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Abstract

Inclusion of means testing into age pension programs allows governments to better direct benefits to those most in need and to control funding costs by providing flexibility to control the participation rate (extensive margin) and the benefit level (intensive margin). The former is aimed at mitigating adverse effects on incentives and to strengthen the insurance function of an age pension system. In this paper, we investigate how means tests alter the trade-off between these insurance and incentive effects and the consequent welfare outcomes. Our contribution is twofold. First, we show that the means test effect via the intensive margin potentially improves the insurance aspect but introduces two opposing impacts on incentives, the final welfare outcome depending upon the interaction between the two margins. Second, conditioning on the compulsory existence of pension systems, we find that the introduction of a means test results in nonlinear welfare effects that depend on the level of maximum pension benefits. More specifically, when the maximum pension benefit is relatively low, an increase in the taper rate always leads to a welfare gain, since the insurance and the positive incentive effects are always dominant. However, when maximum pension benefits are relatively more generous the negative incentive effect becomes dominant and welfare declines.

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1 Introduction

Unlike the U.S.A. and many developed countries in which age pension systems are mainly universal and pay-as-you-go (PAYG), Australia has a unique means tested pension system. It has the following distinct features: (*i*) coverage of the retirement benefits system is not universal in that only a fraction of the retiree population receives age pension retirement benefits; (*ii*) the retirement benefits are dependent on economic status (assets and income) and are directed towards the poorer elderly; (*iii*) the pension benefits are independent of individuals' contribution history; and (*iv*) the tax financing instrument is not restricted to payroll tax revenue collected from the current working population.

Inclusion of means testing into age pension programs allows governments to better direct benefits to those most in need and to control funding costs by providing flexibility to control the participation rate (extensive margin) and the benefit level (intensive margin). This discrimination in retirement benefits is aimed at strengthening risk-sharing across households and generations, and fostering individuals' incentives to work and save as well as minimizing economic distortions. In this paper, we investigate whether means tested pension systems lead to favourable welfare outcomes in a dynamic, general equilibrium model. Our goal is to understand how the design of means testing instruments affects individuals' inter-temporal allocation decisions and to determine the implications for macroeconomic aggregates and welfare.

Similarly to a universal PAYG pension system, a means tested age pension system provides a risk-sharing mechanism across households and generations. It provides individuals with a mechanism to smooth consumption over the life-cycle when market imperfections are present. However, differently from an universal PAYG system, it emphasizes the role of intra-generational redistribution, e.g., risk-sharing within old generations. The distinctive features of a means tested age pension program result in a number of new aspects. First, means testing instruments strengthen the redistributive function of a pension system, with emphasis more on intra-generational risk-sharing. Second, means testing instruments introduce additional effects on the inter-temporal allocation of labor and consumption as individuals reduce savings and work to increase the likelihood of receiving pension benefits in retirement. The judgment regarding the value of a means tested pension program should be based on the welfare effects embodying the trade-off between the insurance and incentive impacts.

To that end, we begin the paper with a two period, partial equilibrium model to demonstrate that means tested pensions create two channels of effects on individuals' incentives: the probability of being a pensioner (extensive margin) and the level of pension benefits (intensive margin). We show that dynamic interactions between these two margins result in opposing effects on savings. More specifically, we show that the extensive margin introduces a new channel of effects that is only embedded in a means tested pension system. On one hand, it tends to encourage agents to save more to prepare themselves for the possibility that they are not eligible for pensions; on other hand, it tends to induce agents to dissave to increase their chances of receiving a pension. Moreover, the direction of the extensive margin effect depends on the strength of the intensive margin effect. If the intensive margin effect is relatively less generous, the extensive margin has a positive effect is positive; otherwise, it has a negative effect. This indicates that the existence of extensive margin effects potentially mitigates the adverse intensive margin effects on savings. The total effect depends on how these interactions combine. The welfare effects of a means tested pension system are dependent on fundamentals including preferences, endowments, market structures and institutional features.

Next, we quantify these effects in a calibrated model of the Australian economy, taking fundamental factors into account. We follow the tradition of the dynamic general equilibrium literature on social security and construct an overlapping generations economy with heterogeneous households facing uninsurable idiosyncratic earnings shocks and mortality risk, a perfect competitive representative firm and a government with a full commitment technology (e.g., Imrohoroglu *et al.*, 1995). We incorporate the main features of Australia's means tested age pension system and calibrate our benchmark model to match key features of the Australian economy. We conduct the following policy experiments: (*i*) First, we compare steady state results of an economy with means tested pension with an economy without a pension; (*ii*) conditioning on the existence of a pension system, we compare steady state results when varying the generosity of the maximum pension and taper rates for the income means test. Our quantitative results are summarized as follows.

First, means testing instruments add new dimensions to the trade-off between the insurance and incentive effects, but the final welfare outcome depends upon how these new aspects interact with other features of the overall social insurance system and upon the nature of the economy.[1] In our first experiment, the results reveal that a non-PAYG pension program with means testing instruments results in lower welfare outcomes than having no pension. This implies that means tested pension systems are not socially desirable in our dynamic, general equilibrium model economy, since the adverse effects on incentives continue to dominate the positive social insurance effects of pensions even when they are means tested. Consequently, when the pension program is completely removed efficiency gains from increases in savings and labor supply result in higher consumption and welfare. This finding is similar to that in the PAYG social security literature.

[1] Empirical evidence on the links between earnings tests on savings and labor decisions is well documented. Neumark and Power (1998, 2000) estimate the effects of means-tested Supplemental Security Income for old age individuals in the U.S.A. and find that these retirement benefits reduce savings and labor supply of those likely to participate in the program when approaching retirement age. There are also a number of studies exploring the effects of labor-earning tests on early retirement and the elderly's working hours in the U.K. Disney and Smith (2002) find that an abolition of the earning test induces older male workers to work 4 more hours a week. This empirical result is also consistent with the result of previous study by Friedberger (2000). These studies confirm that earnings tests significantly affect savings and labor supply decisions in older ages, especially around the mandatory age from which individuals are eligible for retirement benefits. Previous analyses of the effects of means-tested, non-pension benefits (e.g., see Hubbard at el (1995), Powers (1998), Gruber and Yelowitz (1999), Heer (2002) and Chow et. al. (2004)) also found that the asset test reduces saving incentives of low income households.

Second, conditioning on the existence of a pension system, the introduction of means testing results in non-linear welfare effects of changes in the generosity of the pension system and taper rates. When the maximum pension benefits are relatively small, the introduction of income tests (raising taper rates) always leads to a welfare gain as the positive welfare effects from strengthening risk-sharing and mitigating self-insurance disincentives are always dominant. However, once the pension benefits become more generous, the negative incentive effects become more pronounced as taper rates are increased. The underlying economic mechanism behind it is that the economic distortions of taper rates as implicit taxes on life-cycle savings and labor supply are more severe when pension are more generous. There is a trade-off between to these opposing forces, the final welfare outcome depending on the strength of the negative incentive effects of taper rates relative to the positive insurance and incentive effects. We find that there is an optimal of taper rate that balances these two forces, conditioning on the level of maximum pension benefits.

Our paper contributes to several strands of the macroeconomics and public finance literature. First, our work is closely related to an emerging literature analyzing the effects of means tested pensions on savings, labor supply and welfare in a life cycle framework. Sefton and van de Ven (2009) use a calibrated multi-period overlapping generations model to analyze the effects of a means tested pension reform on life cycle savings and labor decisions in the U.K. They find that tightening of the taper rate for the income test encourages poor individuals to save more and to delay retirement, while generating opposite effects on the savings and retirement decisions of the rich. Selton, van de Ven and Weale (2008) conduct a welfare analysis and find that means tested pensions are socially preferred to a universal pension in the U.K. as they deliver better welfare outcomes. Kumru and Piggott (2009) also find a welfare gain from introducing means tests in the U.K. social security system. Kudrna and Woodland (2011) analyze the general equilibrium effects of changing taper rates of the Australian pension system in a deterministic overlapping generations model. Maattanen and Poutvaara (2007) study welfare implication of introducing labor earnings tests to the PAYG social security system in the U.S.A. and find negative welfare effects because the adverse effects of the labor earnings tests on the elderly's labor supply are significantly large. It is noteworthy that these papers emphasize the effects of taper rates working through the intensive margin, i.e., imposing an implicit tax, while abstracting from an important channel of effects via the extensive margin. In contrast, our research extends these papers by highlighting the importance of the extensive margin effects. We show that the interactions between taper rates and the maximum pension benefit via the extensive margin results in opposing effects on individuals' incentives. Subsequently, the welfare effects of changes in taper rates vary significantly over the levels of maximum pension benefits.

Our study is also related to the literature that undertakes dynamic, general equilibrium analyses of social security systems. That literature focuses upon universal PAYG social security systems and consistently finds negative welfare outcomes when accounting for general equilibrium effects. It implies that the adverse effects on incentives tend to dominate the insurance role so that the introduction of an unfunded PAYG social security system usually lowers welfare. The adverse effects of unfunded social security in dynamic, general equilibrium models have been well documented (e.g., see Auerbach and Kotlikoff (1987), Hubbard and Judd (1987), Imrohoroglu, Imrohoroglu and Jones (1995), Conesa and Krueger (1999), Krueger (2006) and Fuster, Imrohoroglu and Imrohoroglu (2007)). Note that this literature focuses on the U.S.A. social security system in which the coverage is universal, and it therefore excludes the effects coming from the extensive margin. Our study is complementary to that literature as we study a pension system in which the extensive and intensive margins are both relevant. We show that interactions between these two margins are important and potentially lead to welfare gains.

Our paper is also linked to the literature on social insurance with means testing. This literature has focused mainly on disability insurance. Diamond and Mirrlees (1978) and Diamond and Mirrlees (1986) conclude that optimal benefits are structured so that the healthy are indifferent as to whether to mimic the disabled or continue working. In a more recent work on optimal disability insurance, Golosov and Tsyvinski (2006) also argue that disability insurance benefits should be asset-tested to prevent individuals from claiming benefits when, optimally, they should not. This paper follows a similar approach but focus on a pension program. Specifically, we analyze the role of means testing in enhancing the social insurance function of public pensions rather than disability insurance. Nevertheless, we reach a similar conclusion that the means testing could be used to foster savings and working longer. However, we find that this statement is not universally correct for a pension program, since we identify some cases in which the introduction of means tests makes the adverse intensive margin effects more severe and results in a negative overall welfare effect.

The paper is structured as follows. In section 2 we present a simple model to highlight the role played by the intensive and extensive margins arising from a mean-tested age pension and to derive some analytical results. In section 3 we set up a dynamic, general equilibrium model that embodies endogenous retirement, earnings uncertainly and a means tested pension system. Section 4 describes details of our calibration of the model to the Australian economy and age pension scheme. Section 5 contains the discussion of a range of policy experiments and results relating to alternative means test parameters. We present conclusions in section 6. The Appendix provides mathematical details for the theoretical model, and the fiscal policy specification and solution algorithm for the dynamic general equilibrium model.

2 A simple model economy with a means tested pension

In this section, we specify a theoretical model and use it to highlight how the inclusion of means testing into the pension benefit formula influences individuals' incentives to save over the life cycle. In doing so, we are able to emphasize the essential role played by means testing on the intensive and extensive margins related to pension receipts by the elderly.

To this end, we consider a simple partial equilibrium economy comprised of agents living for two periods with endowments of w_1 and w_2 in period 1 and 2, respectively. At the beginning of period 1 an agent receives income w_1 and makes a decision on consumption and savings to maximize expected utility, taking the income distribution $f(w_2)$ in period 2 and the government pension policy as given.[2] The individual agent's optimization problem is

$$\max_{c_1,\, c_2,\, s} \{u(c_1) + pEu(c_2) : c_1 + s = w_1 - g(\tau, w_1) \text{ and } c_2 = w_2 + (1+r)\, s_1 + P\}, \quad (1)$$

where p is survival probability, c_1 is consumption when young, s is saving, c_2 is consumption when old, P is an individual specific pension benefit, r is the market rate of return on savings and $g(\tau, w_1)$ is the tax function with tax rate τ.[3] The individual's standard first order necessary condition for an optimal solution is given by $-u'(w_1 - g(\tau, w_1) - s) + (1 + r)pEu'(w_2 + (1+r)\, s_1 + P) = 0$. The optimal savings decision rule, derived by solving this equation, is a function of the initial endowments (assumed the same for all agents for simplicity), the distribution of endowments when old and the age pension benefit and is indicated by $s^* = s(w_1, f(w_2), P)$.

To aid the exposition, we assume that individuals have quadratic preferences given by $u(c) = -c^2/2 + \chi c$, where $\chi > 0$, and that income in period 2 follows a uniform distribution $f(w_2) = 1/w_2^{\max}$. Thus, the expected wage income when old is $E(w_2) = w_2^{\max}/2 \equiv \overline{w}_2$. For ease of exposition, we also assume that rate of return on investment is $r = 0$ and that the survival probability is $p = 1$, guaranteeing that the economy is dynamically efficient so that the pension system fails to yields a higher rate of return. We next consider alternative designs of a public pension program.

Universal PAYG pension and savings. We begin with a universal PAYG pension program in which the government collects tax revenue from incomes of the young in period 1 (whence $g(w_1, \tau) = \tau w_1$) and transfers to every old agent an equal amount of pension benefit, $P = P^{\max}$, i.e., a universal pension. Optimal savings for an agent is simply given by

$$s^* = \frac{(1-\tau)\ w_1 - \overbrace{[\overline{w}_2 + P^{\max}]}^{\text{Expected Income}}}{2}. \quad (2)$$

Variable $\overline{w}_2 = w_2^{\max}/2$ is the average (expected) endowment income when old and the whole term $[\overline{w}_2 + P^{\max}]$ is the expected income when old. Public pensions discourage individuals save for retirements as individual's optimal saving negatively responds to the expected wage income in period 2, $\partial s^*/\partial P^{\max} < 0$. Particularly the more pension benefits individuals receive in period 2 the less they will save in period 1. This is a classic crowding-out effect resulting from

[2] In the following, we consider a typical agent and so do not distinguish between agents.

[3] We abstract from the labor/leisure decision to keep the model sufficiently simple to highlight the channels by which the design of a means-test pension distorts the savings decision. The labour/leisure choice could readily be included, but at the cost of simplicity.

the introduction of a public pension program. Note that, to focus on the effects of pension program, we abstract from the tax financing instrument in this simple example.

The extensive and intensive margins of means tested pensions. We now examine the salient features of means testing instruments. We find that these instruments result in two separate channels of effects: (*i*) the number of agents participating in a public pension program (extensive margin); and (*ii*) the level of pension benefits (intensive margin). The latter is comprehensively analyzed in the PAYG social security literature, while the former is relatively new and only appears when means testing is introduced.

Here, we investigate how these two margins can influence an individual's savings decision. We consider the simplest means tested pension program, in which the government is allowed to discriminate between income groups to determine the receipt public pension benefits; that is, the government uses an income test to determine individuals' pension benefits. To get some intuition, we start with the very simple means testing rule

$$P = \begin{cases} P^{\max} & \text{if } w_2 < \overline{y}_2, \\ 0 & \text{if } w_2 \geq \overline{y}_2, \end{cases} \tag{3}$$

where $\overline{y}_2 \in (0, w_2^{\max})$ is the threshold level of income (here labour or endowment income only) separating pension recipients from non-recipients. This rule state that all agents with income endowments in period 2, w_2, below the income threshold $\overline{y}_2$ are eligible for an equal amount of pension benefits. This pension rule separates the elderly population into two group: one defined as relatively poor and one as relatively rich. With means testing and a uniform distribution for endowments when old, individuals face the probability of $(\overline{y}_2/w_2^{\max}) < 1$ of being a pensioner. By discriminating amongst retirees by income, the government is better able to target poor retirees. Moreover, it also tends to encourage young individuals to save more for their old age compared to a universal pension scheme. Note that, since the government excludes individuals' savings from the testable income, it can directly control the number of individuals participating in the pension program.

With these means testing instruments, the government has two pension policy parameters that it can adjust: first, adjusting the income test threshold, $\overline{y}_2$, to determine the number of pensioners (*extensive margin*) and, second, adjusting $P^{\max}$ to vary the generosity of pension benefits (*intensive margin*). In this simple model, the extensive margin disappears when the government sets $\overline{y}_2 = w_2^{\max}$ or $\overline{y}_2 = 0$. The household's optimal saving rule is

$$s^* = \frac{[w_1 - g(\tau, w_1)] - \overbrace{\left[\overbrace{\underbrace{\left[\frac{\overline{y}_2}{2} + P^{\max}\right]}_{\text{Average income}} \underbrace{\left(\frac{\overline{y}_2}{w_2^{\max}}\right)}_{\text{Probability}}}^{\text{Pensioner}} + \overbrace{\underbrace{\left[\frac{\overline{y}_2}{2} + \overline{w}_2\right]}_{\text{Average income}} \underbrace{\left(1 - \frac{\overline{y}_2}{w_2^{\max}}\right)}_{\text{Probability}}}^{\text{Non-Pensioner}} \right]}^{\text{Expected Income}}}{2}. \quad (4)$$

We now examine how means testing complicates the way that public pension programs influence individuals' saving incentives. We find that means testing adds another source of uncertainty to income in period 2 as the expected income in period 2 depends on the income threshold, $\overline{y}_2$, that is set by the government, and this influences the individual's saving decision. More specifically, as the government adjusts the income test threshold it affects the probability of being a pensioner and expected income, and thereby affects the individual's incentive to save.

To identify these channels through which means testing instruments impact upon individuals' incentives, we take the first derivatives of the saving function with respect to the maximum pension benefit, $\partial s^*/\partial P^{\max}$, and the income test threshold, $\partial s^*/\partial \overline{y}_2$. The former reflects the effect from the *intensive margin,* while the latter captures the effect from the *extensive margin* (hereafter called the intensive margin and extensive margin effects, respectively).

Not surprisingly, we find that the effect through the intensive margin is negative as $\partial s^*/\partial P^{\max} < 0$. We conclude that a public pension program crowds out private savings via the intensive margin even with means testing. However, we find that the sign of the extensive margin effect is ambiguous, since $\frac{\partial s^*}{\partial \overline{y}_2} = \frac{1}{2}\frac{1}{w_2^{\max}}(\overline{w}_2 - P^{\max}) \lesseqgtr 0$. Indeed, it is dependent on the magnitude of the maximum pension benefit, $P^{\max}$, relative to the average income in period 2, $\overline{w}_2$. This distance $(\overline{w}_2 - P^{\max})$ also measures the generosity of the public pension program, i.e., relative strength of the intensive margin effect. As $P^{\max}$ becomes relatively more generous, the strength of the intensive margin effect becomes relative larger. For example, when the maximum pension benefit is higher than the average income in period 2, $P^{\max} > \overline{w}_2$, the pension system is very generous. The direction of the extensive margin effect depends on the strength of the intensive margin effect. If the intensive margin effect is relatively less generous $(P^{\max} < \overline{w}_2)$ the extensive margin effect is $\frac{\partial s^*}{\partial \overline{y}_2} = \frac{1}{2}\frac{1}{w_2^{\max}}(\overline{w}_2 - P^{\max})$, which is positive; otherwise, it is negative. This result indicates that the existence of extensive margin effects potentially mitigates the adverse intensive margin effects on savings. However, the final effect on saving is not clear as it depends on how these two margins interplay.

Taper rate and the intensive and extensive margins. We now consider a more complex means testing rule under which the pension payment depends continuously upon the individual's income. Under this specification, That is, the pension benefit declines by ω for each additional unit of income received, where ω is a taper rate and $0 \leq \omega \leq 1$. Analytically, the pension benefit is determined by

$$P = \begin{cases} P^{\max} - \omega w_2 & \text{if } w_2 < \overline{y}_2, \\ 0 & \text{if } w_2 \geq \overline{y}_2, \end{cases} \tag{5}$$

where the maximum income threshold is now determined by $\overline{y}_2 = P^{\max}/\omega$.

The corresponding optimal savings function is given by

$$s^* = \frac{[w_1 - g(\tau, w_1)] - \overbrace{\left[\overbrace{\underbrace{\left[\frac{\overline{y}_2}{2} + P^{\max} - \omega\frac{\overline{y}_2}{2}\right]}_{\text{Average income}} \underbrace{\left(\frac{\overline{y}_2}{w_2^{\max}}\right)}_{\text{Probability}}}^{\text{Pensioner}} + \overbrace{\underbrace{\left[\overline{w}_2 + \frac{\overline{y}_2}{2}\right]}_{\text{Average income}} \underbrace{\left(1 - \frac{\overline{y}_2}{w_2^{\max}}\right)}_{\text{Probability}}}^{\text{Non-Pensioner}} \right]}^{\text{Expected Income}}}{2}. \tag{6}$$

The expected income in period 2 now depends on three pension policy design parameters - the maximum benefit, $P^{\max}$, taper rate, ω,and income test threshold, $\overline{y}_2$. If an individual is a pensioner, $\frac{\overline{y}_2}{2}$ is the expected labor income endowment they will receive; therefore, total expected income in period 2 is $\left[(1-\omega)\frac{\overline{y}_2}{2} + P^{\max}\right]$, in which the additional term $\left(-\omega\frac{\overline{y}_2}{2}\right)$ reflects the effects of the taper rate, i.e., an implicit tax on individuals' income. A non-pensioner's expected income is $\left[\frac{w_2^{\max} + \overline{w}_2}{2}\right] = \left[\overline{w}_2 + \frac{\overline{y}_2}{2}\right]$.

Inclusion of the taper rate in the pension benefit formula provides the government with an additional tool to affect both the extensive and intensive margins of a pension program. The government can vary taper rates to determine the progressivity of the pension payment schedule. First, the government may use the taper rate to adjust the level of pension benefits, which directly tunes down the negative intensive margin effect on savings. Furthermore, it affects the extensive margin effect as the taper rate appears in the derivative $\frac{\partial s^*}{\partial \overline{y}_2} = \frac{1}{2}\frac{1}{w_2^{\max}}\left(\overline{w}_2 - (P^{\max} - \omega\frac{\overline{y}_2}{2})\right)$. Again, we find that the saving effect via the extensive margin is dependent on the relative strength of the intensive margin. The extensive margin effect is positive ($\frac{\partial s^*}{\partial \overline{y}_2} > 0$) only if the pension benefit is relatively less generous enough, relative to average income in period 2, ($P^{\max} - \omega\overline{y}_2 < \overline{w}_2$). Compared to the previous means testing policy, the presence of the taper rate weakens the strength of intensive margin effect. More specifically, the government can increase the taper rate to amplify the positive extensive margin effect, taking the level of the maximum benefit as given. Consequently, this increases the likelihood that the extensive margin effect is positive.

The extensive margin and savings. In previous cases we implicitly assume that the government can fully observe labor income endowment in period 2 so that it can freely set the income test threshold. We now relax this assumption to consider the case where the government can only observe total income. The government includes interest income from saving when old as a part of testable income. The pension payment then becomes

$$P = \begin{cases} P^{\max} - \omega \left(w_2 + rs \right) & \text{if } w_2 + rs < \overline{y}_2, \\ 0 & \text{if } w_2 + rs \geq \overline{y}_2, \end{cases} \tag{7}$$

where $w_2 + rs$ is the testable income, which includes two components: labor income endowment in period 2 and interest from saving in period 1.

Optimal saving is now implicitly given by equation

$$s = \frac{\overbrace{\left[1 + (1-\omega) r\right] \overbrace{\left[\frac{\widehat{w}_2}{2} + P^{\max} - \omega \frac{\widehat{w}_2}{2} \right]}^{\text{Average income}} \overbrace{\pi^p}^{\text{Probability}}}^{\text{Pensioner}} - \overbrace{(1+r) \overbrace{\frac{(w_2^{\max} + \widehat{w}_2)}{2}}^{\text{Average income}} \overbrace{(1-\pi^p)}^{\text{Probability}}}^{\text{Non-Pensioner}}}{1 + \underbrace{\left[1 + (1-\omega) r\right]^2 \pi^p}_{\text{Pensioner}} + \underbrace{(1+r)^2 \left(1 - \pi^p\right)}_{\text{Non-Pensioner}}} \tag{8}$$

where the probability of being a pensioner is $\pi^p = \frac{\widehat{w}_2}{w_2^{\max}}$, $\widehat{w}_2 = \overline{y}_2 - rs$ and the tax function is assumed to be $g(w_1, \tau) = \tau w_1$. Note that the probability of being a pensioner, π^p, is now dependent on the individual's saving, since the wage rate that separates pensioners from non-pensioners, $\widehat{w}_2$, depends on the level of saving [4]. Under this new means testing policy, the government can no longer directly control the number of pensioners in the economy, since the testable income used by the government to determine the number of agents eligible for the pension program is now dependent on the level of labor income endowment in old age and optimal savings of the agents when young, i.e., $w_2 + rs$.

By including interest income from saving in the income test, the government is providing another (this time, direct) channel through which the means test impacts upon the saving decision. Under the two previously considered means tests for the age pension the policy instruments affected the saving decision of the young indirectly through their impacts upon expected future income. While these indirect impacts remain operational, the new channel or impact upon the saving decision is direct. Higher saving directly reduces the probability of becoming a pensioner (extensive margin) and, if the individual is a pensioner, directly reduces the pension payment (intensive margin).

In responding to a means tested pension policy, individuals optimize their saving for retirement taking into account the effect of saving upon the expected pension payment through the effect on both the intensive and extensive margins. Individuals can manage their savings decision to increase the probability of being a pensioner by decreasing saving. In that sense, the effect of the means test on savings through the extensive margin tends to be negative. On

[4] See Appendix 7.1 for a complete equilibrium solution.

other hand, decreasing the probability of being a pensioner lowers expected income in period 2, which may encourage individuals to save more. Thus, this aspect of the means test leads to two opposing effects on self-insurance incentives to save. The final effect on savings depends on which effect is dominant and how the intensive margin effect interacts with the extensive margin effect.

Discussion. We demonstrate that means test pensions create two channels of effects on individual incentives: the probability of being a pensioner (extensive margin) and the level of pension benefits (intensive margin). We have demonstrated that dynamic interactions between these two margins result in opposing effects on savings and that the total effect depends in how these interactions play out in the economy. Importantly, these interaction will depend on fundamentals of an economy like preferences, endowments, market structure and institutional settings. To make a judgment on the effects of means tested pension program one should seriously account for these fundamentals. In the next section, we develop a dynamic, general equilibrium economy model in which we take into account these factors.

3 A dynamic general equilibrium model

We consider an overlapping generations dynamic general equilibrium model, which consists of heterogeneous households, a perfect competitive representative firm, and a government with full commitment technology.

Demographics. The economy is populated by agents (households) whose ages are denoted by $j \in [1, ..., J]$. Each period a continuum of agents of age 1 are born. The population grows at an exogenous annual rate, n. All agents face an age-dependent survival probability, sp_j, and live at most J periods. When the demographic pattern is stationary, as assumed here, the population share of the cohort age j is constant at any point in time and can be recursively defined as $\mu_j = \mu_{j-1} sp_j / (1+n)$. The share of agents who do not survive to age j is $\widetilde{\mu}_j = \mu_{j-1} (1 - sp_j) / (1+n)$.

Preferences. All agents have identical lifetime preferences over consumption $c_j \geq 0$ and leisure l_j, where household leisure time per period for household j is constrained by $0 \leq l_j \leq 1$. Preferences are time-separable with a constant subjective discount factor β and are given by the expected utility function

$$E\left[\sum_{j=1}^{J} \beta^j \ sp_j \ u(c_j, l_j)\right]. \tag{9}$$

Instantaneous utility obtained from consumption and leisure is defined as

$$u(c_j, l_j) = \left[\left((1 + dp_j)^{\xi} c_j\right)^{\gamma} (l_j)^{1-\gamma}\right]^{1-\sigma} / 1 - \sigma, \tag{10}$$

where γ is the weight on utility from consumption relative to that from leisure, σ is the coefficient of relative risk aversion, dp_j is the number of dependent children at age j and ξ is the demographic adjustment parameter for consumption.

Endowment. Agents are endowed with 1 unit of labor time in each period of life that has efficiency (or working ability) denoted by e_j. The value of an agent's period effective labor services is $h_j = (1 - l_j) e_j$. When the agent chooses to allocate all time to leisure ($l_j = 1$), the agent exits the labor market and has retired. There is no mandatory retirement age so agents may stay in the labor force as long as they choose. The retirement age is endogenously determined. However, retirement is not required to be irreversible since households may re-enter the labor market. The efficiency unit e_j is age dependent and follows a Markov switching process with $\pi_j \left(e_{j+1|}e_j\right)$ denoting the conditional probability that a person of working ability e_j at age j will have working ability e_{j+1} when at age $j+1$. According to this specification, agents have working abilities that vary by age and change stochastically over the life cycle; they therefore face idiosyncratic earnings risk, which is assumed to be non-insurable.

Technology. The production sector consists of a large number of perfectly competitive firms, which is formally equivalent to one aggregate representative firm that maximizes profits. The production technology of this firm is given by a constant returns to scale production function $Y = F(K, L) = AK^{\alpha}L^{1-\alpha}$, where K is the input of capital, L is the input of effective labor services (human capital) and A is the total factor productivity, assumed to be growing at a constant rate, g. Capital depreciates at rate δ. The firm chooses capital and labour inputs to maximize its profit according to $\max_{K,L} \left\{AK^{\alpha}L^{1-\alpha} - qK - wL\right\}$, given rental rate, q, and market wage rate, w.

Means tested pension. In the benchmark economy, the government operates a means tested pension system similar to the current Australian system. The age pension (social insurance) system is not universal but targets households who have low private retirement incomes through the use of income and assets means tests. The amount of pension benefit $\mathcal{P}(a_j, y_j)$ receive at age j varies across individuals and depends on the asset and income tests as

$$\mathcal{P}(a_j, y_j) = \min\left\{\mathcal{P}^a(a_j), \mathcal{P}^y(y_j)\right\}, \tag{11}$$

where $\mathcal{P}^a(a_j)$ is the asset test pension and $\mathcal{P}^y(y_j)$ is the income test pension. Accordingly, the pension benefit is the smaller of the two pension rates; the strictest test binds. The pension benefit arising from the asset test is given by

$$\mathcal{P}^a(a_j) = \begin{cases} P^{\max} & \text{if } a_j \leq \overline{a}_1, \\ P^{\max} - \omega_a(a_j - \overline{a}_1) & \text{if } \overline{a}_1 < a_j < \overline{a}_2, \\ 0 & \text{if } a_j \geq \overline{a}_2, \end{cases} \tag{12}$$

where $\overline{a}_1$ and $\overline{a}_2 = \overline{a}_1 + P^{\max}/\omega_a$ are the asset thresholds and ω_a is the asset taper rate indicating the amount by which the pension is decreased for each additional unit of asset above the low asset threshold. Similarly, the pension benefit based on the income test is given by

$$\mathcal{P}^y(y_j) = \begin{cases} P^{\max} & \text{if } y_j \leq \overline{y}_1, \\ P^{\max} - \omega_y \left(y_j - \overline{y}_1\right) & \text{if } \overline{y}_1 < y_j < \overline{y}_2, \\ 0 & \text{if } y_j \geq \overline{y}_2, \end{cases} \tag{13}$$

where $\overline{y}_1$ and $\overline{y}_2 = \overline{y}_1 + P^{\max}/\omega_y$ are the income thresholds, ω_y is the income taper rate indicating the amount by which the pension is reduced for each additional unit of income above the low income threshold, $\overline{y}_1$.

Market structure. Markets are incomplete and households cannot insure against the idiosyncratic labor income and mortality risks by trading state contingent assets. They can, however, hold one-period riskless assets to imperfectly self-insure against idiosyncratic risks. We assume that agents are not allowed to borrow against future income, i.e., $a_j \geq 0$ for all j. The economy is assumed to be small in the sense that all agents in the economy take the world prices for traded goods and the world interest rate on bonds, r, as given and independent of the amount of trade in these goods and bonds. The free flow of financial capital ensures that the domestic interest rate is equal to the world interest rate, which is assumed to be constant. An implication is that the rental price of capital is then given by $q = r + \delta$.

Household problem. Households are heterogeneous with respect to their state variables including age, working ability and asset holdings. Let $x_j = (e_j, a_j)$ denote the household's state variables at age j. At the beginning of age j the household realizes its individual state $x_j = (e_j, a_j)$ and chooses its optimal consumption, c_j, leisure time, l_j, or working hours, $(1 - l_j)$, and the end-of-period asset holdings, a_{j+1}, taking the transition law for working ability, $\pi_j \left(e_{j+1|}e_j\right)$, conditional survival probabilities, sp_j, the wage and interest rates, and government tax and pension policies as given.

Households have three sources of incomes: labor earnings, savings and transfers. First, if households decide to work they supply $(1 - l_j)\, e_j$ units of effective labor service to the labor market, attract a wage rate w_t and so earn a gross wage income or labor earnings of $(1 - l_j)\, e_j w_t$. Second, households have the cash balance from savings income available to spend in the amount $(1 + r)a_j$. Third, eligible households may receive age pension transfers from the government in amount P_j. Specifically, agents who are $J_1 = 65$ years of age or older are entitled to receive the age pension. There is a maximum amount of pension income, $P^{\max}$, but the actual amount of pension benefits varies across individuals and depends on the asset and income tests as $P_j = \mathcal{P}(a_j, y_j)$, where assessable income for the pension income test is simply labour and interest earnings, $y_j = e_j\,(1 - l_j)\, w + ra_j$. Finally, households receive accidental bequests, b_j, as a lump-sum transfer from the government.

Formally, the life-cycle expected utility maximization problem of agent i can be expressed recursively as

$$V^j(x_j) = \max_{c_j,\, l_j,\, a_{j+1}} \left\{ u(c_j, l_j) + \beta s p_j E\left[V^{j+1}(x_{j+1}) | e_j \right] \right\} \tag{14}$$

subject to the following constraints for every $j \in J$

$$\begin{aligned} a_{j+1} &= \frac{1}{(1+g)} \left[a_j + e_j (1 - l_j) w + r a_j + b_j + \mathcal{P}(a_j, y_j) - T(y_j) - (1 + \tau^c) c_j \right], \quad (15) \\ a_1 &= 0, \ a_J = 0, \ a_j \geq 0, \\ 0 &< l_j \leq 1, \end{aligned}$$

where $E\left[V^{j+1}(x_{j+1}) | e_j\right]$ is the expected value function, $T(y_j)$ is income tax payment and τ^c is the consumption tax rate. Note that individual quantity variables, except for working hours, are normalized by the steady state per capita growth rate, g.

Fiscal policy. The government levies taxes on consumption and income to finance general government consumption and the age pension program. The consumption tax rate is set at τ_c. Income tax is progressive and compactly written as

$$T(y_j) = T_k + \tau_k (y_j - \overline{y}_k), \quad y_j \in [\overline{y}_k, \overline{y}_{k+1}], \tag{16}$$

where the parameters of this tax function are the marginal tax rates, τ_k, the tax payment thresholds, T_k, and the tax bracket income thresholds, $\overline{y}_k$. It is assumed that $\tau_1 = 0$, $T_1 = T_2 = 0$ and $T_k = T_{k-1} + \tau_k \left(\overline{y}_k - \overline{y}_{k-1}\right)$. This specification corresponds to a standard segmented-linear income tax schedule with an initial tax free threshold and marginal tax rates that rise with taxable incomes. The income tax is set so that the consolidated government budget constraint is satisfied every period, whence

$$\overbrace{\sum_j T(y_j)\,\mu(x_j)}^{\text{income tax revenue}} + \overbrace{\sum_j c_j(x_j)\,\mu(x_j)}^{\text{consumption tax revenue}} = \overbrace{\sum_j \mathcal{P}^y(x_j)\mu(x_j)}^{\text{pension payment}} + \overbrace{G}^{\text{General government expenditures}}, \tag{17}$$

where, $\mu(x_j)$ is the measure of agents in state x_j.

Equilibrium. Given government policy settings for tax rates and the age pension system, the population growth rate, world interest rate, a steady state competitive equilibrium is such that

(a) a collection of individual household decisions $\{c_j(x_j), l_j(x_j), a_{j+1}(x_j)\}_{j=1}^{J}$ solve the household problem (14);[5]

(b) the firm chooses labour and capital inputs to solve the profit maximization problem;

(c) the total lump-sum bequest transfer is equal to the total amount of assets left by all the

[5] In the following, endogenous variables for the household of age j are shown with dependence on the vector of state variables, $x_j = (e_j, a_j)$, at that age.

deceased agents, $B = \sum_{j \in J} \widetilde{\mu}_j \int_\Phi a_j(x_j)\, d\Lambda_j(x_j)$;

(d) the current account is balanced and foreign assets, FA, freely adjust so that $1 + r = R^w$;

(e) the markets for capital and labor clear

$$\begin{aligned} K &= \sum_{j \in J} \mu_j \int_\Phi a_j(x_j)\, d\Lambda_j(x_j) + B + FA, \\ H &= \sum_{j \in J} \mu_j \int_\Phi (1 - l_j) e_j(x_j)\, d\Lambda_j(x_j), \end{aligned}$$

and factor prices are determined competitively, i.e., $w = F_L(K, L)$, $q = F_K(K, L)$ and $r = q - \delta$; and

(f) the government budget constraint defined in Eq. (17) is satisfied.

4 Calibration

This section describes the calibration and parameterization of the model. We calibrate our benchmark model to match the Australian economy and report the values of key parameters of the benchmark model in Table 1.

	Parameters	Model	Observation/Comment/Source
Preferences			
	Annual discount factor	$\beta = .99$	to match I/Y
	Inverse of inter-temporal elasticity of substitution	$\sigma = 4$	
	Share parameter for leisure	$\gamma = 0.18$	to match labor supply profile
Technology			
	Annual growth rate	$g = 0.025$	2.65%
	Total Factor Productivity	$A = 1$	
	Share parameter of capital	$\alpha = 0.4$	
	Annual depreciation rate	$\delta = 0.055$	5.5%
Demography			
	Maximum lifetime	$J = 14$	equivalent to 70 years
	Maximum working periods	$J_w = 9$	equivalent to 45 years
	Annual population growth	$n = 0.012$	1.2%
Government			
	Income taxes	$\tau_j, T_j, \overline{y}_j$	tax schedules in 2007
	Medicare levy	$\tau^{Med} = 0.015$	1.5%
	Consumption tax	τ^c	endogenous
	Pensions	$P^{\max}$, *taper*	pension rules in 2007
	Government consumption	$\Delta_G = 0.14$	to match government size

Table 1: Preference and policy parameters

Demographics. One model period corresponds to 5 years. Households become economically active at age 20 ($j = 1$) and live up to the maximum age of 90 years (equal to the maximum model period $J = 14$). The survival probabilities are calculated from life tables for Australia. The annual growth rate of the new born agents (households) is assumed to be 1.2%, which is the long-run average population growth in Australia.

Working abilities. We use estimates of labor productivities and other key life cycle profiles obtained using data drawn from the Household, Income and Labour Dynamics in Australia (HILDA) longitudinal survey (see Wooden and Watson (2002) for more details) for our model calibration. HILDA is a broad social and economics longitudinal survey, with particular attention paid to family and household formation, income and work. We use data from the first 7 waves of HILDA surveys in this paper.

Working ability corresponds to the hourly average wage rate, defined as gross labor income divided by total hours worked. We estimate age-dependent hourly wage rates from HILDA data. The Markov transition matrix that characterizes the dynamics of working abilities over life cycle is estimated by a counting method. To make the transition matrix more persistent we use the average of these estimates. We also make an assumption that labor productivities from 65 decline at a constant rate, reaching zero at age 80 years.[6]

Preferences. The utility function has the constant relative risk aversion (CRRA) form. We follow previous studies (e.g., Auerbach and Kotlikoff, 1987) and set the relative risk aversion coefficient to $\sigma = 4$, which implies an inter-temporal elasticity of substitution of 0.25. We follow Nishiyama and Smetters (2007) and set $\xi = 0.6$. The number dependent children dp_j is calculated from HILDA data, using the average numbers of children of ages $0 - 19$ in each age group, j. We calibrate γ to match work hours on average. The subjective discount factor β is calibrated to match Australia's net investment to GDP ratio, which has averaged around 0.27 since 1990 according to Australian Bureau of Statistics (ABS) data.

Technology. We set the capital share of output $\alpha = 0.4$. The depreciation rate for capital is determined by the steady state condition and is $\delta = 0.055$. The average annual GDP per capita growth rate in Australian is 3.3 percent so we set $g = 0.033$. The total factor of productivity A is a scaling parameter.

Fiscal policy. We use the tax and pension policy parameter values in 2007 to calibrate fiscal policy in the model. The maximum pension is set at $P^{\max} = \$13,314.60$. The income test threshold income is set at $\overline{y}_1^p = \$3,328$ and the income taper rate is $\omega_y = 0.4$. For the asset test, the design is relatively more complicated. There are separate asset tests for renters and homeowners in Australia. In our model, there is no difference between residential and non-residential assets so we are not able to directly use the statutory asset test thresholds. Instead, we choose $\overline{a}_1$ to match the fraction of pensioners at age 65 years. Assets over this

[6] More details on the data and estimation methods provided in the Technical Appendix available at https://sites.google.com/site/chungqtran.

threshold reduces pension by \$1.50 per fortnight for every \$1000 above the limit, implying a taper rate for asset tests is $\omega_a = 0.0015$.

The government collects tax from consumption and income to cover spending on pension and other government spending programs. The consumption tax rate is set at 10 percent, which is the statutory goods and services (GST) rate in Australia. The details of pension and income tax schedule are reported in the Appendix.

Small open economy. The budget constraint for the small open economy may be expressed in steady state form as $0 = rFA + TB$, where FA and TB are the net holding of foreign assets and trade balance respectively. The right hand side is the current account balance consisting of net interest receipts plus the balance of trade (value of exports minus the value of imports) and the left hand side is net capital flows, which are zero. In a steady state, the stock of foreign asset holding is constant and so $0 = rFA + TB$, meaning that there is a current account balance with interest on foreign assets (if $FA < 0$) matched by a positive trade balance. We normalize the world price to 1 and assume that the world (and domestic) interest rate is $r = 5\%$. The Australian trade balance in the last 15 years is about -1.3 percent of GDP. Using this fact in the context of a steady state, the net foreign asset is calculated as $FA = TB/r = 0.013 \times Y/r > 0$, which implies that Australia is a net investor in the world capital market. However, data on Australia's international position reveals the opposite - Australia is a net borrower from the world capital market. Since our benchmark economy is in steady state, it cannot accommodate both facts. In the model, we assume that Australia is a net borrower with 19% of total national assets being foreign-owned.

5 Policy simulations and analysis

In this section, we first present the calibration result of the benchmark model and discuss how our model solution matches the data describing the Australian economy. Next, we specify, present and discuss various policy experiments constructed to explore the implications of alternative designs of a means tested pension for macroeconomic variables and household welfare.

5.1 Benchmark model

Our benchmark model economy is able to match some key features of the Australian economy. We summarize our calibration results in Figure 1.

Asset profiles. In our life-cycle model with income uncertainty and incomplete markets, individuals accumulate assets in early stages of a life cycle. As seen in panel 1 of Figure 1, our model is able to generate a hump-shaped pattern of asset holdings over the life-cycle that broadly matches in the data drawn from the HILDA panel data set.[7] However, individuals draw

[7] Although HILDA is a longitudinal survey, not all questions are asked in every wave. Since waves 2 and 6

down savings faster in the model than observed in the data because they do not have other motives to save, such as for bequests or to accommodate other life cycle shocks. De Nardi, French and Jones (2010), for example, show that bequest motives and health expenditure shocks are the main determinants of savings behavior of elderly American households. Also, we do not have compulsory retirement savings via superannuation or housing in our model. Incorporating these factors would potentially improve the match between model and data generated asset profiles.

Labor market behavior. Our model can match the observed life cycle pattern of labor market behavior and does a good job of capturing life cycle trends in labor force participate rates. However, it generates more young individuals participating in the labor force in early stages of the life cycle. This is primarily due to the assumption of no bequest motive. Since agents are born with no assets our model, there is very little wealth effect on labor supply decisions at young ages. Consequently, the new born agents optimally choose to work to maintain consumption. However, as agents accumulate more assets in middle and older ages, our model captures the labor force participation rates quite well. Agents between ages 20 and 40 years, on average, supply around 30 hours of work per week. Starting from the late 40s, agents decrease work hours and when they reach 70 years of age there is virtually no labor supplied. The model also captures the observed life cycle pattern of labor earnings.

collect information on household assets, we construct the age profiles of asset holdings based on data from these two waves.

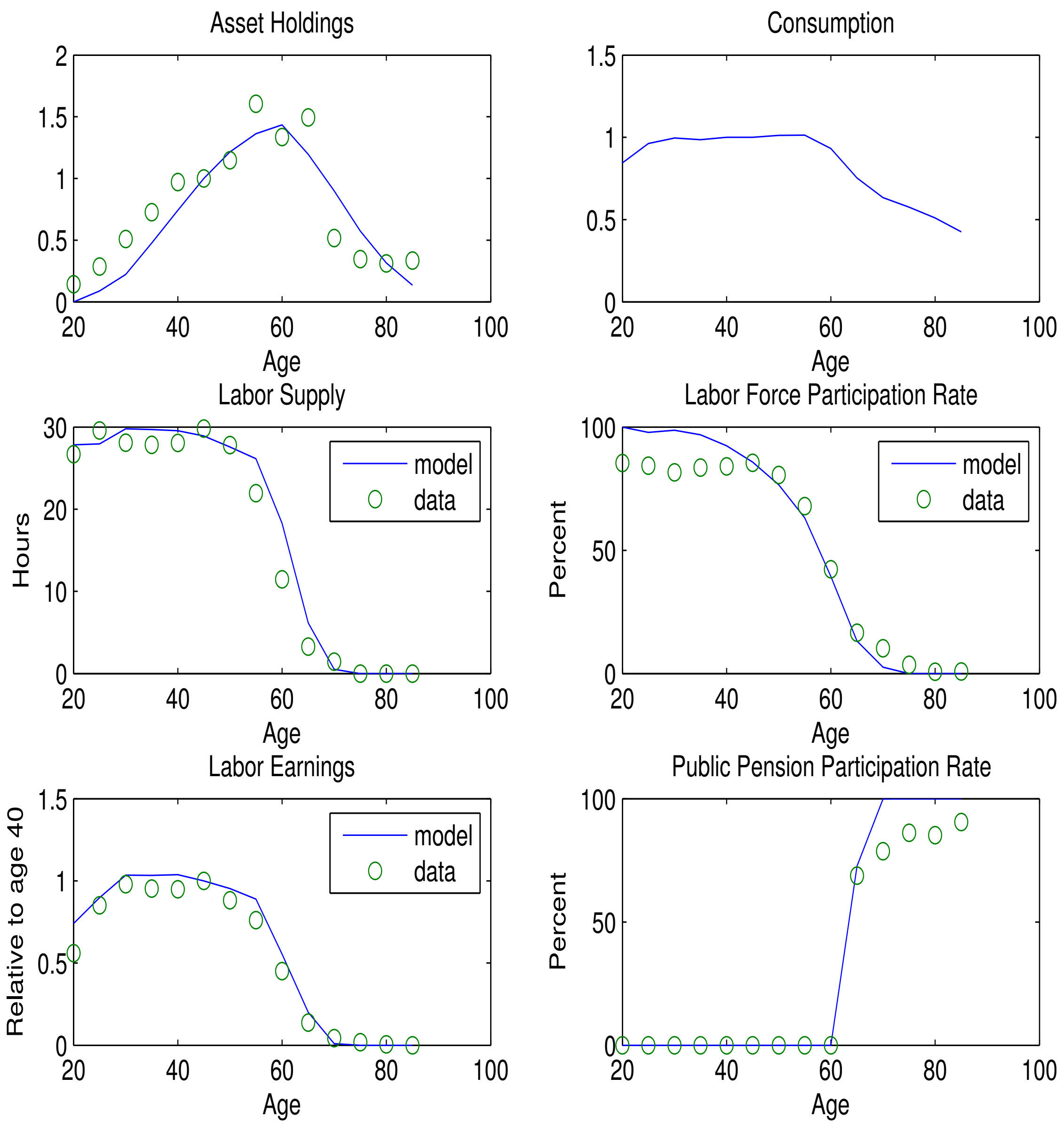

Figure 1: The bechmark model and the data

5.2 Policy experiments

We now examine how the salient features of a means tested pension influence individuals' incentives to work and save, macroeconomic aggregates and welfare. Our primary focus is upon the choice of parameters of the Australian age pension system and, more specifically, upon whether they can be optimally chosen by the government to maximize the steady state expected lifetime utility that accrues to an individual.

The design of a means tested pension program as described above involves the setting of three policy parameters: the maximum pension benefit that an age pensioner may receive, the threshold below which the maximum pension is, in fact, received, and the taper rate that reduces the pension above the threshold level. While the Australian system, as modeled here, has two tests - the income and asset tests - each of which has three such parameters, our policy experiments will simplify the analysis by concentrating on the design of the income test alone, keeping the assets test unchanged. In short, our concern is with the choice of values of the maximum pension, the income test threshold and the income test taper rate.

Thus, our design of a means tested pension program involves setting three policy parameters: the maximum pension benefit, $P^{\max}$, the income threshold, $\overline{y}_1$, and income taper rate, ω_y. To further simplify the analysis, we restrict attention to the study of the effects of social security reforms along just two dimensions: the maximum pension benefit, $P^{\max}$, and the taper rate, ω_y. For convenience, we recall the income test pension payment function

$$\mathcal{P}^y(y) = \begin{cases} P^{\max} & \text{if } y \leq \overline{y}_1, \\ P^{\max} - \omega_y\,(y - \overline{y}_1) & \text{if } \overline{y}_1 < y < \overline{y}_2, \\ 0 & \text{if } y \geq \overline{y}_2, \end{cases} \tag{18}$$

where y is assessable income.

In order to understand how a choice of these two policy instruments influence individuals' inter-temporal allocations of consumption and hours of work, the insurance-incentive trade off and welfare consequences, we implement a number of hypothetical policy reforms. We start from the benchmark economy with the maximum pension benefit $P^{\max,\,\text{benchmark}}$ set equal to 25% of average labor income and the taper rate set at $\omega = 0.4$. We then consider alternative model economies in which we change the values of these two policy parameters.

The effects of maximum pension benefits. In a general equilibrium model, changes in the levels of maximum pension benefits affect not only the generosity of pension benefits (intensive margin) but also the number of pensioners in the economy (extensive margin). However, the effects via the former tend to be strong. To understand the effects of the maximum benefits we simulate a number of alternative model economies in which we vary the levels of the maximum pension benefits, while keeping the taper rate unchanged at its benchmark level.

Technically, we index the maximum pension benefit in an alternative economy to that in the benchmark economy as

$$P^{\max}(\varphi) = \varphi P^{\max,\text{ benchmark}}, \tag{19}$$

where $P^{\max}(\varphi)$ denotes the maximum pension benefits in the economy after the reform and $\varphi \geq 0$ is a parameter. Note that there are several special cases: when $\varphi = 0$ the government closes the pension program, and when $\varphi = 1$ it is the benchmark economy. In our experiments, setting $\varphi < 1$ implies a lower maximum pension benefit than in the benchmark economy, while $\varphi > 1$ implies a higher maximum pension benefit. Any financial discrepancy between the government's consolidated tax revenues and expenditures are financed by a higher or lower income rate

We report the main aggregate and welfare effects of these experiments in Table 2. The first column specifies the maximum pension benefits relative to the maximum pension in the benchmark economy. Note that we normalized capital, labor, output (but not expected utility) in the benchmark model ($\varphi = 1$) to 100 and so the entries in the table show these variables relative to 100 for the benchmark model. We format the benchmark values in italics in Table 2.

$P^{\max}(\varphi) = \varphi P^{\max,\text{ benchmark}}$ φ	Capital	Labor	Output	Expected Utility
0.0	**351.3**	**112.6**	**177.5**	**-0.3666**
0.2	314.1	111.6	168.8	-0.3837
0.4	250.1	109.3	152.2	-0.4086
0.6	187.9	106.2	133.4	-0.4346
0.8	136.2	102.8	115.1	-0.4617
1.0	*100.0*	*100.0*	*100.0*	*-0.4867*
1.25	69.3	96.5	84.5	-0.5193
1.5	47.6	93.4	71.3	-0.5602

Table 2: Aggregate effects when adjusting maximum pension benefits, keeping the taper rate unchanged at the benchmark level (0.4)

In all the experiments reported in Table 2, we consistently find that capital stock, labor supply and output monotonically increase as the government decreases the generosity of pension benefits. This indicates that public pension programs result in adverse effects on individuals' incentives to save and work, thus crowding out savings, labor supply and output. Conversely, cutting the generosity of a public pension program improves efficiency and hence income. We also run the extreme experiment in which the government closes down the public pension program ($\varphi = 0$), shown by the bolded row in Table 2. We find that when the public pension program is completely removed ($\varphi = 0$), efficiency gains from completely removing economic distortions of public pensions on savings and labor supply lead to the highest attainable income. These large crowding out effects on savings found in our experiments are primarily due to our small open economy model assumption. Since the domestic interest rate is equal to the world interest rate, which is assumed constant, general equilibrium interest rate adjustments are removed.

We now turn our attention to the welfare effects. As established in the previous literature, a social security system is often justified as a mechanism for sharing longevity and income risks (social insurance) across households and generations, which potentially improves welfare when markets imperfections are present. On other hand, however, social security systems are often criticized as being detrimental to capital accumulation, labor supply and growth because they distort savings and labor supply decisions (through adverse incentives), resulting in efficiency and welfare losses. The welfare outcomes of a social security system depends how the system trades off the insurance effect against the incentive effect.

In our quantitative experimental results reported in column 5 of Table 2, we find that decreasing the generosity of pension benefits (reducing φ) always leads to increases in the expected utilities of individuals so that expected utility is maximized when the public pension ceases ($\varphi = 0$). This indicates that the adverse effects on incentives always dominate the insurance effect even when means testing is present. It seems that means testing strengthens risk-sharing and incentives via extensive margin effects, but fails to overturn the negative intensive margin effects.

We conclude that a means tested pension is not socially desirable in our dynamic general equilibrium economy as expected utility is highest in an economy with no public pension. This is perhaps not surprising as we learnt from previous studies that general equilibrium adjustments magnify the crowding out effects of social security systems without means testing and that negative welfare outcomes are likely. Indeed, the PAYG social security literature using a dynamic general equilibrium model consistently finds negative welfare effects because the adverse effects on incentives dominate the insurance effect (Auerbach and Kotlikoff (1987) and Imrohoroglu *et al* (1995)), leading to the recommendation that governments privatize their PAYG social security systems. In that sense, our finding for an age pension scheme with means testing is consistent with the previous results in the literature of general equilibrium analysis of social security without means testing.

The effects of taper rates. We now consider the implications of alterations in the taper rate for the income test, keeping the maximum pension level unchanged. We start our analysis with the benchmark economy and vary the taper rate, ω_y, over the interval between 0 and 1. Any financial discrepancy between the government's consolidated tax revenues and expenditures are financed by a higher or lower income tax rate. Specifically, our experiments include two special cases. When the taper rate is nil, $\omega_y = 0$, the government provides a universal pension. On other hand, when the taper rate is unity, $\omega_y = 1$, the government imposes a 100 percent tax rate on pensioners' incomes above the income threshold - any extra income obtained is taxed so there is no incentive to earn extra income from working more or to have extra interest income.

As already argued, the introduction of a taper rate to the pension design results in two opposing effects. First, since the resulting means test targets lower income agents (extensive margin), it mitigates self-insurance disincentives, lowers the deadweight loss of tax financing, and strengthens intra- and inter-generational risk-sharing. Second, it creates economic distortions as it imposes a higher implicit income tax (by the amount of the taper rate) on savings and labor incomes of pensioners. When the former effect is dominant, the welfare effects are positive; otherwise, the welfare effects will be negative. In this experiment, we examine how these two effects interplay. Note that in these experiments we only focus on the effects triggered by taper rates as we keep the maximum pension level unchanged.

We report the results of these experiments in Table 3. Column 1 specifies the various values of the income taper rate. Columns 2 to 5 present the values of aggregate variables including capital, labor, output and expected utility. Again, we normalized the values of aggregate variables in the benchmark economy ($\omega_y = 0.4$) to 100, which are shown in italics in row 5, and report those in alternative economies relative to the benchmark.

Taper rates (ω_y)	Capital	Labor	Output	Expected Utility
0.0	92.3	99.6	96.6	-0.4867
0.1	95.9	99.7	98.1	-0.4826
0.2	98.2	99.7	99.1	-0.4802
0.3	99.9	100.04	99.99	**-0.4785**
0.4	*100.0*	*100.0*	*100.0*	*-0.4787*
0.5	99.3	99.92	99.7	-0.4797
0.6	97.5	99.8	98.9	-0.4820
0.7	96.8	99.8	98.6	-0.4829
0.8	96.1	99.81	98.2	-0.4840
0.9	95.2	9.82	97.9	-0.4851
1.0	94.6	99.8	97.7	-0.4860

Table 3: Aggregate effects when adjusting the taper rate

First, we analyze whether the current means tested pension system in Australia would deliver a more favourable outcome than a universal pension system like the one in the U.S. We compare row 2 ($\omega_y = 0$) and row 6 ($\omega_y = .4$) and find that removing the income test results in a lower capital stock and labor supply, causing output to drop by 3.5 percent. We also find that expected utility is lower in the economy with a universal pension system than in the benchmark economy, meaning that newly born agents would prefer to live in the benchmark economy. We conclude that the means tested pension system in the benchmark economy is socially preferred to the universal pension system.

Second, we consider a wider range of alternative means tested systems and find changing taper rates results in non-linear effects on individuals' behavior and macroeconomic aggregates. When the government raises the taper rate from 0.4 to 1, there is a decrease in the capital stock and labor supply. This suggests that the economy is in a region in which the adverse effects of the taper rate as an implicit tax dominate the effect of the taper rate via the extensive margin.

Raising the taper rate therefore discourages individuals from saving more or working longer, as they face a higher effective marginal income tax rate on earnings in old ages. This result is consistent with previous work on means tested pensions by Selton, van de Ven and Weale (2008), who analyze a calibrated multi-period overlapping generations model of the U.K. and find that the pension reform encourages poorer individuals to save more and to delay retirement, while generating opposite effects on the savings and retirement decisions of richer individuals.

We find that the welfare effects have a hump-shaped pattern. Starting from the benchmark taper rate, $\omega_y = 0.4$, the expected utility for a household decreases as taper rates are increased. This implies that the adverse incentive effects of the more stringent income test dominant the insurance effect. On other hand, however, we find the opposite outcome as the taper rate is reduced from 0.4 to 0. Looking at the whole range for the taper rate, we observe that the introduction of, and increase in, a small taper rate at first improves expected utility for the household, reaches a maximum, and then decreases welfare at higher taper rates.

This non-linear pattern of welfare effects of changes in the income taper rate clearly indicates a trade off between the insurance and incentive aspects of means testing. When the economy is in a region where the insurance effects are dominant, increases in the taper rate induce more self-insurance by working longer hours and increasing saving, which, in turn, lead to efficiency gains and a positive welfare outcome. However, when the taper rate becomes bigger, distortions arising from having higher effective marginal tax rates become more severe, which, in turn, reduce savings and labor supply. Aggregate capital, labor supply and income decrease and welfare subsequently decreases.

The point at which expected utility reaches a maximum is around $\omega_y = 0.3$. This indicates that the introduction of means testing (via a taper rate) is socially desirable in our model, conditioning the pre-existence of a pension system with the benchmark level of the maximum pension benefit. Sefton and Ven (2009) conduct a welfare analysis in a partial equilibrium model of the U.K. pension system and also find that means tested pensions are socially desirable. Our analysis of the Australian pension system in a general equilibrium framework also reaches a similar conclusion. This suggests that the conclusions of Sefton and Ven obtained with a partial equilibrium model might well be confirmed when accounting for dynamic general equilibrium adjustments.

Interactions between maximum pension benefits and taper rate. We now turn our attention to interactions between maximum pension benefits and taper rate, and derive implications for the insurance-incentive trade off and welfare. We numerically characterize two steady states economies with two different levels of the maximum pension benefits: low $\varphi = .5$ and high $\varphi = 1.5$. In each alternative economy, the government keep the maximum pension benefits unchanged and the government varies the taper rate between 0 and 1.

We report the effects of alternative taper rates and maximum pension benefits in the design of means testing of the age pension on the aggregate capital stock and labor supply in Table 4.

	Capital			Labor		
Taper rate	Low	Benchmark	High	Low	Benchmark	High
0.0	1.3490	.7842	.4003	.1729	.1623	.1556
0.1	1.4605	.8148	.4075	.1741	.1630	.1561
0.2	1.5315	.8346	**.4088**	.1748	.1637	.1564
0.3	1.5760	.8491	.4081	.1753	.1641	**.1565**
0.4	1.5974	**.8499**	.4050	.1754	**.1643**	.1565
0.5	1.6125	.8443	.4011	.1756	.16427	.1564
0.6	1.6161	.8284	.3941	.1758	.16424	.1563
0.7	1.6257	.8230	.3829	.1759	.16421	.1563
0.8	1.6309	.8165	.3775	.1760	.1642	.1562
0.9	1.6371	.8093	.3729	.1761	.1641	.1561
1.0	**1.6405**	.8039	.3667	**.1762**	.1640	.1560

Table 4: Aggregate capital stocks and labor when adjusting tapter rates in three different economies: low, benchmark and high maximum pension benefits.

We find that the effects of changes in the taper rates on the aggregate capital stock and labor supply vary significantly across the economies. In the economy where the level of maximum pension benefits is relatively low, the taper rate that maximizes the capital stock and labor supply is 1, which is much higher than in the benchmark economy. On other hand, in the economy where the level of maximum pension benefits is relatively high, the taper rate that maximize the levels of aggregate capital and labor is around 0.3. This indicates that the effects of means testing on incentives to work and to save are dependent of the levels of maximum pension benefits. When the levels of maximum pension benefits are relatively low, tightening the taper rate leads to an increase in the capital stock and labor supply. The intuition for this result can be explained by the prediction in our simple model. That is, when the pension benefits $P^{\max}$ are relatively less generous the positive extensive margin effect is positive and always dominates the negative intensive margin effects. On other hand, in the economy where the levels of maximum pension benefits are relatively generous (benchmark or high) there is a trade off between two opposing forces. The positive extensive margin effect tends to be a dominant force when the rate rates are small, but loses ground to the negative intensive margin effects as the taper rate becomes sufficiently high (0.4 or above in the benchmark economy). This result confirms that the existence of the extensive margin embedded in a means tested pension system potentially mitigates the adverse intensive margin effects on savings.

We now analyze the welfare outcome in which the interactions between the insurance and incentive effects are taken into account. Our results for the effect of these different policy settings upon expected utility are summarized in Table 5.

We find that the welfare effects of varying the taper rate are different across the three economies and, hence, dependent upon the levels of the maximum pension benefit. In the first economy where the maximum pension benefits are relatively less generous (Low), increases in the taper rate lead to monotone increases in capital stock, labor supply, national income and,

	Maximum Pension		
Taper rate	Low (50%)	Benchmark (100%)	High (150%)
0	-.4086	-.4867	-.5602
0.1	-4012	-.4826	-.5580
0.2	-3969	-.4802	**-.5577**
0.3	-.3947	**-.4785**	-.5581
0.4	-.3940	*-.4787*	-.5593
0.5	-.3935	-.4797	-.5607
0.6	-.3927	-.4820	-.5631
0.7	-.3917	-.4829	-.5669
0.8	-.3911	-.4840	-.5688
0.9	-.3907	-.4851	-.5706
1.0	**-.3900**	-.4860	-.5727

Table 5: The effects on expected utilities when adjusting tapter rates in three different economies: low, benchmark and high maximum pension benefits.

therefore, expected utility. This implies that the effects of higher taper rates in mitigating self-insurance disincentives and strengthening risk-sharing are always dominant so that the welfare effects are always positive. The optimal taper rate in this economy is $\omega_y = 1$. There is no clear trade off between insurance and incentive effects as the taper rate increases. However, as pointed in the previous analysis, the positive extensive margin effects tend to be a dominant force.

In the third economy where the maximum pension benefits are assumed to be 150% more generous than in the benchmark economy (High), we again find a hump-shaped pattern of welfare effects. This is indicative of the two opposing effects of means testing at work: mitigating self-insurance disincentives and strengthening risk-sharing versus distortions of higher effective marginal income tax rates of the higher taper rate. When the former is dominant the welfare effects are positive; otherwise, they are negative. The insurance and incentive effects are evenly balanced around $\varphi = 0.2$, which is the optimal taper rate in this economy. Note that the taper rate that delivers the best welfare outcome is not necessarily the one that results in highest levels of capital stock, labor supply and output. The difference is partly due to the fact that means testing strengthens the social insurance role of the pension system.

To enable a more detailed examination of the welfare and macroeconomic implications of alternative pension design parameters, we simulate a number of alternative economies for a wider range of maximum pension benefits. We summarize the results of these policy experiments on aggregate variables in Tables 8, 9, 10 and 11. Table 11 shows that the level of expected utility is greatest when the taper rate is unity for age pension replacement rates up to 0.6, indicating that it is optimal for pensioners to only receive the pension for incomes less than the income threshold. The optimal taper rate is 0.5 when the replacement rate is 0.8, drops to 0.3 for replacement rates of unity and 1.25, and further to 0.2 when the replacement rate is 1.5. Thus, the optimal taper rate falls as the pension becomes more generous. Overall, we find from these tables that the interaction between the maximum pension benefit and the taper rate magnifies the disincentive effects of the taper rate as an implicit tax on life-cycle savings and labor supply.

In summary, our results point out the importance of accounting for the interaction between these two pension policy instruments and of analyzing the economic mechanisms that explain these nonlinear effects. Our results point to a conclusion that the welfare effect of introducing and increasing an income test taper rate is nonlinear and dependent of the level of the maximum pension benefit. The interaction between these two policy variables is important as it has different implications for individuals' inter-temporal allocation of resources, macroeconomic aggregates and welfare.

6 Conclusion

Inclusion of means testing into the pension benefit formula allows governments to have additional policy instruments to affect the number of public pensioners (extensive margin) and the benefit level (intensive margin). The former is aimed at strengthening risk sharing across individuals and generations and to mitigate the adverse effects of self-insurance incentives. In this paper, we analyzed the welfare implications of these salient features of age pension design for the trade off between insurance and incentive effects. We find that the extensive margin strengthens the insurance effect but introduces two opposing effects on incentives, and that the magnitude of the positive extensive margin effect depends on relative strength of the intensive margin. The final welfare outcome depends how two opposing effects on incentives play out in the economy.

We investigate these trade-offs in a dynamic general equilibrium model with heterogeneous agents that is calibrated to the Australian economy. We find that the introduction of a taper rate leads to positive welfare outcomes and that the pattern of welfare effects varies, depending on the level of maximum pension benefits. More specifically, when the maximum pension benefit is relatively less generous, increases in taper rates always leads to a welfare gain as the insurance effect together with the positive incentive effect are always dominant. However, when the maximum pension benefits are relatively more generous, there is an optimal taper rate at which the insurance and positive incentive effects efficiently trade off with the negative incentive effects and at which expected utility is maximized. Importantly, our results reveal that the interactions between the levels of maximum pension benefits and taper rates are critical in forming the direction of the welfare effects.

Our results carry important policy implications. Countries that are interested in introducing means testing to their currently universal pension systems should take into account the potential interactions between the choice of taper rates and the choice of the levels of maximum pension benefit. Our results highlight the point that the effects of a higher taper rate on savings, labor supply and household welfare are nonlinearly dependent on the level of the maximum pension benefit.

References

Auerbach, J. Alan and Laurence J. Kotlikoff. 1987. *Dynamic Fiscal Policy.* Cambridge University Press.

Conesa, Juan and Dirk Krueger. 1999. "Social Security Reform with Heterogeneous Agents." *Review of Economics Dynamics* 2:757–795.

De Nardi, Mariacristina, Eric French and B. John Jones. 2010. "Why Do the Elderly Save? The Role of Medical Expenses." *Journal of Political Economy* 118(1):39–75.

Diamond, Peter and James Mirrlees. 1978. "A model of social insurance with variable retirement." *Journal of Public Economics* 10(3):295–336.

Diamond, Peter and James Mirrlees. 1986. "Payroll-Tax Financed Social Insurance with Variable Retirement." *Scandinavian Journal of Economics* 88(1):25–50.

Fuster, Luisa, Ayse Imrohoroglu and Selahattin Imrohoroglu. 2007. "Elimination of Social Security in a Dynastic Framework." *Review of Economic Studies* 74 (1):113–145.

Golosov, Mikhail and Aleh Tsyvinski. 2006. "Designing Optimal Disability Insurance: A Case for Asset Testing." *Journal of Political Economy* 114 (No 2):257–279.

Hubbard, R. Glenn and Kenneth Judd. 1987. "Social Security and Individual Welfare." *American Economic Review* 77(4):630–646.

Imrohoroglu, Ayse, Selahattin Imrohoroglu and Douglas H. Jones. 1995. "A Life Cycle Analysis of Social Security." *Economic Theory* 6(1):83–114.

Krueger, Dirk. 2006. "Public Insurance against Idiosyncratic and Aggregate Risk: The Case of Social Security and Progressive Taxation." *CESifo Economic Studies* 52:587–620.

Kudrna, George and Alan Woodland. 2011. "An Intertemporal General Equilibirum Analysis of the Australian Age Pension Means Test." *Journal of Macroeconomics* 33:61–79.

Kumru, Cagri and John Piggott. 2009. "Should Public Retirement Provision Be Means-tested?" ASB Research Paper No. 2009 AIPAR 01.

Maattanen, Niku and Panu Poutvaara. 2007. "Should Old-age Benefits Be Earnings-tested?" IZA Discussion Papers 2616.

Nishiyama, Shinichi and Kent Smetters. 2007. "Does Social Security Privatization Produce Efficiency Gain?" *Quarterly Journal of Economics* 122:1677–1719.

Sefton, James and Justin van de Ven. 2009. "Optimal Design of Means-Tested Retirement Benefits." *The Economic Journal* 119(541):461–481.

Selton, Jam, Justin van de Ven and Martin Weale. 2008. "Means Testing Retirement Benefits: Fostering Equity or Discouraging Savings?" *Economic Journal* 118(528):556–590.

Wooden, M., S. Freidin and N. Watson. 2002. "The Household, Income and Labour Dynamics in Australia (HILDA) Survey: Wave 1." *Australian Economic Review* 35:339–348.

7 Appendix

7.1 Solving the simple model

We provide a solution for a model in which savings is incorporated in the income test formula and the government finances its pension program via a tax on the labor income of the young.

Household. The individual agent's optimization problem is

$$\max_{c_1,\ c_2,\ s} \left\{ u(c_1) + pEu(c_2) \text{ st. } c_1 + s = (1-\tau) w_1 \text{ and } c_2 = w_2 + (1+r) s + P \right\},$$

where P is the pension benefit defined as

$$P = \begin{cases} P^{\max} - \omega [w_2 + rs] & \text{if } w_2 + rs < \overline{y}_2 \\ 0 & \text{if } w_2 + rs \geqq \overline{y}_2. \end{cases}$$

Let $y_2 = w_2 + rs$ be testable income and follows an uniform distribution. Assuming that $u(c) = -\frac{c^2}{2} + \chi c$ is the functional form for individual preferences, the individual's first order necessary condition for optimality is

$$-c_1 + \chi = pE\left[u'(c_2) \frac{\partial c_2}{\partial s} \right],$$

where

$$\begin{aligned} E\left[(c_2)^{-\sigma} \frac{\partial c_2}{\partial s} \right] &= \int_{y_2^{\min}}^{y_2^{\max}} (-c_2 + \chi) \left(\frac{\partial c_2}{\partial s} \right) f(y_2)\, dy_2, \\ &= \chi - \int_{y_2^{\min}}^{y_2^{\max}} c_2 \left(\frac{\partial c_2}{\partial s} \right) f(y_2)\, dy_2 \\ f(y_2) &= \frac{1}{w_2^{\max}} : \text{uniform} \sim [y_2^{\min} = rs, y_2^{\max} = rs + w_2^{\max}]. \end{aligned}$$

The individual's consumption in period 2 is

$$c_2 = \begin{cases} (1-\omega) w_2 + [1 + (1-\omega) r] s + P^{\max} & \text{if } P > 0 \\ w_2 + (1+r) s & \text{if } P = 0, \end{cases}$$

and the first derivative with respect to saving is

$$\frac{\partial c_2}{\partial s} = \begin{cases} 1 + (1-\omega) r & \text{if } P > 0 \\ (1+r) & \text{if } P = 0. \end{cases}$$

Using this expression for consumption when old, expected marginal utility may be expressed as

$$E\left[(c_2)^{-\sigma}\frac{\partial c_2}{\partial s}\right] = \chi - [1+(1-\omega)r]\,p\int_{y_2^{\min}}^{\overline{y}_2}[(1-\omega)y_2+s+P^{\max}]\,f(y_2)\,dy_2 - (1+r)\,p\int_{\overline{y}_2}^{y_2^{\max}}[y_2+s]\,f(y_2)\,dy_2.$$

The individual's first order necessary condition becomes

$$(1-\tau)\,w_1 - s = \begin{cases} [1+(1-\omega)r]\,p\left\{\int_{y_2^{\min}}^{\overline{y}_2}[(1-\omega)y_2+s+P^{\max}]\frac{1}{w_2^{\max}}dy_2\right\} \\ \quad + (1+r)\,p\left\{\int_{\overline{y}_2}^{y_2^{\max}}[y_2+s]\frac{1}{w_2^{\max}}dy_2\right\}. \end{cases}$$

Let $\widehat{w}_2 = \overline{y}_2 - rs$ denote the level of income endowment in period 2 that separates pensioners from non-pensioners, taking saving, s, as given. Noting that $dy_2 = dw_2$, we obtain the expression

$$\begin{aligned}
(1-\tau)\,w_1 - s &= \begin{cases} \frac{[1+(1-\omega)r]p}{w_2^{\max}}\left\{\int_0^{\widehat{w}_2}[(1-\omega)w_2+[1+(1-\omega)r]s+P^{\max}]\,dw_2\right\} \\ \quad + \frac{(1+r)p}{w_2^{\max}}\left\{\int_{\widehat{w}_2}^{\overline{w}_2^{\max}}[w_2+(1+r)s]\,dw_2\right\} \end{cases} \\
&= \begin{cases} \frac{[1+(1-\omega)r]p}{w_2^{\max}}\left[(1-\omega)\frac{(w_2)^2}{2}+([1+(1-\omega)r]s+P^{\max})\,w_2\right]\Big|_0^{\widehat{w}_2} \\ \quad + \frac{(1+r)p}{w_2^{\max}}\left[\frac{(w_2)^2}{2}+(1+r)sw_2\right]\Big|_{\widehat{w}_2}^{w_2^{\max}}, \end{cases} \\
&= \begin{cases} \frac{[1+(1-\omega)r]p}{w_2^{\max}}\left\{\left[(1-\omega)\frac{(\widehat{w}_2)^2}{2}\right]+[[1+(1-\omega)r]s+P^{\max}]\,\widehat{w}_2\right\} \\ \quad + \frac{(1+r)p}{w_2^{\max}}\left[\frac{\left(w_2^{\max}\right)^2-(\widehat{w}_2)^2}{2}+(1+r)s\,(w_2^{\max}-\widehat{w}_2)\right], \end{cases} \\
&= \begin{cases} [1+(1-\omega)r]^2\,p\frac{\widehat{w}_2}{w_2^{\max}}s + \frac{[1+(1-\omega)r]p}{w_2^{\max}}\left[(1-\omega)\frac{(\widehat{w}_2)^2}{2}+P^{\max}\widehat{w}_2\right] \\ \quad + (1+r)^2\,p\frac{w_2^{\max}-\widehat{w}_2}{w_2^{\max}}s + \frac{(1+r)p}{w_2^{\max}}\frac{\left(w_2^{\max}\right)^2-(\widehat{w}_2)^2}{2}, \end{cases} \\
&= \begin{cases} \left[[1+(1-\omega)r]^2\,p\frac{\widehat{w}_2}{w_2^{\max}}+(1+r)^2\,p\frac{w_2^{\max}-\widehat{w}_2}{w_2^{\max}}\right]s \\ + \frac{[1+(1-\omega)r]p}{w_2^{\max}}\left[(1-\omega)\frac{(\widehat{w}_2)^2}{2}+P^{\max}\widehat{w}_2\right] + \frac{(1+r)p}{w_2^{\max}}\frac{\left(w_2^{\max}\right)^2-(\widehat{w}_2)^2}{2}. \end{cases}
\end{aligned}$$

This equation may be solved for the optimal level of saving function, yielding the implicit expression

$$
\begin{aligned}
s &= \frac{(1-\tau)\, w_1 - \frac{[1+(1-\omega)r]p}{w_2^{\max}}\left[(1-\omega)\,\frac{(\widehat{w}_2)^2}{2} + P^{\max}\widehat{w}_2\right] - \frac{(1+r)p}{w_2^{\max}}\frac{\left(w_2^{\max}\right)^2-(\widehat{w}_2)^2}{2}}{1+[1+(1-\omega)\,r]^2\, p\frac{\widehat{w}_2}{w_2^{\max}} + (1+r)^2\, p\frac{w_2^{\max}-\widehat{w}_2}{w_2^{\max}}}, \\
&= \frac{(1-\tau)\, w_1 - [1+(1-\omega)\,r]\, p\left[\frac{\widehat{w}_2}{2} + P^{\max} - \omega\frac{\widehat{w}_2}{2}\right]\left(\frac{\widehat{w}_2}{w_2^{\max}}\right) - (1+r)\, p\frac{\left(w_2^{\max}+\widehat{w}_2\right)}{2}\left(1-\frac{\widehat{w}_2}{w_2^{\max}}\right)}{1+[1+(1-\omega)\,r]^2\, p\frac{\widehat{w}_2}{w_2^{\max}} + (1+r)^2\, p\left(1-\frac{\widehat{w}_2}{w_2^{\max}}\right)},
\end{aligned}
$$

where $\widehat{w}_2 = \overline{y}_2 - rs$.

Government. The government budget clearing condition is

$$
\begin{aligned}
\frac{w_1\tau}{p} &= \int_{y_2^{\min}}^{\overline{y}_2} Pf\,(y_2)\, dy_2, \\
&= \int_{y_2^{\min}}^{\overline{y}_2} (P^{\max} - \omega y_2)\, f\,(y_2)\, dy_2, \\
&= \int_0^{\widehat{w}_2} (P^{\max} - \omega\,(w_2 + rs))\, f\,(w_2)\, dw_2, \\
&= \frac{1}{w_2^{\max}}\left(P^{\max}w_2 - \omega\left(\frac{w_2}{2} + rs\right) w_2\right)|_0^{\widehat{w}_2}, \\
\frac{w_1\tau}{p} &= \overbrace{\frac{\widehat{w}_2}{w_2^{\max}}}^{\#\text{ of pensioners}}\ \overbrace{\left(P^{\max} - \omega\left(\frac{\widehat{w}_2}{2} + rs\right)\right)}^{\text{Average pension benefit}},
\end{aligned}
$$

where s is optimal saving and $\widehat{w}_2 = \overline{y}_2 - rs$.

Equilibrium. The equilibrium conditions for this simple economy reduce to

$$
s^* = \frac{(1-\tau^*)\, w_1 - [1+(1-\omega)\,r]\, p\left[\frac{\widehat{w}_2^*}{2} + P^{\max} - \omega\frac{\widehat{w}_2^*}{2}\right]\left(\frac{\widehat{w}_2^*}{w_2^{\max}}\right) - (1+r)\, p\frac{\left(w_2^{\max}+\widehat{w}_2^*\right)}{2}\left(1-\frac{\widehat{w}_2^*}{w_2^{\max}}\right)}{1+[1+(1-\omega)\,r]^2\, p\frac{\widehat{w}_2^*}{w_2^{\max}} + (1+r)^2\, p\left(1-\frac{\widehat{w}_2^*}{w_2^{\max}}\right)} \tag{20}
$$

$$
\tau^* = \frac{p}{w_1}\left[\frac{\widehat{w}_2^*}{w_2^{\max}}\left(P^{\max} - \omega\left(\frac{\widehat{w}_2^*}{2} + rs^*\right)\right)\right], \tag{21}
$$

$$
\widehat{w}_2^* = \overline{y}_2 - rs^*. \tag{22}
$$

These equilibrium conditions simultaneously determine the solutions for $(s^*, \tau^*, \widehat{w}_2^*)$. The first is the optimal saving function. The second equation determines the tax rate, τ^*, that ensures a government budget balance. The final equation determines the period 2 (extensive margin) wage rate, $\widehat{w}_2^*$, that separates pensioners from non-pensioners. Note that $P^{\max}$, ω and $\overline{y}_2$ are exogenously set by the government.

7.2 Fiscal policy in the dynamic general equilibrium model

Means tested pension. The Australian government runs a means tested age pension program. The maximum pension is set at $P^{\max} = \$13,314.60$ in 2007, which is technically is calculated by the formula $P^{\max} = 0.25 \times MTAWE$, where $MTAWE$ is the Male Total Average Weekly Earnings. We assume that is $MTAWE$ is the average labor income $\overline{y}$ and the replacement rate $\Psi = 0.25$. In our benchmark model, the maximum pension is defined by $P^{\max} = 0.25\overline{y}$. In 2007-8 the income test threshold is set at \$3328 and incomes over these amounts reduce pension by \$0.4 for every \$1. We therefore choose $\overline{y}_1 = \$3328$ and $\omega_y = 0.4$. The pension benefit using the income test is given by

$$\mathcal{P}^y(y_j) = \begin{cases} 13,314.6 & \text{if } j \geq 60 \text{ and } y_j \leq \overline{y}_1 = 3328, \\ \max\left[0, \ (13314.6 - 0.4\,(y_j - \overline{y}_1))\right] & \text{if } j \geq 60 \text{ and } y_j > \overline{y}_1 = 3328. \end{cases}$$

There are two separate asset tests for renters and homeowners in Australia. For renters, the asset test threshold was \$171,750 in 2007. For homeowners, residential assets are excluded from the assets test and so the lower bound threshold for the asset test for homeowners is set higher at \$296,250. Assets above the asset test threshold reduce the age pension by \$1.5 per fortnight for every \$1000 above the limit, which implies a taper rate for the asset test of $\omega_a = 1.5/1000 = 0.0015$. In our model, there is no difference between residential and non-residential assets, so we are not able to use the statutory asset test threshold directly. Instead, we choose $\overline{a}_1$ to match the observed fraction of pensioners at age 65 years. The pension benefit using the asset test is given by

$$\mathcal{P}^a(a_j) = \begin{cases} 13314.6 & \text{if } j \geq 65 \text{ and } a_j \leq \overline{a_1}, \\ \max\left[0, (13314.6 - 0.0015\,(a_j - \overline{a}_1))\right] & \text{if } j \geq 65 \text{ and } a_j > \overline{a}_1. \end{cases}$$

The government collects tax from consumption and income to cover spending on pensions and other government spending programs. The consumption tax rate is set at the statutory 10 percent.

Income tax function. The Australian income tax schedule is progressive. We use the tax schedules for 2007-8 in the benchmark model so that the tax function is given by

$$T(y) = \begin{cases} 0 & \text{if } y < 6,000 \\ 0.15\,(y - 6,000) & \text{if } 6,000 < y \leq 25,000 \\ 3,600 + 0.3\,(y - 30,000) & \text{if } 25,000 < y \leq 75,000 \\ 17,100 + 0.4\,(y - 75,000) & \text{if } 75,000 < y \leq 150,000 \\ 47,100 + 0.45\,(y - 150,000) & \text{if } y > 150,000, \end{cases}$$

where y is taxable income.

Senior Tax offset. The maximum amount of senior tax offset is \$2230 and for every income dollar above the income limit of $\$24,876$ the tax offset reduces by 12.5 cents so we set $SATO^{\max} = \$2,230$, $\overline{y}_1^{sato} = \$24,876$ and $\omega_{sato} = 0.125$. The senior tax offset function is given by

$$SATO(y) = \begin{cases} 2230 & \text{if } j \geq J_1 \text{ and } y \leq \overline{y}_1^{sato} = 24,876 \\ \max\left[0, 2230 - .125\left(y - \overline{y}_1^{sato}\right)\right] & \text{if } j \geq J_1 \text{ and } y > \overline{y}_1^{sato} = 24,876 \end{cases}$$

for households of pensionable age $j \geq J_1$ and zero otherwise.

7.3 Algorithm to solve the dynamic general equilibrium model

We follow the algorithm in Auerbach and Kotlikoff (1987) to solve the model. The general procedure to solve for general equilibrium is summarized as follows:

1. Discretize the state space of assets as $[a_0, ..., a_{\max}]$.
2. Guess an initial wage rate, w, and endogenous government policy variables while taking the world interest rate as given.
3. Work backwards from period J to period 1 to obtain decision rules for consumption, savings, labor supply, and the value and marginal value functions of the household.
4. Iterate forwards to obtain the measure of households across states, using the household decision rules and the laws of motion for working ability shocks and mortality shocks and taking the distribution of agents of age 1 as given.
5. Aggregate labor supply and clear the labor market to get a new wage rate; balance the government budget to determine endogenous government variables.
6. Check the relative change in aggregate variables after each iteration and stop the algorithm when the change is sufficiently small (10^{-4} percent). Otherwise, repeat steps from 3 to 6.

7.4 Additional Tables and Graphs: Policy Simulations

Maximum Pension - φ=	Y	K	H	W	τ_C%	Gini	Welfare
0.0	3.859	2.012	0.955	2.389	0.017	0.548	-0.396
0.2	3.653	1.777	0.946	2.281	0.025	0.560	-0.406
0.4	3.336	1.450	0.932	2.116	0.040	0.578	-0.425
0.6	3.046	1.188	0.914	1.969	0.059	0.589	-0.443
0.8	2.774	0.973	0.894	1.835	0.081	0.599	-0.462
1.0	2.526	0.795	0.875	1.706	0.104	0.605	-0.482
1.2	2.341	0.669	0.864	1.601	0.125	0.614	-0.499
1.4	2.178	0.574	0.849	1.516	0.147	0.623	-0.515

Table 6: Aggregate variables: varying maximum pension benefits while keeping taper rate unchanged

Taper Rates	Y	K	H	W	τ_C%	Gini	Welfare
0.000	2.554	0.811	0.879	1.717	0.118	0.593	-0.479
0.100	2.558	0.816	0.878	1.722	0.113	0.595	-0.478
0.200	2.560	0.820	0.876	1.727	0.110	0.597	-0.478
0.300	2.577	0.831	0.878	1.734	0.107	0.599	-0.476
0.400	2.564	0.823	0.876	1.729	0.106	0.600	-0.478
0.500	2.547	0.811	0.875	1.719	0.106	0.602	-0.480
0.600	2.537	0.804	0.875	1.714	0.105	0.603	-0.481
0.700	2.526	0.795	0.875	1.706	0.104	0.605	-0.482
0.800	2.519	0.790	0.875	1.702	0.104	0.607	-0.483
0.900	2.512	0.784	0.874	1.697	0.103	0.609	-0.484
1.000	2.494	0.771	0.874	1.686	0.103	0.611	-0.486

Table 7: Aggregate variables: varying taper rates while keeping maximum pension benefits unchanged

	0	.2	.4	.6	.8	ψ=1	1.2	1.4
0	240.677	196.981	161.570	135.668	114.664	98.628	86.369	75.173
.1	240.677	205.170	169.826	141.171	116.291	99.193	86.338	74.732
.2	240.677	208.212	173.434	144.793	119.852	99.706	86.428	74.361
.3	240.677	210.910	174.579	145.780	120.157	101.011	86.268	74.079
$\omega = .4$	240.677	212.001	174.378	146.243	120.380	100.000	85.399	73.265
.5	240.677	213.400	174.856	145.746	119.880	98.541	84.631	72.543
.6	240.677	214.867	175.039	145.471	118.880	97.676	82.447	71.014
.7	240.677	215.989	176.292	144.442	118.280	96.620	81.364	69.803
.8	240.677	216.433	177.005	144.313	117.451	96.019	80.199	69.126
.9	240.677	217.241	177.190	143.949	116.382	95.337	79.101	67.602
1	240.677	217.560	177.376	144.289	115.246	93.726	78.148	66.481

Table 8: Aggregate capital : all experiments

	0	.2	.4	.6	.8	ψ=1	1.2	1.4
0	106.495	104.704	103.026	101.592	100.299	99.353	98.608	98.052
.1	106.495	105.291	103.562	102.007	100.672	99.621	98.795	98.168
.2	106.495	105.294	103.875	102.348	100.985	99.861	98.956	98.256
.3	106.495	105.328	103.961	102.524	101.131	100.008	99.070	98.326
$\omega = .4$	106.495	105.401	103.992	102.583	101.189	100.000	99.081	98.340
.5	106.495	105.490	104.038	102.604	101.192	99.993	99.048	98.298
.6	106.495	105.552	104.063	102.601	101.151	99.959	99.025	98.264
.7	106.495	105.603	104.105	102.555	101.092	99.923	98.970	98.173
.8	106.495	105.644	104.146	102.550	101.057	99.876	98.904	98.116
.9	106.495	105.645	104.168	102.527	101.028	99.815	98.835	98.023
1	106.495	105.633	104.186	102.534	100.958	99.743	98.760	97.958

Table 9: Aggregate labor: all experiments

	0	.2	.4	.6	.8	ψ=1	1.2	1.4
0	149.490	136.560	124.815	115.259	106.766	99.628	93.915	88.376
.1	149.490	139.083	127.426	117.161	107.245	99.773	93.728	87.986
.2	149.490	140.043	128.726	118.401	108.626	99.851	93.650	87.558
.3	149.490	140.860	129.270	118.804	108.655	100.522	93.442	87.258
$\omega = .4$	149.490	141.255	129.318	119.060	108.724	100.000	93.018	86.733
.5	149.490	141.704	129.545	118.981	108.582	99.340	92.680	86.363
.6	149.490	142.127	129.647	119.031	108.267	98.964	91.850	85.603
.7	149.490	142.493	130.132	118.823	108.187	98.515	91.323	84.969
.8	149.490	142.627	130.439	118.802	107.879	98.263	90.750	84.634
.9	149.490	142.887	130.501	118.721	107.501	97.973	90.196	83.860
1	149.490	142.974	130.585	118.864	107.051	97.264	89.716	83.294

Table 10: Aggregate income: all experiments

	0	.2	.4	.6	.8	ψ=1	1.2	1.4
0	83.088	86.759	90.516	93.866	97.203	100.193	102.888	105.849
.1	-0.000	85.986	89.466	93.001	96.882	100.074	102.900	105.996
.2	-0.000	85.720	89.094	92.484	96.218	99.983	102.896	106.144
.3	-0.000	85.495	89.014	92.395	96.216	99.712	103.010	106.268
$\omega = .4$	-0.000	85.430	89.084	92.377	96.217	100.000	103.268	106.598
.5	-0.000	85.316	89.070	92.506	96.354	100.392	103.509	106.858
.6	-0.000	85.188	89.089	92.589	96.586	100.635	104.151	107.403
.7	-0.000	85.076	88.956	92.787	96.730	100.926	104.524	107.854
.8	-0.000	85.110	88.918	92.836	96.916	101.097	104.907	108.111
.9	-0.000	84.998	88.913	92.924	97.159	101.287	105.260	108.693
1	-0.000	84.976	88.909	92.894	97.423	101.724	105.584	109.120

Table 11: Aggregate welfare: all experiments