u_t^* is the natural rate of unemployment and $(u_t - u_t^*)$ the unemployment gap. A secular trend in unemployment is $\bar{u}_t$ while z_t is the deviation of the natural rate from this secular trend. It is necessary to measure both $\bar{u}_t$ and u_t^*. The former is measured using data on flows into and out of unemployment over time for six demographic groups. The process involves a factor model with a common component. Although this model is short, Pagan and Robinson (2022) observe that, if there were a large number of groups, one could recover the factor, here $\bar{u}_t$. For any finite number of groups, however, the system is short. Subsequently for the measurement of the NAIRU the estimated $\bar{u}_t$ is treated as being observed, and as is typically done (e.g. in McCririck and Rees 2017), a Phillips curve is used to relate inflation to the unemployment gap, with the latter here assumed to follow an exogenous AR(2) process, while z_t follows an AR(1). There are now excess shocks in Crump *et al.*'s system, and so the system is short.

A characteristic of Crump *et al.* (2019) is the careful handling of inflation expectations in the Phillips curve, including the use of survey forecasts from professional forecasters, as in Hirose and Kurozumi (2021). However, equating forecasts and expectations to eliminate a short system in any model that incorporates a NAIRU would be making a questionable assumption, as it has been shown in the past that the expectations of households and professional forecasters differ (see, for example, Dräger *et al.*, 2016). Crump *et al.* (2019) instead allow for a measurement error between these survey expectations and the expectations in the model, but then the system is short again, although the degree of recovery of the shocks may be improved.[22] We note here also that exploring further how forecasts and expectations data can be used in the estimation of stars seems to be a productive area for future research.

[22] This is also done by Alichi *et al.* (2017).

5.2 Stochastic Volatility (SV)

The second increasingly common model feature which results in a short system is the integration of an error term into the macroeconomic model that treats time varying volatility as a stochastic volatility process. To see how this materializes, consider the following simple example. Suppose that there is a single variable and it has conditional volatility that is specified to follow an SV process. This produces the following model:

$$y_t = B_1 y_{t-1} + \exp\{.5h_t\}\varepsilon_t \tag{51a}$$

$$h_t = \mu + \beta h_{t-1} + \omega_t. \tag{51b}$$

Although estimation of the parameters can be complex and is important in practice, let us assume here that we have parameter estimates or know the true values as we have done throughout the paper. Then, define:

$$\begin{aligned} \zeta_t &= y_t - B_1 y_{t-1} \\ \Leftrightarrow \zeta_t^2 &= \exp\{h_t\}\varepsilon_t^2, \end{aligned}$$

so that

$$\begin{aligned} \log(\zeta_t^2) &= h_t + \log(\varepsilon_t^2) \\ &= \mu + \beta h_{t-1} + \omega_t + \log(\varepsilon_t^2). \end{aligned} \tag{52}$$

Computing smoothed shocks gives an SSF form

$$\log(\zeta_t^2)^D = \mu + \beta E_T h_{t-1} + E_T\omega_t + E_T \log(\varepsilon_t^2) \tag{53}$$

$$E_T h_t = \mu + \beta E_t h_{t-1} + E_T \omega_t. \tag{54}$$

Because there is only one observable $\log(\zeta_t^2)$ in (51), the system is short and both the shocks ε_t and ω_t cannot be recovered.[23] Note here again that it has been assumed that parameters are either known or estimates of them are available. The SSF *has* to hold — it is an implication of the SV model. The inclusion of a SV process therefore can be problematic when the model is used to interpret, rather than summarize, the data using the shocks, as is the case when estimating stars.

Is there an alternative to the SV model? Yes, of course. Other major classes of models for capturing conditional volatility, namely (E)GARCH (Bollerslev, 1986, and Nelson, 1991), are not short and are well capable of capturing the same type of time varying volatility behaviour in macroeconomic variables as the SV model.[24] For example, one might use an EGARCH model taking the form:

$$y_t = B_1 y_{t-1} + \exp\{.5h_t\}\varepsilon_t$$
$$h_t = \mu + \beta h_{t-1} + \alpha \varepsilon_{t-1}^2,$$

and avoid recoverability issues introduced by an SV process error specification.

6 Eliminating Shocks: Smooth-Transition Models for a Star

Are there alternative ways to handle short systems when estimating stars? Pagan and Robinson (2022) canvassed the idea of adding in extra observables to short systems. The other alternative considered was to delete shocks. One clearly does not want to do that if the shock has some economic content, but if the sole purpose is to account for an exogenous process, then there are other ways of handling that.

Consider, for example, the standard assumption that the NAIRU u_t^* evolves as an ex-

[23] This scenario is exactly the same as the one in Section 2.2, albeit with the second shock being log transformed.
[24] One might recall that Engle's (1982) empirical application was for the modelling of U.K. inflation.

ogenous $I(1)$ process. Many adopt this formulation on the grounds that there have been movements in the NAIRU. This then seems to provide a *"free lunch"*, or more charitably, an agnostic approach, as no stance needs to be taken on when these shifts took place, and how the star variable changed between these shifts. Instead it is constantly changing over time.

As an alternative to that, one could specify a process for u_t^* that allows for a finite number of changes. A simple approach would be that of Okimoto (2019), who uses a smooth transition model to describe the evolution of the star variable trend inflation π_t^*.[25] With a sample of T observations, the aim of this approach is to capture the evolution of the star as undergoing a smooth transition from the value at the beginning of the sample μ_1 to that at the end μ_2 using a deterministic function that depends on (t/T).[26] There are many such functions that could be applied, one of which is the exponential function employed by Okimoto (2019):

$$\begin{aligned} \pi_t^* &= \mu_1 + G(s_t; c, \gamma)(\mu_2 - \mu_1) \\ G(s_t; c, \gamma) &= \frac{1}{1 + \exp(-\gamma(s_t - c))}, \gamma > 0 \\ s_t &\equiv \frac{t}{T}. \end{aligned} \tag{55}$$

A similar approach was used by Murphy (2020) in the context of Australian models, albeit with a different transition function $G(\cdot)$ in (55).[27] We believe that further analysis of the approach of modelling star variables using smooth-transition models, such as evaluation of their real-time reliability — akin to Orphanides and Van Norden (2002) for unobserved-component models — is warranted to better understand their potential usefulness for policy.

[25]See the survey in van Dijk *et al.* (2002) on smooth-transition models.
[26]One could allow for knot points in the sample as well, just as one does with spline functions.
[27]Lye and McDonald (2021) has elements of this. Recently Gao *et al.* (2022) have proposed a related approach for TVP SVAR models.

7 Conclusion

Stars are frequently cited in speeches by central bank officials and the financial press when addressing the appropriateness of the current policy stance. Moreover, estimates of stars are routinely published by central banks and organizations such as the OECD in their Economic Outlook report. Recently, authors from the World Bank have produced an extensive cross-country database of stars such as the growth rate of potential output (see Kilic Celik *et al.*, 2023). In general, substantial resources are devoted to estimating stars, which highlights their importance in the conduct of macroeconomic policies.

Federal Reserve Chairman Jerome H. Powell once commented that conventional wisdom is that monetary policy involves navigating by stars like ships of the past, but shifting stars makes that challenging (Powell, 2018). In that regard, Sablik (2018, p. 3) records that New York Fed President John C. Williams (one of the authors of the LW model) bemoaning the challenges of using the natural real rate as a guide for policy by saying: *'As we have gotten closer to the range of estimates of neutral what appeared to be a bright point of light is really a fuzzy blur'*. These comments illustrate some of the issues related to parameter uncertainty, shifts in stars, and wide confidence intervals surrounding estimates of stars. They significantly complicate the conduct of macroeconomic policy. And they are well known.

The point of this article is more fundamental. Drawing on the recent theoretical literature on shock recovery, we simply ask whether the models used to estimate stars can in fact recover the true star from the observed data, or nearly so. This would seem to be a minimal desirable property of any model. We address this question in the most favorable setting possible, namely, when the models used to measure the star variables are correctly specified and all their parameters are known. In many cases, the answer to this question is no.

Understanding the limitations of models which play a critical role in the conduct of macroeconomic policy is important. Whether a model can recover the variable it is intended to measure is paramount, yet it is not routinely discussed. Just as presenting confidence intervals around stars is standard practice for demonstrating the statistical uncertainty surrounding the estimates, the extent of recoverability of the star variable also needs to become typical disclosure information. We have shown how this can be communicated simply as a correlation between the estimated (first difference) in the star variable and its actual value, which is easily calculated using the Kalman filter and smoother. This correlation should be routinely reported alongside the star estimates to policymakers.

One conclusion from this paper is that our ability to navigate economic policy by the stars is even more limited than we thought. A second is that re-thinking how star variables are modelled more broadly could be promising, and we provide one possible account of how that might be done. At present, star variables are generally handled as an exogenous stochastic variable. This is a purely statistical approach and whether the model can recover the star variable is ignored. More generally, there is a trend to incorporate greater flexibility into macroeconomic models, frequently by introducing additional shocks, and these inevitably lead to short systems. While the aim of providing a better description of the data is admirable, it is necessary to recognize that this has limitations. Free lunches are rarely available in econometrics.

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Appendix A: Parameter Values

Laubach and Williams (2003)

$$\sigma_1 = .387,\ \sigma_2 = .731,\ \sigma_3 = .323,\ \sigma_4 = .605,\ \sigma_5 = .102$$
$$\alpha_1 = 1.51,\ \alpha_2 = -.57,\ b_y = .043,\ \alpha_r = -.098,\ c = 1.068.$$

LW give the sum of $\alpha_1 + \alpha_2$ and not α_1, α_2. The values here come from Table 3 of Buncic (2022).

Holston, Laubach and Williams (2017)

The values here come from Table 3 of Buncic (2022).

$$\begin{array}{lllll} \sigma_1 = 0.3338 & \sigma_2 = \ \ 0.7862 & \sigma_3 = \ \ 0.1742 & \sigma_4 = \ \ 0.5739 & \sigma_5 = 0.1230 \\ \alpha_1 = 1.5399 & \alpha_2 = -0.5986 & a_r = -0.0679 & b_\pi = 0.6708 & b_y = \ \ 0.0756 \end{array}$$

Holston, Laubach and Williams (2023)

$$\begin{array}{rrrrr} \sigma_1 = 0.4516 & \sigma_2 = \ \ 0.7873 & \sigma_4 = \ \ 0.5000 & \sigma_5 = 0.1453 & \sigma_3 = \ \ 0.1181 \\ \alpha_1 = 1.3872 & \alpha_2 = -0.4507 & a_r = -0.0790 & b_\pi = 0.6800 & b_y = \ \ 0.0733 \\ \kappa_{2020Q2-Q4} = 9.0326 & \kappa_{2021} = \ \ 1.7908 & \kappa_{2022} = \ \ 1.6760 & c = 1.1283 & \phi = -0.0854, \end{array}$$

where the κ terms capture the temporary increase in the shock variances and ϕ denotes the coefficient on the Oxford COVID tracker.

McCririck, and Rees (2017)

These are the posterior mean values reported in Table A2 on page 17 of their paper.

$$\sigma_1 = .32,\ \sigma_2 = .80,\ \sigma_3 = .34,\ \sigma_4 = .55,\ \sigma_5 = .05,\ \sigma_6 = .15,\ \sigma_7 = .07$$
$$\alpha_1 = 1.48,\ \alpha_2 = -.54,\ \alpha_r = .06,\ \beta_1 = .41,\ \beta_2 = -.33,\ \beta = .64.$$

Schmitt-Grohe and Uribe (2022)

We thank Martin Uribe for providing these. We put $\alpha = 0$ as we did not receive a posterior mean value for that. The posterior median reported in the paper is very close to 0.

$$B = \begin{bmatrix} .2627 & .0187 & -.5031 \\ .3129 & .3292 & -.1170 \\ .2268 & -.0977 & .5048 \end{bmatrix}, \ C = \begin{bmatrix} -.0956 & 0 & -.2603 & 1 & -.0051 \\ -.4892 & 0 & .5632 & .8727 & .3651 \\ 1.3964 & 1.0 & -.0309 & .2579 & -.2184 \end{bmatrix}$$

$$\begin{aligned} \rho_1 &= .2426,\ \rho_2 = .3298,\ \rho_3 = .2619,\ \rho_4 = .4254,\ \rho_5 = .3110 \\ \sigma_1 &= .4824,\ \sigma_2 = .6250,\ \sigma_3 = 1.3624,\ \sigma_4 = 1.0913,\ \sigma_5 = .4723 \\ \delta &= 8.3292,\ \sigma_y = \sqrt{1.2304},\ \sigma_\pi = \sqrt{.4862},\ \sigma_i = \sqrt{.3208}. \end{aligned}$$

Hirose and Kurozumi (2021)

Their model is more complex as it has news shocks in each of the three structural equations, more expectations, stochastically varying technology and an extra effect of growth on inflation. Because of these differences the parameters are adjusted.

$$\begin{aligned} \omega_j &= 0.6; \tau = 0.4; \sigma_1 = 2.66; \sigma_4 = 1.3 \\ \gamma &= .8; \kappa = .2; \sigma_2 = 1.4 \\ \phi_r &= .7; \phi_\pi = 1.5; \phi_y = .5; \sigma_3 = .05 \end{aligned}$$